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Research on Backstepping Control of Simplified Uncertain Supersonic Missile Model

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Abstract

A new kind of adaptive backstepping method is proposed for simplified missile model of pitch channel with time-varying and uncertain parameters. Backstepping control is an effective method for coping with system uncertainties. Adaptive method is integrated with backstepping method and a Lyapunov function is constructed to guarantee the whole system is stable. Through the theoretical analysis and numerical simulation, comparison between PID control and backstepping control shows that the backstepping control for the uncertain missile system has a better control effect. And compared with the PID control method, backstepping control has better robustness.

Keyword: *Uncertain supersonic missile, Backstepping control, PID control, Second order system*

1. Introduction

Second order system has very complex dynamics and many complex engineering objects such as missile, airplane and rocket, can be viewed as a second order system for designers. So research on second order system is meaningful and it is enough[1-5]. Model uncertainties are always exist and they are caused by environment changes or random reason [6-8]. For example, air dynamic coefficients changes as its speed and height and air density changes. Especially for supersonic missiles or hypersonic missiles, model parameters will change in a very big range. Also, the weight change as flue consume, that will also affect the parameters of simplified missile model. Common PID control or optimal control or feedback control can not provide satisfied performance as parameters change or uncertainties are very big. There are many papers tried adaptive method to solve those uncertainties [9-18]. But adaptive method is always integrated with other method, such as adaptive robust control or adaptive sliding mode control and else.

Backstepping method is a kind of feedback control method and it is based on Lyapunov stability theory. It is an effective and robust method for coping with system uncertainty by constructing control law step by step cleverly [9-11]. In this paper, a kind of adaptive backstepping control law is designed for simplified missile model of pitch channel. The adaptive law can solve the uncertain problems that is not convenient for backstepping method itself. Also, detailed simulations with PID control and adaptive backstepping control method were done to show the effectiveness of the proposed method.

2. Problem Description

The simplified linear model of supersonic missile pitch channel can be written as following second order system:

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$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \tag{1}$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z \tag{2}$$

where a_{ij} is air dynamic coefficient of missile, α is attack angle of missile, ω_z is the rotate speed of pitch angle and.

The control objective is to design a control law such that the attack angle α can track the desired angle α^d. Without loss of generality, assume $\alpha^d = 1$.

3. PID Control Law Design

The structure of PID control is showed as following Figure 1. The system is constructed by PID controller and control object. And the object is controller by PID controller which is consisted by proportional item, differential item and integral item.

PID controller is a kind linear controller, it is composed by the error signal defined by the difference between the desired value x_1^d and output of system x_1 as follows:

$$e(t) = x_1 - x_1^d \tag{3}$$

The PID control law is designed as

$$u(t) = k_p\left(e(t) + \frac{1}{T_I}\int_0^t e(t)dt + \frac{T_D e(t)}{dt}\right) \tag{4}$$

It can be written as a transfer function as

$$G(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{5}$$

Where k_p is the coefficient of proportional item and T_I is the coefficient of integral item and T_D is the coefficient of differential item.

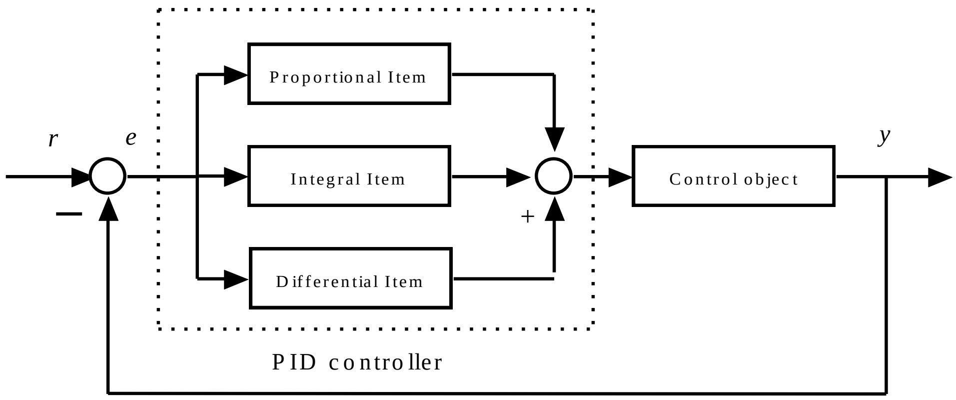

Figure 1. Structure of PID Control System

4. Backstepping Law Design

Consider the first subsystem as follows:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z \tag{6}$$

