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Infinite-Horizon Optimal Control for Deploying a Tethered Sub-satellite

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Abstract

An infinite-horizon optimal control problem for the deployment of a tethered sub-satellite under complex nonlinear constraints is presented. The nonlinearities in the system model and the constraints of states and control are taken into consideration. Legendre-Gauss-Radau pseudo spectral method is described for solving the infinite-horizon optimal control problem numerically. By using a smooth, monotonic domain transformation technique, an infinite-horizon optimal control problem is approximated with a finite-horizon problem. The resulting problem in the finite interval is transcribed to a nonlinear programming problem. The proposed methods yield a series of approximations to the state on the entire horizon. The optimal control law can be determined via a nonlinear programming, without any iteration. A numerical example is included to demonstrate the high effectiveness and hard real-time of the proposed schemes.

Keywords: *Tethered satellite; Nonlinear; Infinite-Horizon; Radau Pseudospectral Method*

1. Introduction

The tethered Satellite System (TSS) has found many advantages over other space technologies. Numerous promising applications of tether technology have been proposed, including tether-assisted re-entry, space transportation, air braking, and electrodynamic drives [1, 2]. In a tether application to be successful, safely controlling the deployment of a tethered sub-satellite is a significant challenging task with complex constraints for the nonlinear nature in the dynamics of the TSS [3]. The tether dynamics is very complex and the performance requirements generally demand high accuracy in a tether application. The traditional solution method for the problem relies on solving the associated Hamilton-Jacobi-Bellman equation, while the classical feedback control is very difficult to meet the demands for the nonlinear problem. Optimal control may be ideal for the deployment control problem, it is feasible to solve the solution of the problem efficiently and reliably for real-time implementation.

Over the past decades, significant attention has been attracted to study optimal control of tethered satellite systems. Gläβel. [4] presented an adaptive neural network to control the deployment of a tether-assisted re-entry mission in the case of two degrees of freedom. Steindl [5] studied optimal control problems of two degrees of freedom for tethered satellites in deployment. In this study, they designed the optimal open-loop control rule

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using the Maximum Principle to achieve a controlled the deployment. Williams [6] developed a deployment control strategy for the YES2 mission. The open-loop optimal trajectories were determined by pseudo spectral method. The linear closed-loop controllers were designed using a receding horizon control that tracked the optimal trajectory. Williams [7] also presents a study on utilizing space tether technology for the rendezvous and capture of payloads under in-plane and demonstrates the feasibility of the application. Williams and Trivailo [8] made a comparison of various cost functions for the optimal control of a tethered satellite system under in-plane motion. Wen. [9] designed a nonlinear optimal control law for the deployment process of a sub-satellite system using only tether tension. The control scheme used a rapid re-computation of the open-loop optimal control based on a Legendre pseudo spectral method. Wen. [10] presents a nonlinear optimal control scheme for the retrieval of a tethered sub-satellite model which is formulated over an infinite-horizon. Two Legendre pseudo spectral algorithms including direct method and indirect method are explored to find an optimal trajectory that returns to the mother satellite.

In this paper, we present the infinite-horizon optimal control for the deployment of a tether sub-satellite under complex nonlinear constraints. The nonlinearities in the system model and the constraints of states and control are taken into consideration. Legendre-Gauss-Radau pseudo spectral method is described for solving the infinite-horizon optimal control problem numerically. By using a smooth, monotonic domain transformation technique, an infinite-horizon optimal control problem is approximated with a finite-horizon problem. The resulting problem in the finite interval is transcribed to a nonlinear programming problem. A numerical example is included to demonstrate the high effectiveness and hard real-time of the proposed schemes.

2. Problem Formulation

2.1 Mathematical Model

In order to effectively analyze the dynamics and control of the tethered satellite, it is necessary that introducing a simplified system model. In the model the tether mass is neglected, the mother satellite S1 and the sub-satellite S2 are considered as point masses connected via an inelastic straight tether. The tether may vary in length in a controlled manner. The control input of the system is the tension control, which is considered to be changes as the tether length in the deployment. This model has often been used to design controllers and verify their performance, which enhances the computational efficiency of control laws.

