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The Optimal Guaranteed Cost Control of Tractor-semitrailer Steering Stability

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Abstract

The Gim tire model was adopted to set up the nonlinear dynamic model of tractor-semitrailer. Choosing the slip angle of tractor, the yaw velocity of tractor, the yaw velocity of semitrailer, and the angle between tractor and semitrailer as control variables, as well as considering the uncertain characteristic of the tire cornering stiffness, the optimal guaranteed cost control scheme was proposed. An optimal guaranteed cost controller of DYC for the tractor-semitrailer stability was developed based on model following technique. Based on the established nonlinear dynamic model, simulation in Matlab/Simulink software environment was described. The simulation results suggest that for the tractor-semitrailer with optimal guaranteed cost controller of DYC, the handling and stability performance on big slip angle is improved. The conclusion can be useful for the system design of tractor-semitrailer.

Keywords: *direct yaw-moment control, parametric uncertainty, optimal guaranteed cost control, Gim tire model*

1. Introduction

Tractor-semitrailer lateral instability problem (trailer swing, jack-knifing, rollover) is a worldwide major road safety problem, many studies report that combination of vehicle accidents are mostly caused by the lateral instability. Therefore, the study of tractor-semitrailer driving stability is important to improve the driving safety of tractor-semitrailer [1].

Direct yaw-moment control can improve vehicle stability considerably especially severe conditions, which has attracted many researchers from home and abroad. The tire cornering stiffness, affected by many aspects, is an important parameter in the vehicle dynamic control system. But most of vehicle stability controllers based on the model following are designed using a certain constant for the tire cornering stiffness parameter. Though quadratic optimal control is widely used in vehicle stability controllers, it can not obtain the optimal performance when the tire cornering stiffness varies in a large range. Fortunately, optimal guaranteed cost control theory provides a good means to solve the parameter uncertainties problem [2-8].

In this paper, a direct yaw-moment controller of the tractor-semitrailer stability based on optimal guaranteed cost control was presented to solve the tire cornering stiffness parameter uncertainties.

2. Nonlinear Tractor-semitrailer Dynamic Model

2.1 Tractor-semitrailer Dynamic Model

The schematic diagram of the tractor-semitrailer model is shown in Figure 1. Dynamics of this model can be represented by Equations (1-4)[1]:

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$$m_1 u(\dot{\beta}_1 + r_1) = F_{yf} + F_{yr1} + F_y \quad (1)$$

$$I_{Z1}\dot{r}_1 = a_1 F_{yf} - b_1 F_{yr1} - c_1 F_y \quad (2)$$

$$m_2 u(\dot{\beta}_2 + r_2) = F_{yr2} - F_y \quad (3)$$

$$I_{Z2}\dot{r}_2 = -b_2 F_{yr2} - c_2 F_y \quad (4)$$

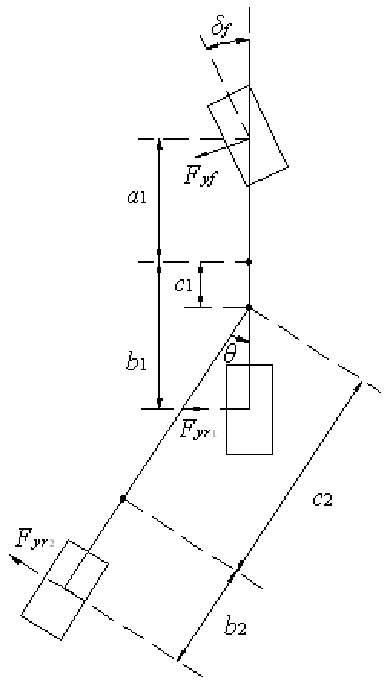

Figure 1. Schematic Diagram of Tractor-Semitrailer Model

In the above equations, m_1 and m_2 stand for the mass of tractor and semitrailer respectively; r_1 and r_2 stand for the yaw rate of tractor and semitrailer respectively; β_1 and β_2 stand for the slip angle of tractor and semitrailer respectively ; I_{z1} and I_{z2} are the moments of inertia of tractor and the semitrailer, respectively; F_{yf} , F_{yr1} and F_{yr2} are the lateral forces produced by each of the front and rear tires, respectively; For the tractor, a_1 and b_1 and c_1 are the distances from the center of gravity to the front and rear axle and hinge point respectively; For the semitrailer, b_2 and c_2 are the distances from the center of gravity to the rear axle and hinge point.