Define a new error variable as $e_\alpha = \alpha - \alpha^d$, then

$$\dot{e}_\alpha = \omega_z - a_{34}\alpha - a_{35}\delta_z \tag{7}$$

Use backstepping design method to design the desired value of ω_z as ω_z^d such that:

$$\omega_z^d = -k_1 e_\alpha - \hat{k}_2 + a_{35}\delta_z \tag{8}$$

Where $\hat{k}_2$ is an adaptive item which is used to approximate information about a_{34}. Substitute it into above equation, it holds:

$$\begin{aligned}\dot{e}_\alpha &= \omega_z^d + e_\omega - a_{34}(e_\alpha + \alpha^d) - a_{35}\delta_z \\ &= -k_1 e_\alpha - \hat{k}_2 + e_\omega - a_{34}(e_\alpha + \alpha^d) \\ &= (-k_1 - a_{34})\ e_\alpha - a_{34}\alpha^d - \hat{k}_2 + e_\omega\end{aligned} \tag{9}$$

Define

$$\bar{k}_1 = -k_1 - a_{34} \tag{10}$$

It is obvious there exists a big enough k_1 such that $\bar{k}_1 < 0$.
Define

$$\tilde{k}_2 = -a_{34}\alpha^d - \hat{k}_2 \tag{11}$$

Then

$$\dot{\tilde{k}}_2 = -\dot{\hat{k}}_2 \tag{12}$$

Choose turning law for estimation of unknown parameter as:

$$\dot{\hat{k}}_2 = k_2 e_\alpha \tag{13}$$

Then the above equation can be written as

$$\dot{e}_\alpha = \bar{k}_1 e_\alpha + \tilde{k}_2 + e_\omega \tag{14}$$

Choose a Lyapunov function as

$$V_1 = \frac{1}{2k_2}\tilde{k}_2^2 \tag{15}$$

Then

$$\dot{V}_1 = \frac{1}{k_2}\tilde{k}_2\dot{\tilde{k}}_2 = -\frac{1}{k_2}\tilde{k}_2\dot{\hat{k}}_2 = -\frac{1}{k_2}\tilde{k}_2 k_2 e_\alpha = -\tilde{k}_2 e_\alpha \tag{16}$$

Consider the second subsystem, then it holds

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z - \dot{\omega}_z^d \tag{17}$$

Design adaptive control law as

$$a_{25}\delta_z = -\hat{a}_{24}\alpha - \hat{a}_{22}\omega_z + \dot{\omega}_z^d - k_3 e_\omega - k_4 \int e_\omega dt \tag{18}$$

Where ω_z^d contains δ_z, so $\dot{\omega}_z^d$ contains $\dot{\delta}_z$ and solve the derivative $\dot{\omega}_z^d$, then it holds:

$$\begin{aligned} \dot{\omega}_z^d &= -k_1 \dot{e}_\alpha - \dot{\hat{k}}_2 + a_{35}\dot{\delta}_z \\ &= -k_1(\omega_z - a_{34}\alpha - a_{35}\delta_z) - k_2 e_\alpha + a_{35}\dot{\delta}_z \end{aligned} \tag{19}$$

And

$$\begin{aligned} a_{25}\delta_z &= -(\hat{a}_{24} - k_1 a_{34})\alpha - (\hat{a}_{22} + k_1)\omega_z \\ &-k_2 e_\alpha + k_1 a_{35}\delta_z + a_{35}\dot{\delta}_z - k_3 e_\omega - k_4 \int e_\omega dt \end{aligned} \tag{20}$$