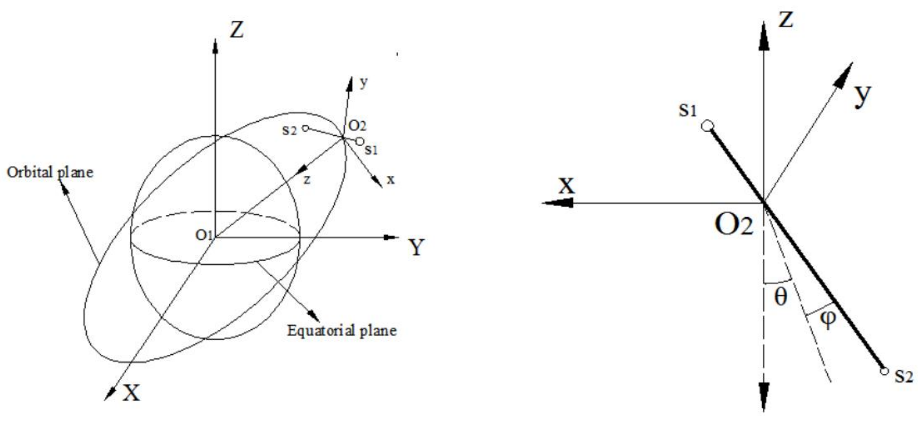


Figure 1. Simplified Tethered Satellite System Model

The simplified system model is shown in Figureure1. The inertial coordinates(*X,Y,Z*) are attached to the center of the Earth *O1*, the orbital coordinates(x,y,z) are centered at *O2*. It is assumed that the center of mass of the TSS remains in a circular orbit. The equations of motion for the system may be derived via the second Lagrange equation in spherical coordinates. Considering the system motion of in-plane and out-plane, the dimensionless system dynamics equations are given.

$$\ddot{\theta} + 2(\dot{\theta} - 1)(-\dot{\varphi}\tan\varphi + \dot{\Lambda} / \Lambda) + 3\sin\theta\cos\theta = Q_\theta / m_2\omega_f^{\ 2} L_f^2 \cos^2\varphi \tag{1}$$

$$\ddot{\varphi} + 2\dot{\varphi}\dot{\Lambda} / \Lambda + [(\dot{\theta} - 1)^2 + 3\cos^2\theta]\sin\varphi\cos\varphi = F \tag{2}$$

$$\ddot{\Lambda} - \Lambda\{(\dot{\varphi})^2 + \cos^2\varphi[(\dot{\theta} - 1)^2 + 3\cos^2\theta] - 1\} = -u \tag{3}$$

The state of TSS is represented by dimensionless vector $x = (\theta, \varphi, \Lambda, \dot{\theta}, \dot{\varphi}, \dot{\Lambda})$, which respectively represents in-plane libration angle, out-of-plane libration angle, length of the tether, in-plane angle velocity, out-of-plane angle velocity and tether motion velocity. m_1 is the mass of the mother satellite, and m_2 is the mass of sub-satellite, the assumption $m_2 << m_1$ is made that the orbit center of the system is the mass center of the mother satellite. $\Lambda = l(t) / L_f$ is the dimensionless unstrained length of the tether, ***l(t)*** and $\boldsymbol{L_f}$ is the current length and reference length of the tether respectively ($\boldsymbol{L_f}$ = 1km). ω_f is the orbit angular velocity , $v = \omega_f * t$ is the dimensionless time , $\dot{\Lambda} = [dl(t) / L_f] * [dt / dv] = dl(t) / [\omega_f * L_f]$ is the dimensionless time derivative. Q_θ is the generalized forces, $F = F_t / m_2\omega_f^{\ 2} L_f$ is the dimensionless thrust force, F_t is the thrust force, $u = T / m_2\omega_f^{\ 2} L_f^2$ is the dimensionless tether tension, *T* is the tether tension along the tether.