2.2. Gim Tire Model

Gim tire model can be represented by Equation (5, 6):

$$\begin{cases} F_x = C_s s_s l_n^2 + \mu_x F_z (1 - 3l_n^2 + 2l_n^3) & s_s < s_{sc} \\ F_x = \mu_x F_z & s_s \geq s_{sc} \end{cases} \quad (5)$$

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$$\begin{cases} F_y = C_\alpha s_\alpha l_n^{\ 2} + \mu_y F_z (1 - 3l_n^{\ 2} + 2l_n^{\ 3}) & s_\alpha < s_{\alpha c} \\ F_y = \mu_y F_z & s_\alpha \geq s_{\alpha c} \end{cases} \tag{6}$$

In the above equations, meaning of each parameter can be seen at reference [9].

3. Optimal Guaranteed Cost Controller Design

3.1. Linear Tractor-semitrailer Dynamic Model

Considering the linear tire, the lateral forces can be written by Equations (7-9):

$$F_{yf} = C_f \alpha_f \tag{7}$$

$$F_{yr1} = C_{r1} \alpha_{r1} \tag{8}$$

$$F_{yr2} = C_{r2} \alpha_{r2} \tag{9}$$

Where,

$$\alpha_f = \beta_1 + \frac{a_1 r_1}{u} - \delta_f$$

$$\alpha_{r1} = \beta_1 - \frac{b_1 r_1}{u}$$

$$\alpha_{r2} = \beta_1 + \theta + \frac{c_1 r_1}{u} - \frac{(c_2 + b_2) r_2}{u}$$

$$\dot{\theta} = r_1 - r_2$$

In the above equations, C_f 、 C_{r1} are the front and rear tire cornering stiffness of tractor respectively; C_{r2} is the tire cornering stiffness of semitrailer; α_f α_{r1} is the front and rear tire slip angle of tractor respectively; α_{r2} is the tire slip angle of semitrailer; θ is the tractor and semitrailer center line angle.

Taking the front steering angle δ_f and corrective yaw moment M as the inputs, the tractor slip angle β_1, tractor yaw rate r_1, semitrailer yaw rate r_2 and tractor and semitrailer center line angle θ as the state variables, combining the equations (1)-(4) and (7)-(13), the state response can be written as:

$$A_{ac} \dot{X}_{ac} = B_{ac} X_{ac} + C_{ac1} U_1 + C_{ac2} U_2 \tag{10}$$

Where,

$$X_{ac} = \begin{bmatrix} \beta_1 & r_1 & r_2 & \theta \end{bmatrix}^T, \quad U_1 = \begin{bmatrix} \delta_f \end{bmatrix}, \quad U_2 = \begin{bmatrix} M \end{bmatrix}$$

$$C_{ac1} = \begin{bmatrix} -C_f & -C_f a_1 & 0 & 0 \end{bmatrix}^T, \quad C_{ac2} = \begin{bmatrix} 0 & 1 & I_{z2} / I_{z1} & 0 \end{bmatrix}^T$$

$$A_{ac} = \begin{bmatrix} (m_1 + m_2)u & -m_2 c_1 & -m_2 c_2 & 0 \\ -m_2 u c_1 & Iz_1 + m_2 c_1^2 & m_2 c_1 c_2 & 0 \\ -m_2 u c_2 & m_2 c_1 c_2 & Iz_2 + m_2 c_2^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_{ac} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & C_{r2} \\ b_{21} & b_{22} & b_{23} & -C_{r2} c_1 \\ b_{31} & b_{32} & b_{33} & -C_{r2}(b_2 + c_2) \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

$$b_{11} = C_f + C_{r1} + C_{r2}$$

$$b_{12} = \frac{C_f a_1 - C_{r1} b_1 - C_{r2} c_1}{u} - m_1 u$$

$$b_{13} = \frac{-C_{r2}(b_2 + c_2)}{u} - m_2 u$$

$$b_{21} = C_f a_1 - C_{r1} b_1 - C_{r2} c_1$$

$$b_{22} = \frac{C_f a_1^2 + C_{r1} b_1^2 + C_{r2} c_1^2}{u} + m_2 u c_1$$

$$b_{23} = \frac{C_{r2}(b_2 + c_2) c_1}{u}$$

$$b_{31} = -C_{r2} b_2 - C_{r2} c_2$$

$$b_{32} = \frac{C_{r2} c_1 b_2 + C_{r2} c_1 c_2}{u} + m_2 u c_2$$

$$b_{33} = \frac{C_{r2}(b_2 + c_2)^2}{u}$$

3.2. Optimal Guaranteed Cost Controller

Figure 2 represents the principle of the optimal guaranteed cost controller [10].

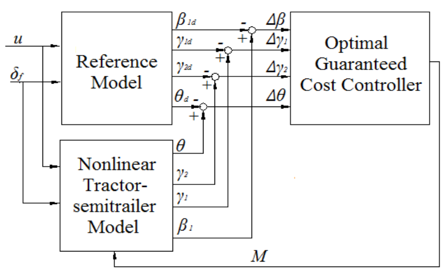

Figure 2. Schematic Diagram Of The Optimal Guaranteed Cost Controller Principle

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Taking the reference tractor slip angle β_{1d}, reference tractor yaw rate γ_{1d}, reference semitrailer yaw rate γ_{2d} and reference tractor and semitrailer center line angle θ_d as the state variables, the reference vehicle model state response can be written as:

$$A_d \dot{X}_d = B_d X_d + C_d U_1 \tag{11}$$

Where $A_d = A_{ac}$, $B_d = B_{ac}$, $C_d = C_{ac1}$.