Define

$$T = a_{25}\delta_z - k_1 a_{35}\delta_z - a_{35}\dot{\delta}_z \tag{21}$$

Then

$$T = -(\hat{a}_{24} - k_1 a_{34})\alpha - (\hat{a}_{22} + k_1)\omega_z - k_2 e_\alpha - k_3 e_\omega - k_4 \int e_\omega dt \tag{22}$$

Define

$$\tilde{a}_{24} = a_{24} - a_{25}\hat{a}_{24} \tag{23}$$

$$\tilde{a}_{22} = a_{22} - a_{25}\hat{a}_{22} \tag{24}$$

Then

$$\dot{\tilde{a}}_{24} = -a_{25}\dot{\hat{a}}_{24} \tag{25}$$

$$\dot{\tilde{a}}_{22} = -a_{25}\dot{\hat{a}}_{22} \tag{26}$$

Substitute the control law into the above equation, it holds:

$$\dot{e}_\omega = \tilde{a}_{24}\alpha + \tilde{a}_{22}\omega_z + \tilde{a}_{25}\dot{\omega}_z^d - a_{25}k_3 e_\omega - a_{25}k_4 \int e_\omega dt \tag{27}$$

Design the turning laws for the estimation of unknown parameters as

$$\dot{\hat{a}}_{24} = k_5 e_\omega \alpha \tag{28}$$

$$\dot{\hat{a}}_{22} = k_6 e_\omega \omega_z \tag{29}$$

Choose a Lyapunov function as

$$V_2 = \frac{1}{2k_5}\tilde{a}_{24}^2 + \frac{1}{2k_6}\tilde{a}_{22}^2 \tag{30}$$

Then

$$\dot{V}_2 = -e_\omega \alpha \tilde{a}_{24} - e_\omega \omega_z \tilde{a}_{22} \tag{31}$$

Choose a Lyapunov function as

$$V_3 = \frac{1}{2} k_4 \left(\int e_\omega dt \right)^2 \tag{32}$$

Then

$$\dot{V}_3 = k_4 e_\omega \int e_\omega dt \tag{33}$$

Choose a Lyapunov function as

$$V_4 = \frac{1}{2} e_\alpha^2 + \frac{1}{2} e_\omega^2 \tag{34}$$

Then

$$\dot{V}_4 = e_\alpha \dot{e}_\alpha + e_\omega \dot{e}_\omega \tag{35}$$

Where

$$e_\alpha \dot{e}_\alpha = \overline{k}_1 e_\alpha e_\alpha + \tilde{k}_2 e_\alpha + e_\alpha e_\omega$$

$$e_\omega \dot{e}_\omega + k_4 a_{25} e_\omega \int e_\omega dt = \tilde{a}_{24} \alpha e_\omega + \tilde{a}_{22} \omega_z e_\omega + \tilde{a}_{25} \dot{\omega}_z^d e_\omega - a_{25} k_3 e_\omega e_\omega \tag{36}$$

Choose a Lyapunov function as

$$V_5 = \frac{1}{2 k_7 a_{25}} \tilde{a}_{25}^2 \tag{37}$$

Then

$$\dot{V}_5 = -\tilde{a}_{25} e_\omega \dot{\omega}_z^d \tag{38}$$

Choose a big Lyapunov function for the whole system as

$$V = V_1 + V_2 + V_3 + V_4 + V_5 \tag{39}$$

Then

$$\dot{V} = \overline{k}_1 e_\alpha^2 + e_\omega e_\alpha - k_3 e_\omega^2 \tag{40}$$

The equation can be reduced with a inequality as

$$\dot{V} \le \overline{k}_1 e_\alpha^2 + \frac{1}{2} e_\alpha^2 + \frac{1}{2} e_\omega^2 - k_3 e_\omega^2 \tag{41}$$

It is obvious that there exist big enough positive k_1 and k_3 such that

$$\dot{V} \le 0 \tag{42}$$

Then the system is easy to be proved to be stable according to Lyapunov stable theorem.