Through a linear transformation, equations (1)(2)(3) can be expressed by the following state-Space equations form.

$$\dot{x}_1 = x_4 , \quad \dot{x}_2 = x_5 , \quad \dot{x}_3 = x_6 \tag{4}$$

$$\dot{x}_4 = 2(x_4 - 1)(x_5 \tan x_2 - x_6 / x_3) - 3\sin x_1 \cos x_1 \tag{5}$$

$$\dot{x}_5 = -2x_5x_6 / x_3 - [(x_4 - 1)^2 + 3\cos^2 x_1]\sin x_2 \cos x_2 + u_b \tag{6}$$

$$\dot{x}_6 = [x_5^{\ 2} + (x_4 - 1)^2 \cos^2 x_2 + 3\cos^2 x_2 \cos^2 x_1 - 1]x_3 - u_a \tag{7}$$

Where the dimensionless state vector $x = (x_1, x_2, x_3, x_4, x_5, x_6) = (\theta, \varphi, \Lambda, \dot{\theta}, \dot{\varphi}, \dot{\Lambda})$, the dimensionless control inputs vector $U = (u_a, u_b) = (F, u)$

In this research, the optimal control strategy is adopted to ensure that the sub-satellite is deployed to the target position accurately by controlling the tether tension and thrust force.

2.2. Optimal Control Problem

The paper is devoted to finding an optimal trajectory that deploys the sub-satellite from the initial position to target position not only accurately but also quickly. The infinite-horizon optimal control problem is to find the suitable perturbed state vectors $\delta x(t)$ and perturbed control vectors $\delta u(t)$, satisfying equations (4)(5)(6)(7), while minimizing the following performance indexes.

$$J = \frac{1}{2}\int_0^\infty g(\delta x(t), \delta u(t), t)dt = \frac{1}{2}\int_0^\infty \left[\delta x^T(t) Q \delta x(t) + \delta u^T(t) R \delta u(t)\right]dt \tag{8}$$

subject to the following constraints:
State equations:

$$\delta \dot{x} = f(\delta x(t), \delta u(t), t) = A\delta x(t) + B\delta u(t) \tag{9}$$

The initial conditions:

$$\delta x(t = 0) = \delta x_0 \tag{10}$$

Where $\delta x(t)$ are the perturbed state variables, $\delta u(t)$ are the perturbed control variables. ***A*** represents the system state influence matrix. ***B*** represents the control influence matrix. ***Q*** is the positive semi-definite weighted matrix. ***R*** is defined as the positive definite weighted matrix. State equations are nonlinear, which should be paid special attention.

3. Radau Pseudo Spectral Optimal Control

Over the last few years, pseudo spectral (PS) methods have increased in popularity in the numerical solution of optimal control problems [11-15], because PS methods have many advantages including high precision operation, fast convergence rate, easy implementation. PS methods are a class of direct collocation where a optimal control problem is transformed into a nonlinear programming problem (NLP) by discretizing the state vectors and control vectors which are approximated by Lagrange polynomials or Chebychev polynomials of order N at Gaussian quadrature points. The most commonly used collocation points are Legendre-Gauss (LG), Legendre-Gauss-Lobatto (LGL), Legendre-Gauss-Radau (LGR), Chebychev-Gauss-Lobatto (CGL) points. These points are obtained from the roots of orthogonal polynomials and its derivatives, such as Legendre polynomials, Chebychev polynomials. All collocation points are defined on the domain [-1, +1], but differ significantly in that the LGL points and CGL points include both of the endpoints, the LGR points include left-endpoints, and the LG points include neither of the endpoints. The co-location points distribution of LG, LGL, LGR points over the computational domain $[-1, +1]$ is shown in Ref [16].

