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Design and Simulation of Fractional Order Control Systems Based on Bode's Ideal Transfer Function

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Abstract

The fractional order calculus theory and its modeling methods have been applied widely in control field. And design of fractional order control systems become hot point of recent years. This paper establishes fractional order systems which parameters are obtained by Bode's ideal transfer function method to get the desired frequency response. Apply this method a new control structure of fractional order pseudo-derivative feedback (FOPDF) is suggested. Control methods are designed for integral first order system and for fractional first order system, and apply to a real high power semiconductor laser diodes constant temperature controlling system and the hydraulic servo systems and the electric drive system, the simulation results indicate that the effectiveness and validity of this method.

Keywords: fractional *order control system; Bode's ideal transfer function, high power Semiconductor Laser Diode, robustness, Al-Alaoui+CFE*

1. Introduction

Fractional calculus deals with derivatives and integrals to an arbitrary order. There are several definitions of fractional derivatives and integrals, the most fundamental definition of a fractional derivative and integral of order α is given by Grünwald-Letnikov definition [1, 3, 5], Grünwald-Letnikov (GL) definition is given as:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t-jh) \qquad (1)$$

Where, $\binom{\alpha}{j} = \frac{\alpha(\alpha+1)...(\alpha+j-1)}{j!} = \frac{\alpha!}{j!(\alpha-j)!}$.

When $\alpha > 0$, means α is derivative order of $f(t)$; when $\alpha < 0$, mean α is integral order of $f(t)$. This definition is wildly used in control field [5].

Design the fractional controller is a main researching area of fractional calculus applying in the control field. And there are many kinds of methods to design a fractional order PID (FOPID) controller, including the phase margin, the amplitude margin, dominant pole method, optimization method, and the Bode's ideal transfer function method. The Bode's ideal transfer function method is easy and effective. It suggested an ideal shape of the open-loop transfer function of the form [5]:

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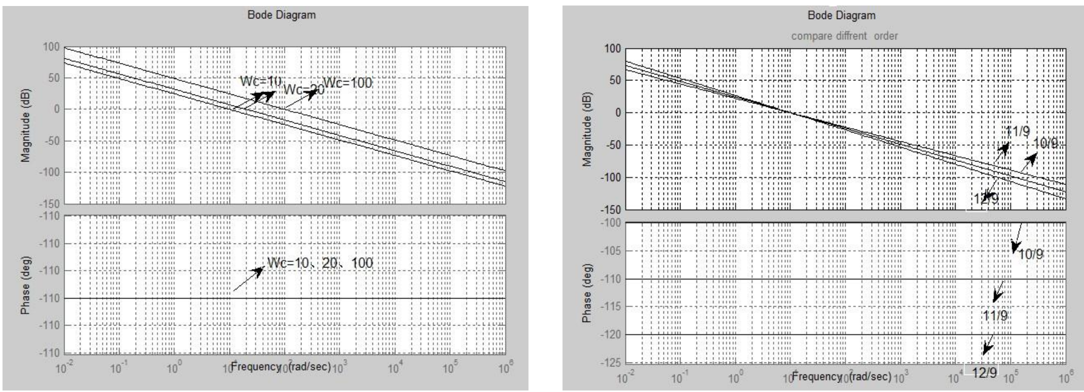
$$G_{opi}(s)=\left(\frac{\omega_c}{s}\right)^{\alpha},\alpha\in R \tag{2}$$

where ω_c is the gain crossover frequency, In fact, the transfer function $G_{opi}(s)$ is a fractional-order differentiator for $\alpha>0$ and a fractional-order integrator for $\alpha<0$.

Its Open-loop characteristics are as follows:

(1) Amplitude-frequency curve is is a straight line of constant slope -20α dB/dec;
(2) Phase curve is a horizontal line at $-\alpha\pi/2$ rad;
(3) The Nyquist curve consists, simply, on a straight line through the origin with $\arg G_{opi}(j\omega)=-\alpha\pi/2$ rad.



a) Frequency and Phase Curves When Fix α =11/9 and Changing ω_c

b) Frequency and Phase Curves when Fix ω_c =10rad/s, and Changing α

Figure 1. Frequency and Phase Curves When Parameter Changing

Figure 1 indicate that if the gain changes ω_c, will changes together but the phase margin of the system remains as a independent value as $\phi_m=(\pi - 2\alpha/\pi)$ rad[5].In this paper we applied the Bode's ideal transfer function method to the determine the parameters of the FOPID system for integral first order system and fractitional first order system. The procedures of design FOPID system

(1) Choosen the controlled plant and its transfer function $G_P(S)$;
(2) Assume gain crossover frequency ω_c, the phase margin Φ_m; Work out the transfer function of FOPID controller as (3):

$$G_c(s)=\frac{G_{opi}(s)}{G_P(S)} \tag{3}$$

(3) Using impulse invariance method get the discretization model of $G_c(s)$ and the simulation block of FOPID controller, Schematic Diagram is as Figure 2 [13].