To track the reference vehicle model, let $X = [\Delta\beta \quad \Delta\gamma_1 \quad \Delta\gamma_2 \quad \Delta\theta]^T = X_{ac} - X_d$, we can get:

$$A_{ac}\dot{X} = B_{ac} X + C_{ac2} U_2 \tag{12}$$

The uncertainty of tyre cornering stiffness can be expressed as follows：

$$\begin{cases} C_f = C_f(1+\Delta_f \rho_f) & (\|\rho_f\| \le 1) \\ C_{r1} = C_{r1}(1+\Delta_{r1}\rho_{r1}) & (\|\rho_{r1}\| \le 1) \\ C_{r2} = C_{r2}(1+\Delta_{r2}\rho_{r2}) & (\|\rho_{r2}\| \le 1) \end{cases}$$

. Then equation (12) can be written as:

$$A_{ac}\dot{X} = (B_{ac} + \Delta B_{ac}) X + (C_{ac2} + \Delta C_{ac2}) U_2 \tag{13}$$

Where $\Delta B_{ac} = DFE_1$, $\Delta C_{ac2} = DFE_2$

$$D = \begin{bmatrix} C_f\Delta_f & C_{r1}\Delta_{r1} & C_{r2}\Delta_{r2} & 0 \\ C_f\Delta_f a_1 & \text{-}C_{r1}\Delta_{r1}b_1 & \text{-}C_{r2}\Delta_{r2}c_1 & 0 \\ 0 & 0 & \text{-}C_{r2}\Delta_{r2}(b_2+c_2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} \rho_f & 0 & 0 & 0 \\ 0 & \rho_{r1} & 0 & 0 \\ 0 & 0 & \rho_{r2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & \frac{a_1}{u} & 0 & 0 \\ 1 & \text{-}\frac{b_1}{u} & 0 & 0 \\ 1 & \text{-}\frac{c_1}{u} & \frac{\text{-}(b_2+c_2)}{u} & 1 \\ 0 & 1 & \text{-}1 & 0 \end{bmatrix}$$

$$E_2 = 0$$

The quadratic cost function associated with system (12) can be defined as:

$$J = \int_0^\infty \left(X^T Q X + U_2^T R U_2 \right) dt \tag{14}$$

Where $Q \rangle 0$, $R \rangle 0$

For system (12) and cost function (14), $U_2 = -YX^{-1}x(t)$ is an optimal state feedback control law if there exists a solution for the following optimization problem:

$$\begin{bmatrix} A_0X + XA_0^T - B_0Y - Y^TB_0^T & D_0 & XE_1^T - Y^TE_2^T & X & Y^T \\ * & -\varepsilon^{-1}I & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -Q^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix} < 0$$

Where $A_0 = A_{ac}^{-1}B_{ac}$, $B_0 = A_{ac}^{-1}C_{ac2}$, $D_0 = A_{ac}^{-1}D$

4. Nonlinear Simulation

The major parameters are as follows： m_1 =6870 Kg ； m_2 =6181 Kg ； I_{Z1} =20441 $Kg \cdot m^2$ ； I_{Z2} =81912 $Kg \cdot m^2$ ； a_1 =1.96 m ； $c_{\alpha r1}$ =143.33 kN/rad ； $c_{\alpha r2}$ =80.312 kN/rad ； $c_{\alpha f}$ =143.33 kN/rad ； b_1 =2.35 m ； b_2 = 3.30 m ； c_1 = 2.05 m ； c_2 =5.23 m .

The tractor-semitrailer speed is 108 km/h. The maximum amplitudes of the sinusoidal input in fig.3 are 0.135rad and 0.103rad, respectively. The road adhesion coefficients are 0.8 (dry road) and 0.6(wet road) respectively. The simulation time is 15s.