Properly choosing the type of collocation points mainly depends on the boundary-value problem that is being solved. For the finite-horizon optimal control problems, boundary conditions at both the initial point and the end point are usually required. Meanwhile LGL points are fixed at both of the boundary points, so it is the most Proper choice. For the infinite-horizon optimal control problems, generally boundary conditions only at initial points are required. The solutions at infinity tend to zero. LGR points are fixed at one boundary points, generally in domain transformation, the initial point $\tau = -1$ corresponds to $t = 0$, the end point $\tau = +1$ is a singularity, which correspond to $t = +\infty$, so LGR points is most appropriate the infinite-horizon problems. The collocation point distribution of LGL and LGR points over the physical domain $[0, +\infty)$ have been shown in Ref [15]. Although the LGL and LGR points are dense at the initial time, the LGR points are further spread out over the time axis at instances beyond the initial time, toward $t = +\infty$ .The LGR points provide a more accurate representation of the infinite horizon by distributing more points toward $t = +\infty$. Consequently, these results strongly suggest that the LGR points are a better-suited choice for the infinite horizon control. The differences among PS methods are summarized in Table 1.

Table 1. The Differences of PS Methods

PS	Collocation Points Type	Collocation Points Range	Interprolation polynominals	Horizon
GPM	LG	(-1,1)	Lagrange	NA
RPM	LGR	[-1,1)	Lagrange	Infinite
LPM	LGL	[-1,1]	Lagrange	Finite
CPM	CGL	[-1,1]	Chebychev	Finite

In the paper, LGR PS is applied for solving the infinite-horizon nonlinear optimal control problems the fundamental idea is as follows. First, by a domain transformation technique, the infinite-horizon is mapped to a finite-horizon. An infinite-horizon problem is approximated with a finite-horizon problem. Second, the nonlinear optimal control problems are discretized and transcribed to a serial of nonlinear programming problems by LGR PS method. Third, the nonlinear programming problems are solved by a sequential quadratic programming.

3.1 Domain Transformation

For the infinite-horizon optimal control problems, a smooth, strictly monotonic transformation is used to map a infinite time domain $t \in [t_{0,} \ \infty]$ to a computational domain $\tau \in \left[\tau_{0,} \ \tau_f\right) = \left[-1, +1\right)$, the affine transformation is found in Ref [15].

$$t = (1+\tau)/(1-\tau) \ \leftrightarrow \ \tau = (t-1)/(t+1) \tag{11}$$

Equations (8-10) are written respecting to the computational domain τ . The minimized performance indexes

$$J = \frac{1}{2}\int_{-1}^{+1} \frac{2}{(1-\tau)^2}\left[\delta x^T(\tau) Q \delta x(\tau) + \delta u^T(\tau) R \delta u(\tau)\right] d\tau \tag{12}$$

Subject to the dynamic equations

$$\dot{x} = \xi(\tau) f(x(\tau), u(\tau), \tau) \tag{13}$$

The initial conditions

$$\delta x(\tau = -1) = \delta x_0 \tag{14}$$

$\xi(\tau)$ is the transformation metric ,which represents the derivative with respect to time τ , and it is defined as:

$$\xi(\tau) = \frac{dt}{d\tau} = \frac{2}{(1-\tau)^2} \tag{15}$$

3.2. Interpolation

The state vectors and control vectors are approximated at LGR points by the Lagrange interpolating polynomials of degree N so that

$$x(\tau) \approx x^N(\tau) = \sum_{k=0}^{N} x(\tau_k) L_k(\tau) \tag{16}$$

$$u(\tau) \approx u^N(\tau) = \sum_{k=0}^{N} u(\tau_k) L_k(\tau) \tag{17}$$

Where $L_k(\tau)$ is a basic of *N*th degree Lagrange interpolation polynomials on the interval [-1, 1). τ_k (*k=0,…,N*) are LGR collection points and the initial point τ_0 = -1, the rest points are the roots of $L_{k-1}(\tau) + L_k(\tau)$, so -1<=τ_k < +1.