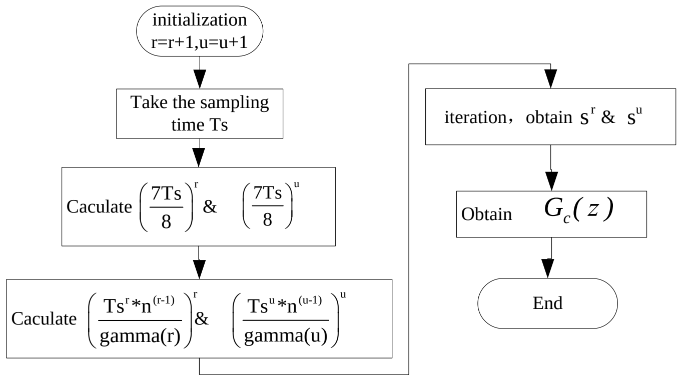


Figure 2. Schematic Diagram of the Controller Discretization Process

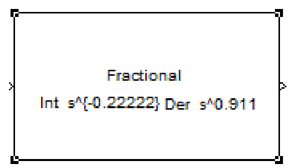


Figure 3. The Simulation Block of Fractional $PI^{\lambda}D^{\mu}$ Controller

(4) Simulate base on Matlab software, and get the unit step responses of the system, analysis dynamic performances.

Pseudo-derivative feedback (short for PDF) control system introduced by Phelan in 1977 [15], this system is simple yet effective. PDF structure provides all the control aspects of PID control, but without system zeros that are normally introduced by a PID compensator. Phelan named this structure "Pseudo-derivative feedback (PDF) control from the fact that the rate of the measured parameter is fed back without having to calculate a derivative. The PDF structure internalizes a pre-filter, one would apply to cancel the zeros introduced in the PI (or PID) equivalent system [17]. The PDF structure is usually introduced into the design of electro-hydraulic servos [16], automatic control systems of electric traction [16], it offers a good disturbance rejection performance and promotes the response speed. In this paper the fractional PID algorithm is introduced into FOPDF systems to promote the property of fractional PID control system，the simulation results to a first-order controlled plant illustrate that the FOPDF control method has superior performances to IOPDF structure. The basic structure of IOPDF is as Figure 4.

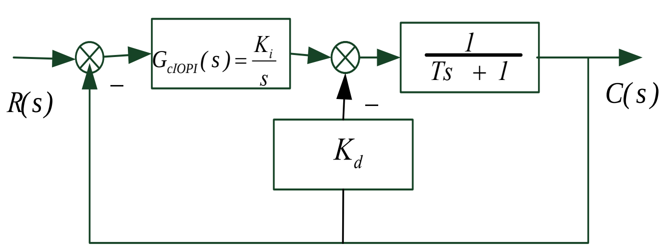


Figure 4. The Basic Structure of IOPDF System

And using Bode's ideal transfer function for FOPDF system should use step (5) to determine the control structure and parameters.

(5) According to the basic structure of IOPDF, establish the FOPDF control system model to a first-order controlled plant 1/(Ts+1), $1/(Ts^{\gamma}+1)$, the model for integer plant is shown in Figure 5. We suggest a basic structure of FOPDF system like Figure 5, where, the FOPI maybe a single FOPI controller or combined by FOPI controllers. The parameter K_d can be tuned of Chenliu method [17], as a basic tuning parameter of design FOPDF control structure. The K_d value is 5.44.Then the internal-loop can be treated as a whole controlled object. The equivalent open-loop block diagram of Figure 5 is shown in Figure 6. From Figure 6 the transfer function of FOPI controller designed based on Bode's ideal transfer function can be expressed as (4):

$$G_{cFOPI}(s)=\frac{G_{opi}(s)}{G_{pop}(s)}=\frac{\left(\frac{\omega_c}{s}\right)^{\alpha}}{\frac{1}{Ts+1+K_d}}=(\omega_c)^{\alpha}\cdot[Ts^{1-\alpha}+(1+K_d)\cdot s^{-\alpha}] \qquad (4)$$