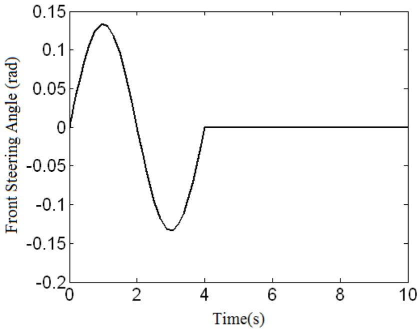

Figure 3. The Input of Front Steering Angle

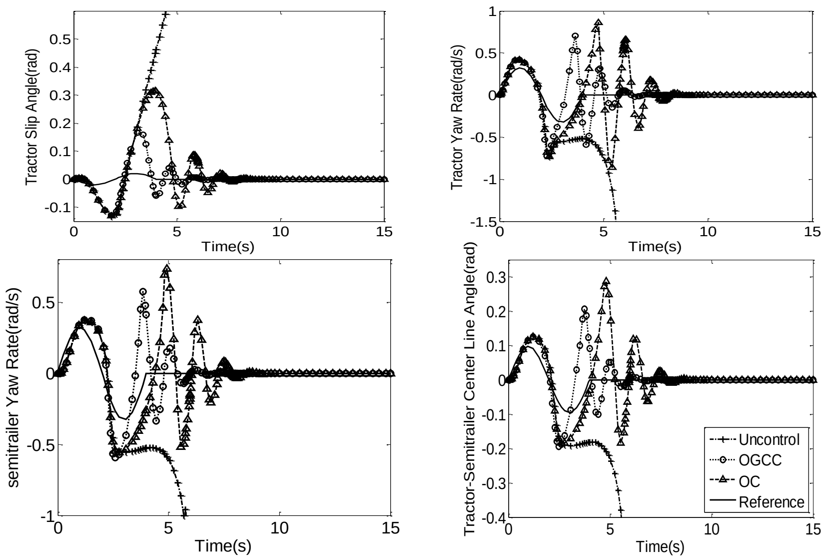

Figure 4. Responses of Dry Road

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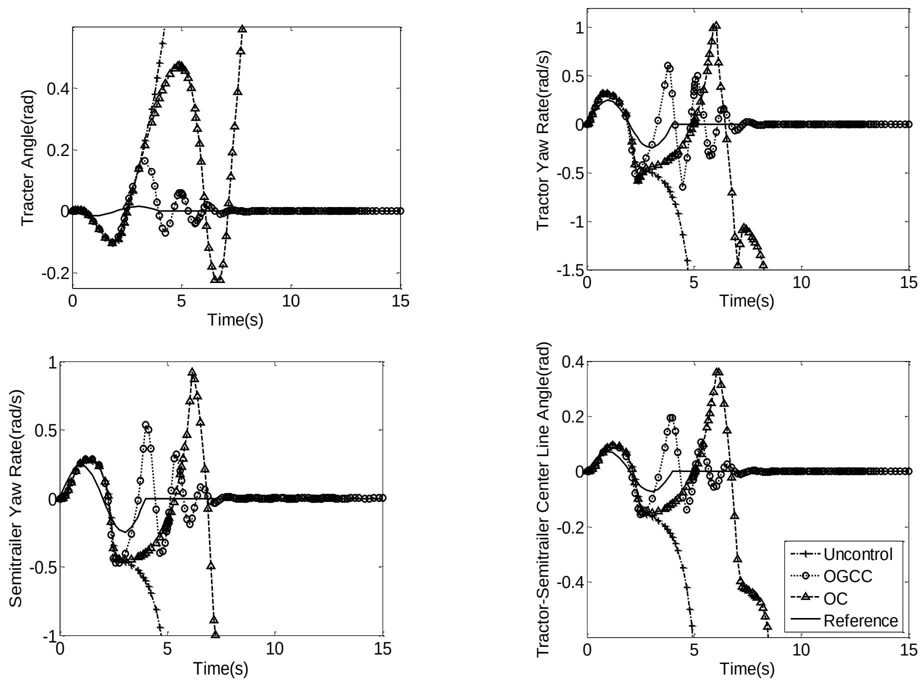

Figure 5. Responses of Wet Road

Simulations are carried out on the nonlinear tractor-semitrailer model presented in section 1 to analyze and evaluate the optimal guaranteed cost control scheme proposed in section 3. We can easily find that when the road adhesion coefficient is 0.8, compared with the conventional optimal control (OC). The optimal guaranteed cost control (OGCC) could not only keep the system stable but also obtain much smaller tractor slip angle and tractor and semitrailer center line angle and have much better yaw rate response of the tractor and the semitrailer. We can also easily find that when the road adhesion coefficient is 0.6, compared with the conventional optimal control, though the conventional optimal control lose stability ultimately, the optimal guaranteed cost control can still keep stable and have good steady-state response.

The uncontrolled vehicle will lose stability quickly in both road adhesion coefficients (0.8 and 0.6).

5. Conclusion

Direct yaw moment control is one of the most efficient methods of vehicle stability control system. Vehicle stability control system is affected greatly by the adhesion coefficient. The optimal guaranteed cost control can handle the systems whose working conditions vary in a large range very well. A yaw moment control scheme is proposed based on the optimal guaranteed cost control in this paper. Simulation results demonstrate that the optimal guaranteed cost control can keep vehicle system stable at the bigger front wheel angle and has better transient and steady state response than the conventional optimal control.

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