$$L_k(\tau) = \prod_{\substack{j=0 \\ k \neq j}}^{N} \frac{\tau - \tau_j}{\tau_k - \tau_j}, \; K=0,\ldots,N \tag{18}$$

3.3. Differentiation

The derivatives of the *i*th state vector is approximated at the LGR points τ_k (*k=0,...,N*) via a serials of differential matrix $\boldsymbol{D_{k,j}}$ by the form:

$$\dot{x}_i(\tau_k) \approx \dot{x}_i^N(\tau_k) = \sum_{j=0}^{N} x_i(\tau_j) \dot{L}_j(\tau_k) = \sum_{j=0}^{N} D_{k,j} x_i(\tau_j) = D_k x_i \tag{19}$$

The dynamic differential equation (13) are approximated at the LGR points

τ_k (*k=0,...,N*) which can be expressed as:

$$\dot{x}(\tau_k) \approx \dot{x}^N(\tau_k) = \xi(\tau_k) f(x(\tau_k), u(\tau_k), \tau_k) = D_k X \tag{20}$$

The differential constraints are transformed into a serials of algebraic equations which is expressed using the differential matrix $\boldsymbol{D_k}$.

Where the vector of state variables *X* is denote as

$$X = [x(\tau_0), x(\tau_1), \ldots., x(\tau_N)]^T \tag{21}$$

Where $\boldsymbol{D_{k,j}}$ are entries of the *N*×(*N*+1) differential matrix ***D***. It has one row for LGR points, the elements in the ***i***th column are the derivatives of the Lagrange polynomials evaluated at each of the collocation points. $\boldsymbol{D_{k,j}}$ is denote as [17]

$$D_{k,j} = \dot{L}_j(\tau_k) \tag{22}$$

$$D_{k,j} = \begin{cases} \dfrac{L_N(\tau_K)}{L_N(\tau_j)} \dfrac{1}{\tau_K - \tau_j} \dfrac{1-\tau_j}{1-\tau_K} & k \neq j \\[2ex] -\dfrac{N(N+2)}{4} & k = j = 0 \\[2ex] \dfrac{1}{2(1-\tau_K)} & k = j \neq 0 \end{cases} \tag{23}$$

3.4. Integration

The integral constraints are also transformed into algebraic equations by the Gauss-Lobatto integral rule [11].The integral in the performance index is approximated using the Gauss-Lobatto integral rule which provides a highly accurate result and equations (12) can be expressed by the following form:

$$
\begin{aligned}
J &= \frac{1}{2}\int_{-1}^{+1} \frac{2}{(1-\tau)^2}\left[\delta x^T(\tau) Q \delta x(\tau) + \delta u^T(\tau) R \delta u(\tau)\right] d\tau \\
&= \sum_{k=0}^{N} \frac{w_k}{(1-\tau_k)^2} g(\tau_k)
\end{aligned} \tag{24}
$$

Where w_k is the LGR integration weights given by[17] $g(\tau_k)$

$$
w_k = \begin{cases} \dfrac{2}{(N+1)^2} & k = 0 \\ \dfrac{1}{(N+1)^2} \bullet \dfrac{1-\tau_K}{[L_N(\tau_K)]^2} & 1 \le k \le N \end{cases} \tag{25}
$$

Where $g(\tau_k)$ is the equations defined by

$$
g(\tau_k) = \delta x^T(\tau_k) Q \delta x(\tau_k) + \delta u^T(t_k) R \delta u(\tau_k) \tag{26}
$$

At LGR points, the infinite-horizon optimal control problems described by the above technologies, the optimal control problems have been transformed into a standard NPL problem. The optimal problem is transformed into a set of algebraic equations forms, Equation (24) and (25) can be expressed in the matrix form.

$$
J = \frac{1}{2} X^T H X \tag{27}
$$

$$
AX = B \tag{28}
$$

I refers to the unit vector of $\boldsymbol{n} \times \boldsymbol{n}$, ***0*** represents the zero vector of $\boldsymbol{n} \times \boldsymbol{n}$. The first equation of the system above expresses the initial condition constraints.