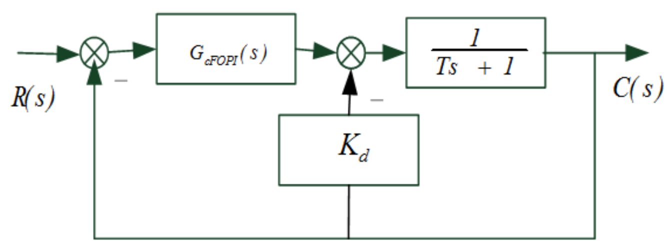


Figure 5. Block Diagram of FOPDF Control System Aimed to Integer First-order Control Object

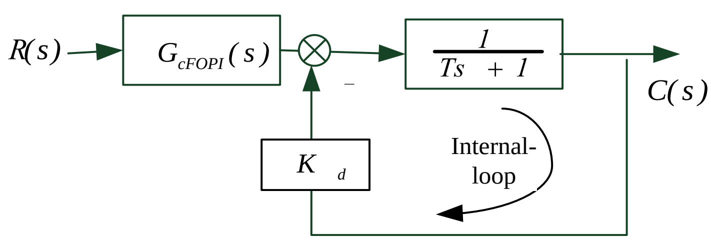


Figure 6. Open-loop Block Diagram of FOPDF Control System Aimed to Integer First-order Control Object

2. Establishing Fractional Order Control System Model

2.1. Establishing FOPID Controller Model Aimed at Integer First-Order Control Plant

Assume the open-loop transfer function of control object is as (5):

$$G_{PI}(s)=\frac{1}{Ts+1} \qquad (5)$$

here, T=0.4s, hypothesisΦ_m=70°, ω_c =10rad/s.

As procedures (2), the transfer function of controller aimed at integer first-order control plant based on Bode's ideal transfer function can be get from formula (6) ：

$$G_{c1}(s)=\frac{G_{opi}(s)}{G_P(S)}=\left(10^{\frac{11}{9}}\right)\left(0.4s^{-\frac{2}{9}}+s^{-\frac{11}{9}}\right) \qquad (6)$$

The simulation model of FOPID controller aimed at integer first-order control plant is shown in Figure 7.

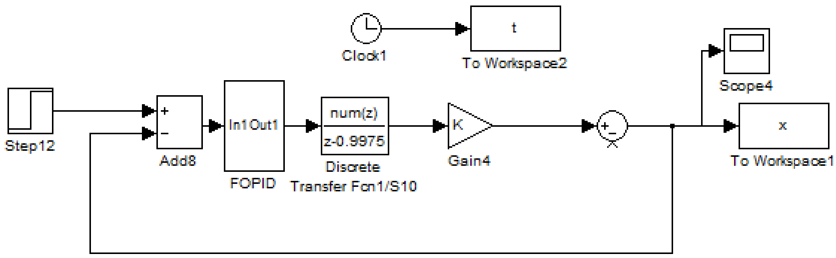


Figure 7. Simulation Model of FOPID Controller Aimed at Integer First-order Control Plant

2.2. Establishing FOPDF Controller Model Aimed at Integer First-Order Control Plant

Assume T is 0.4s, the expected crossover frequency ω_c is 10 rad/s, the phase margin Φ_m is 70^o,then equation of the controller can be determined as (7):

$$G_{c1FOPI}(s)=(10)^{\frac{11}{9}}\cdot[0.4s^{-\frac{2}{9}}+6.44s^{-\frac{11}{9}}] \qquad (7)$$

Establishing FOPDF control system simulation model as Figure 8.