5. Numerical Simulation

5.1 Simulation of PID Control

Choose PID control parameters as

$$k_p = 2, k_i = 5, k_d = 5$$

First, consider the situation of system with no uncertainties, the simulation result can be shown as below figure 2:

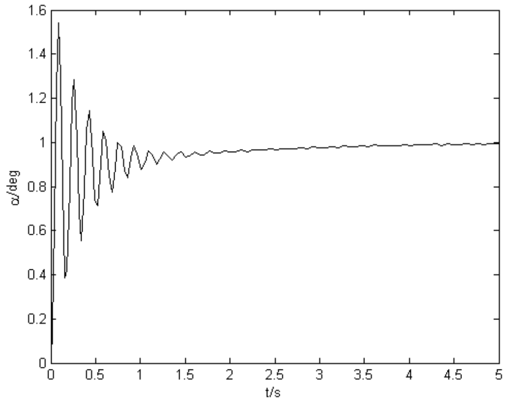

Figure 2. Result of Simulation with no Uncertainty

Second, consider the situation of system with constant uncertainty, the system parameters can be written as

$$A_{22} = a_{22} \cdot (1 + k), A_{24} = a_{24} \cdot (1 + k), A_{25} = a_{25} \cdot (1 + k);$$
$$A_{34} = a_{34} \cdot (1 + k), A_{35} = a_{35} \cdot (1 + k)$$

Where k is a constant and desired value is 1. The simulation result is shown as below figures:

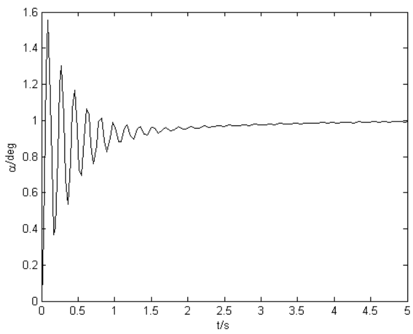

Figure 3 Result Of $k = -10\%$

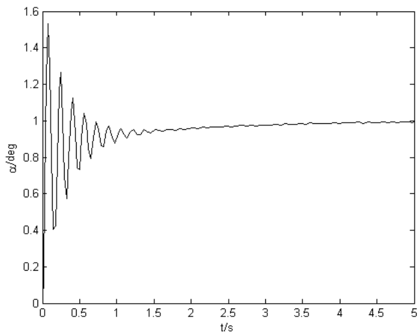

Figure 4 Result Of $k = 10\%$

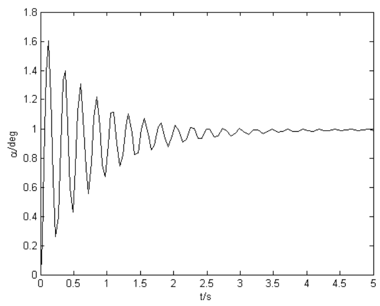

Figure 5 Result Of $k = -50\%$

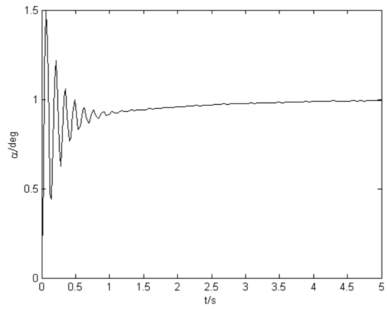

Figure 6 Result Of $k = 50\%$

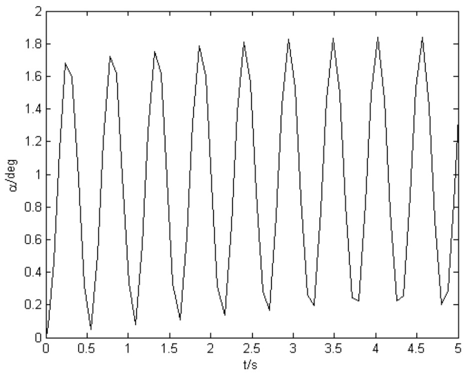

Figure 7. Result Of $k = -90\%$

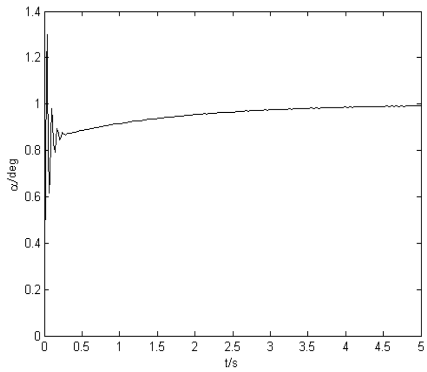

Figure 8. Result Of $k = 500\%$

Third, consider the situation of system with random uncertainties, and assume the random uncertainty is generate by rand function of Matlab software , then it can be written as

$$k_s = 2K * \left(\mathrm{rand}\left(5,1\right) - 0.5\right)$$

Where function rand() generate a random number of interval (0, 1) and $k_{s1}, k_{s2}, k_{s3}, k_{s4}, k_{s5}$ have the same characteristic as k_{s1}. Then parameters can be described as

$$A_{22} = a_{22} \cdot (1 + k_{s1})\ ;\ A_{24} = a_{24} \cdot (1 + k_{s2})\ ;\ A_{25} = a_{25} \cdot (1 + k_{s3})\ ;$$
$$A_{34} = a_{34} \cdot (1 + k_{s4})\ ;\ A_{35} = a_{35} \cdot (1 + k_{s5})$$

Simulation results are shown as following figures:

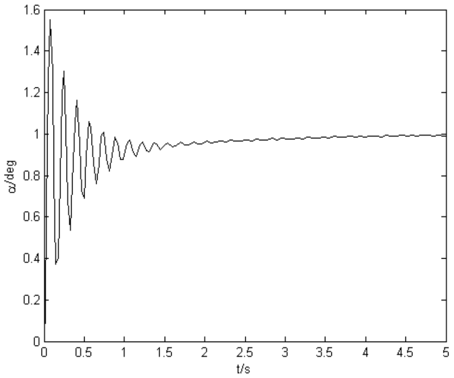

Figure 9. Result Of $k_s \in (-10\%, 10\%)$

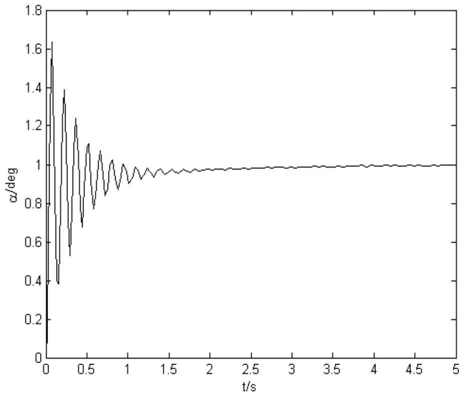

Figure 10. Result of $k_s \in (-50\%, 50\%)$

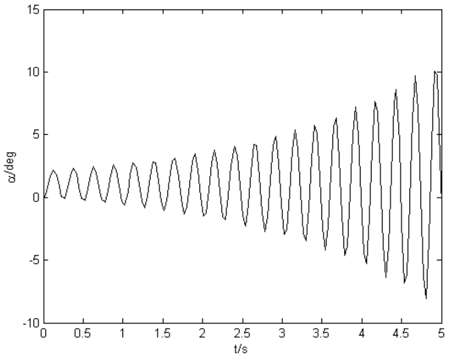

Figure 11. Result of $k_s \in (-90\%, 100\%)$

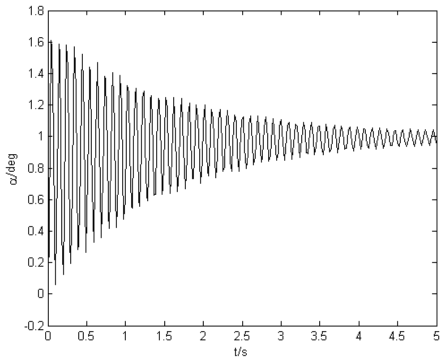

Figure 12. Result of $k_s \in (-90\%, 500\%)$

Simulation results shows that the PID control is stable if the uncertainty is small, but it the uncertainty is increase, especially if the uncertainty is negative, PID control will be unstable or volatile.

5.2 Simulation of Backstepping Control

Choose control parameters as

$$c_1 = 8, c_2 = 5, q_1 = 1, q_2 = 1, q_3 = 1, k_1 = 20, k_2 = 20, k_3 = 20$$

First, consider the situation of system with no uncertainty, the simulation result is shown as below figure 13.

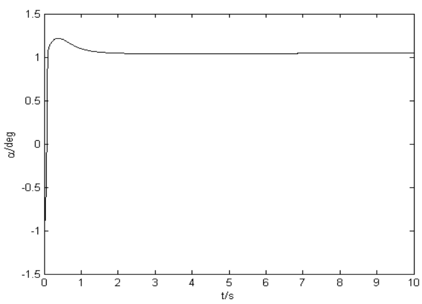

Figure 13. Result of no Uncertainty

Second, consider the situation of system with constant uncertainty, the simulation results are shown as following figure 14 to figure 19.

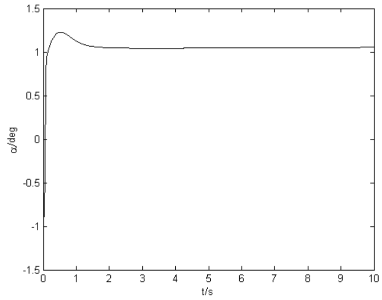

Figure 14. Result Of $k = -10\%$

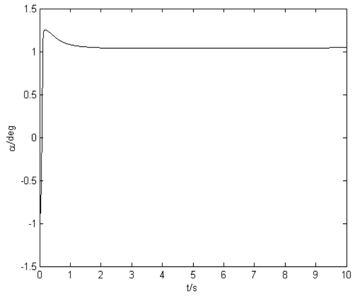

Figure 15. Result Of $k = 10\%$

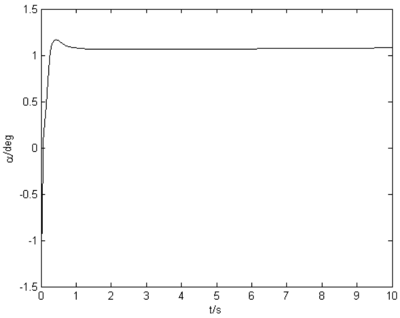

Figure 16. Result of $k = -50\%$

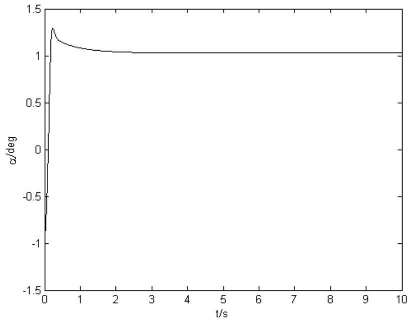

Figure 17. Result of $k = 50\%$

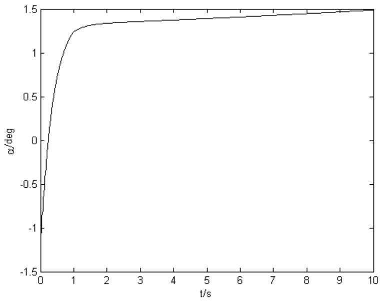

Figure 18. Result of $k = -90\%$

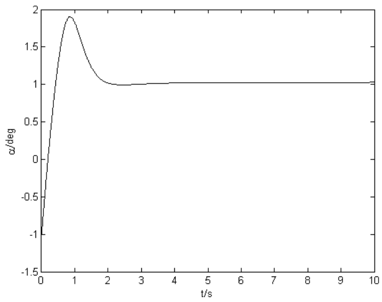

Figure 19. Result of $k = 500\%$

Third, consider the situation of system with random uncertainties, simulation results are shown as following figure 20 to figure 23.