The nonlinear trajectory optimal problem of TSS that described by equations (8)(9)(10) has been discretized and transformed into a NLP by the LGR PS method. Many NLP solvers are available for solving the problem routinely. Then the optimal control law may be obtained by solving the nonlinear programming problem.

4. Numerical Example

The performance of the proposed controllers is verified by the below case studies. In the numerical example, some constraint conditions should be determinate. All the vectors are transformed into dimensionless vectors. The dimensionless deployment length of tether may vary in the interval [$\boldsymbol{L}_{min}$, $\boldsymbol{L}_{max}$], where $\boldsymbol{L}_{min} \geqq 0.1$, $\boldsymbol{L}_{max} \leqq 1$, the initial value of tether length is 0.1.The dimensionless tether tension may vary in the interval [T_{min}, T_{max}], where $\boldsymbol{T}_{min} \geqq 0.01$, $\boldsymbol{T}_{max} \leqq 6$. The tether tension should be greater than zero to avoid the possibility of the tether becoming tangled due to the slack meanwhile the tension should not be great enough to snap the tether. The in-plane libration angle and the out-of-plane libration angle should not change too greatly in order to avoid interfering with the mother satellite.

At the terminal time of the task, the tethered sub-satellite should closely reach the specified target. Meanwhile the angle velocity of in-plane libration and out-of-plane libration are close to zero and the tether tension should remain positive throughout the deployment process. Only 40 LGL points are chosen in order to keep the computational cost low, since this is intended to be run in real time.

The deployment maneuver is considered to begin with the initial states in the two Case.

(1) Case1: control with no disturbance

Initial constraints: $\boldsymbol{X}(t=t_0) = [\theta, \dot{\theta}, \varphi, \dot{\varphi}, \Lambda, \dot{\Lambda}] = [0, 0, 0, 0, 0.1, \mathbf{0}]$;

(2) Case2: control with a initial disturbance

Initial constraints: $\boldsymbol{X}(t=t_0) = [\theta, \dot{\theta}, \varphi, \dot{\varphi}, \Lambda, \dot{\Lambda}] = [0.1, 0, 0.3, 0, 0.1, 0]$;

The weight matrix is chosen for the controller design:

$\boldsymbol{Q}$ = diag[1, 1, 100, 100, 1, 1]； $\boldsymbol{R}$ = diag[1, 0.1]

The error tolerances of the function iteration are set to 1e-7. If the error is satisfied at all LGR points , the iteration will be terminated. The Performance index could make the system energy to be optimal and make the system change smoothly.

Figure 2 shows the time history of the dimensionless control input of tether tension and thrust force in Case1 and Case2. Figure 2 indicates that in Case1 and Case2, the control input all mainly depends on the tether tension, which is controlled accurately and is smooth, while the thrust force is small. This hybrid control method could result in considerable energy savings. At the same time, it could also reduce the complexity of designing the sub-satellite structure.