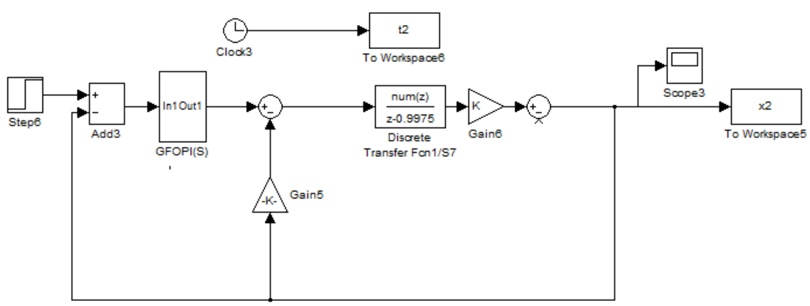


Figure 8. The Simulation Block of FOPDF Aim to Integer First-order Plant Control System

2.3. Establishing FOPDF Control System Model Aimed at Fractional First-Order Control Plant

Assume the open-loop transfer function of control object is (8):

$$G_{P2}(s)=\frac{1}{Ts^{\gamma}+1} \qquad (8)$$

here, T=0.5s, $\gamma=0.5$ hypothesis$\Phi_m=70^o$, ω_c =10rad/s.

As procedures (2) ,the transfer function of controller aimed at fractional first-order control plant based on Bode's ideal transfer function can also be get from formula (9) ：

$$G_{c2}(s)=\frac{G_{opi}(s)}{G_P(S)}=\left(10^{\frac{11}{9}}\right)\left(0.5s^{-\frac{13}{18}}+s^{-\frac{11}{9}}\right) \tag{9}$$

The simulation model of FOPID controller aimed at fractional first-order control plant is shown in Figure 8.While the subsystem block of FOPID controller and Fractional Order system are designed of the simulation blocks shown in Figure 3 applying the parameters in equation (8), (9).

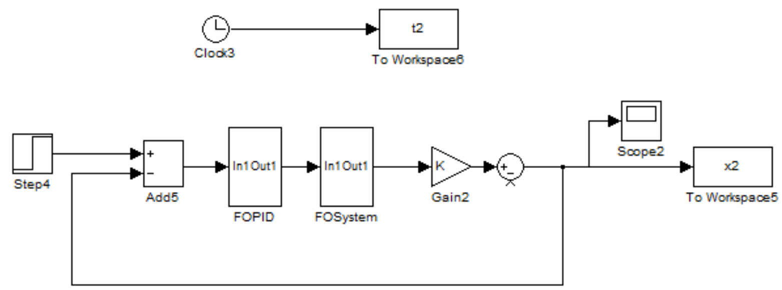


Figure 9. Simulation Model of FOPID Controller Aimed at Fractional First-order Control Plant

2.4. Establishing FOPDF Control System Model Aimed at Fractional First-Order Control Plant

For $1/(Ts^{\gamma}+1)$,assume $\gamma$is 0.5,T is 0.4s,(the other parameters are same as integer first-order control plant) ,the FOPDF structure is the same as integer one, but the equation of the controller is expressed as (10):

$$G_{c2FOPI}(s)=(10)^{\frac{11}{9}}\cdot[0.4s^{-\frac{13}{18}}+6.44s^{-\frac{11}{9}}] \tag{10}$$

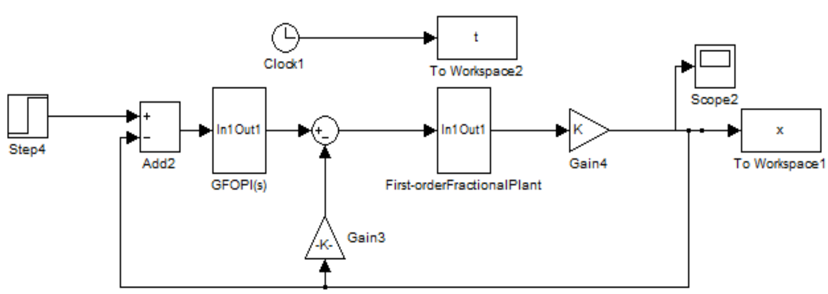
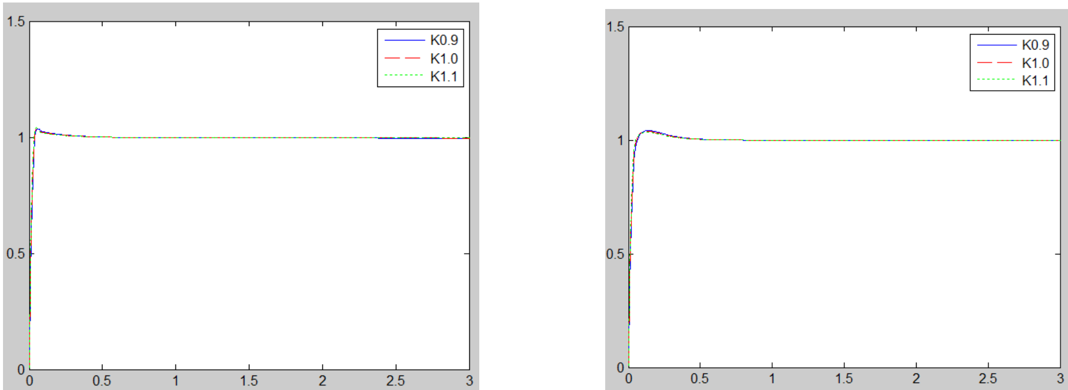