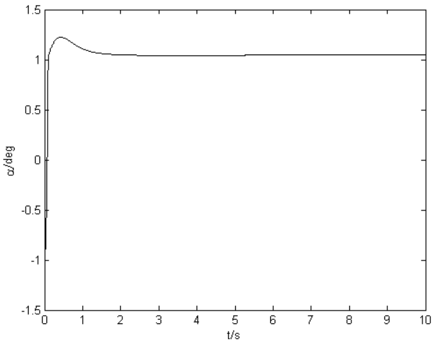

Figure 20. Result of $k_s \in (-10\%, 10\%)$

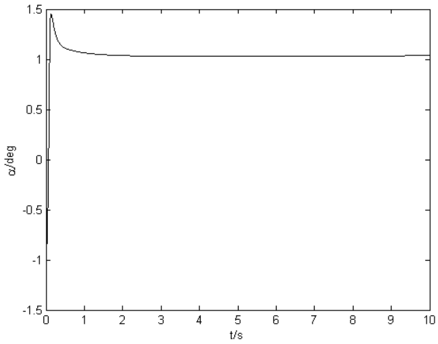

Figure 21. Result of $k_s \in (-50\%, 50\%)$

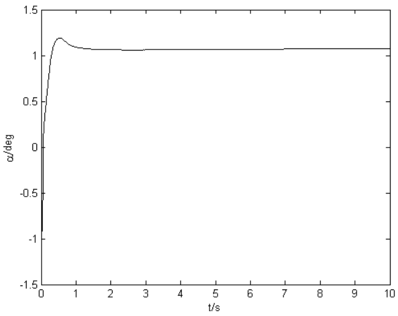

Figure 22. Result of $k_s \in (-90\%, 100\%)$

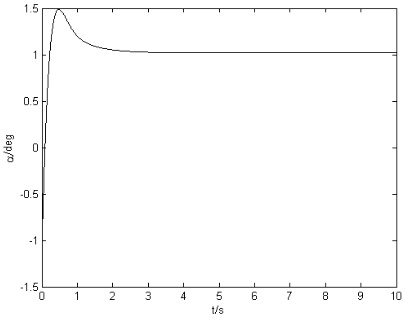

Figure 23. Result of $k_s \in (-90\%, 500\%)$

Simulation result shows that system is stable with backstepping method for all random uncertainties, it means that the method is robust.

5.3 Comparison and Analysis

The above simulation results shows that both two methods can make the missile system stable in some characteristic height with consideration of small uncertainties. But if the uncertainties increase, the PID control is not effective and especially if the uncertainties are random, the control effect of PID control is not as good as backstepping control. And both backstepping control and synchronous control are stable but the synchronous control method has better robustness and it is not necessary to know the system parameter in advance.

6. Conclusion

A new kind of adaptive backstepping control law was proposed for pitch channel control of supersonic missile. And three kinds of parameter uncertainties are considered to testify the robustness of proposed method. Also, the numerical simulation was done with PID control method. And simulation result shows that if the parameters change in a small range, the PID method is as effective as adaptive backstepping method. But if the simplified missile parameters change in a big range, the proposed adaptive backstepping method will have a better performance and it is more robust compared with PID control.

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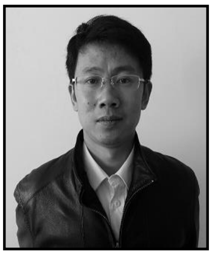

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