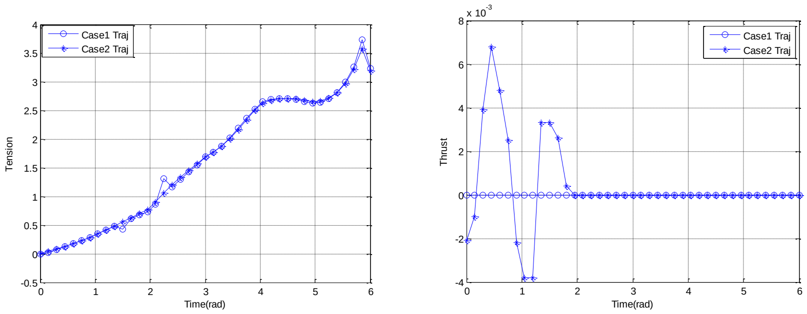
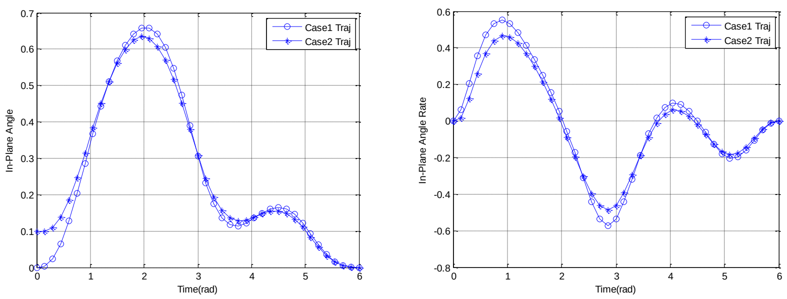


Figure 2. The Dimensionless Optimal Control Input

Figure 3.shows the time history of the dimensionless state variables under case1 and case2. The dimensionless states include in-plane libration angle, out-of-plane libration angle, length of the tether, in-plane angle velocity, out-of-plane angle velocity and tether motion velocity.

In case1, the state variables changed slightly, and out-of-plane angle and the velocities of the out-plane angles were always closed to zero. In case2, the effect of disturbance is added in the initial states. The differences of the moving trajectory between case1 and case2 are obvious in the initial stage, but with the proposed nonlinear optimal control schemes, the initial errors in states are damped extremely quickly. The trajectories in case1 and case2 converge to the almost same states after several sampling times. So to some extent, the control schemes can mitigate the influence of disturbances efficiently. Finally, these simulation results indicate the effect of the proposed control schemes seem to have a significant effect.



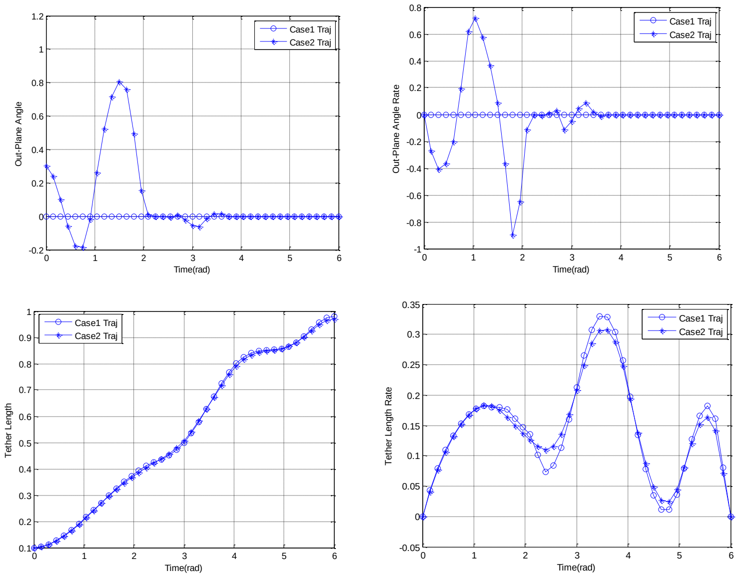


Figure 3. The Optimal Value of the Dimensionless State Variables

5. Conclusions

The paper presents an optimal control schemes for the deployment of a tethered sub-satellite under nonlinear constraints. The infinite-horizon optimal control problem is accurately approximated through LGR PS method. By a domain-transformation technology, the optimal solution on the entire infinite-horizon is obtained without any explicit integration or derivation of the necessary conditions. The control algorithm improving the computational efficiency and accuracy that makes if valid for real time operation. Numerical results show that very few optimization parameters are required to successfully deploy tether with high effectiveness and high accuracy under the proposed nonlinear optimal control schemes.

Acknowledgments

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