Figure10. The Simulation Block of FOPDF Aim to Fractional First-order Plant Control System

3. Simulation and Result Analysis

Unit Step Responses with open-loop system gain K varying (0.9K,K,1.1K) are shown in Figure 11. Figure 11 shows that the controllers designed based on Bode's ideal transfer function have both robustness anti-to the variation of system open-loop gain.

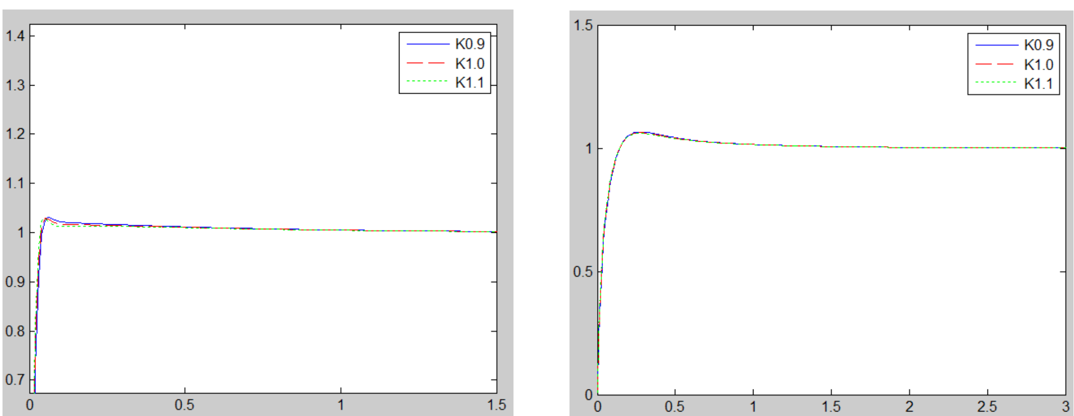


a)Unit Step Responses of Integer First-order First-order Control Plant

b) Unit Step Responses of Fractional Control Plant

Figure 11. Unit Step Responses of FOPID Controller

The parameter of K_d can be tuned to a suitable value to get a desirable step response. The bode diagram of open-loop FOPDF system both for integer or fractional first-order plant are the same, is shown in Figure12 a). Figure 12 b) is partial enlarged unit step responses of FOPDF control system of integer first-order plant, when changing the system gain K. And Figure 12 is unit step responses of FOPDF control system of fractional first-order plant, when changing the system gain K.



a） Partial Enlarge Unit Step Responses of FOPDF Control Structure Aim at Integer First-order Plant

b） Unit Step Responses of FOPDF Control Structure Aim at Fractional First-order Plant

Figure 12. Unit Step Responses of FOPDF Control Systems

4. Conclusions

With the Bode's ideal transfer function method, we obtain closed-loop systems robust to gain variations and the step responses indicating an iso-damping property. And the calculating process is fairly easy to traditional fractional order controller design methods. But, it must be noted that the characteristic has limitation to the gain crossover frequency ω_c and that the phase margin of the resulting closed-loop system is not exactly identical to the prescribed value defined by the slope α at that frequency. This is due to realize a arbitrary order by physical facilities are usually not easily. Simulations illustrate that the order α can be confirmed easily, but the select of ωc must consider a lot. To obtain a good performance, ωc must be as high as possible. And if too high this will hardly to get the PID parameters. And in this paper, we also have presented a new strategy of FOPDF control structure and apply Bode's ideal transfer function method, Chenliu method for parameters tuning. The numerical performances indicate that FOPDF control may result in a more rapid, more accurate and more robust control system. FOPDF method which has iso-damping property is of the practicability and effectiveness. FOPDF control method is likely applied in first order or first order time lag system like the hydraulic servo systems and the electric drive systems, high power semiconductor laser diodes constant temperature controlling systems.

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