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Analysis of Nonlinear Vibration Behavior of the Spacecraft Cable with Coupling Constraints

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Abstract

In order to avoid the problem that cables are divorced from constraints during spacecraft launching, rules of nonlinear vibration behavior of laying cables need to be analyzed, so the common restriction condition of cables inside the spacecraft is selected to study. Based on the inertial coordinate system of cables, force on the cable element is analyzed with elastic mechanics, and then the vibration model of cables with coupled constraints can be obtained. And then, with the application of boundary conditions, the vibration model of spacecraft cables with longitudinal load is obtained, and vibration ordinary differential equations of cable with longitudinal load are obtained by using the Galerkin method. Further, the vibration response of cable with one end effected by longitudinal disturbance is obtained along with the nonlinear vibration model of cables with coupling constraints calculated and analyzed by using the numerical method. The results show that mechanic properties and the reliability of work of cables can be improved by using local constraints on the laying path of spacecraft cables properly.

Keywords: *spacecraft cables, mechanic modeling, coupling constraints, nonlinear vibration, simulation analysis*

1. Introduction

The cable is an important structure to transfer energy and information of the spacecraft. Nonlinear vibration of laying cables may be caused during spacecraft launching, which will cause the problem that cables may be divorced from constraints. Cables account for about 20%~30%[1] of the overall quality of the spacecraft, so unreasonable wiring will increase the failure probability of cable. In order to ensure the reliability and safety of the spacecraft, rules of nonlinear vibration behavior of laying cables need to be analyzed.

Irreparable consequences will be caused when any part malfunctions in the complex internal structure, and the cable is an important part of the spacecraft. The mathematical model consisting of nonlinear equations is established to seek the regularity. The linearization may cause great errors in the engineering, and even makes results of the analysis change away from nature itself, so it is necessary to study the nonlinear vibration behavior of the vibration system.

Many scholars study currently on the nonlinear characteristics of flexible rods, cables and shells with nonlinear vibration theory, but researches on nonlinear vibration characteristics of cables are very few. For example, the nonlinear modal and dynamic response of Euler-Bernoulli beams were studied by Nayfeh and his partners by using the method of multiple scales[2-3].The problems of internal resonances and

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combination resonances of the elastic shell[4]and the board[5]were analyzed by Fu Yi-ming and Zhang Si-jin with the principle of Hamilton. Zhao Yue-yu and Jiang Li-zhong[6] studied on the global bifurcation of dynamical systems of flexible sheets. Luo and Han[7] launched the study on the resonance conditions and the chaos phenomenon of the undamped nonlinear elastic rod with the Chirikov criterion. Zhao Yue-yu[8]did a series of researches on the rule of nonlinear vibration behavior of the elastic cable. Ishihara[9] analyzed natural frequencies and transient responses of the laminate made of fiber reinforcements by using the shear deformation theory. Nonlinear vibration response of the flat cylindrical shell with a concentrated mass by the action of gravity and the periodic excitation was studied by Nagai[10].However cables laying in the spacecraft produce nonlinear vibration easily with the spacecraft vibrating, and then the cable may be free from restraints or twining together to affect the normal work. We can see the fact that the rules of nonlinear vibration behavior of cables inside the spacecraft must be studied.

In summary, this paper takes cables with coupling constraints in the spacecraft as a research object. The unit displacement is described by using the inertial coordinate system, and nonlinear vibration equations of cables with coupling constraints are established. Further, the solution space of this model is transformed into finite dimension with the Galerkin method, and then the qualitative and quantitative analysis of nonlinear characteristics of the object can be launched. Then nonlinear vibration rules of cables with coupling constraints consisted of fixed ends, clamps and planes are analyzed, and results also provide a certain theoretical reference or the study on nonlinear vibration problems of other flexible bodies.

2. Mechanical Analysis and Modeling

The cable model is fixed at both ends as shown in Figure 1.

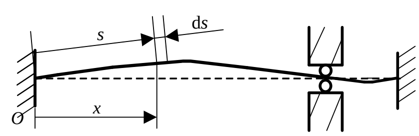

Figure 1. The Cable Model with Local Constraint

The force of the differential section ds is analyzed to establish kinetic equations of the model of cables. The formula is deduced based on the following assumptions:

1) Bending stiffness of cable is small enough that it can be ignored;

2) The axial strain of the micro segment is small enough that it is in the range of elasticity;

3) The cross section of cable is round;

4) Only consider the geometric nonlinearity.

Micro segment ds is taken as object of study, whose distance to the left end of the cable is x . The relationship between x and s is shown as Eq.1, where θ is bending angle of the micro segment:

$$ds = dx / \cos\theta \tag{1}$$

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The stress of the infinitesimal section is analyzed as shown in Figure 2, where the directions of the axis u and w are longitudinal and transverse motion directions respectively.

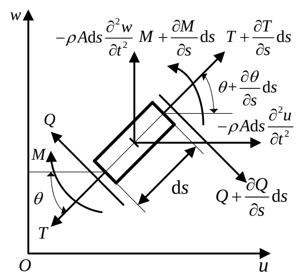

Figure 2. The Stress Analysis of the Infinitesimal Section under the Action of Axial Force

$T(x,t)$ is the axial force acting on section x , and the direction of $T(x,t)$ is along the tangent direction of the neutral layer after deformation. $M(x,t)$ and $Q(x,t)$ are the bending moment and shear force acting on section respectively. Without considering torsion of cables, quasi static relationship between the bending moment and shear force is shown asEq.2:

$$Q = \partial M / \partial s = (\partial M / \partial x)\cos\theta \tag{2}$$

According to the force analysis, the dynamic equilibrium equations are obtained:

$$\rho A\left(\partial^2 u / \partial t^2\right)\mathrm{d}s = \frac{\partial}{\partial s}(T\cos\theta + Q\sin\theta)\mathrm{d}s, \quad \rho A\left(\partial^2 w / \partial t^2\right)\mathrm{d}s = \frac{\partial}{\partial s}(T\sin\theta - Q\cos\theta)\mathrm{d}s \tag{3}$$

where ρ and A are density and cross-sectional area of cables.Eq.2 is substituted into Eq.3, then

$$\begin{cases} \rho A\left(\partial^2 u / \partial t^2\right)\mathrm{d}s = \dfrac{\partial}{\partial x}\left[T\cos\theta + (\partial M / \partial x)\cos\theta\sin\theta\right]\cos\theta \\ \rho A\left(\partial^2 w / \partial t^2\right)\mathrm{d}s = \dfrac{\partial}{\partial x}\left[T\sin\theta - (\partial M / \partial x)\cos\theta\sin\theta\right]\cos\theta \end{cases} \tag{4}$$

The trigonometric functions in Eq.4are expanded with Taylor's formulas, then

$$\sin\theta \approx \partial w / \partial x, \ \cos\theta \approx 1 - (\partial w / \partial x)^2 / 2 \tag{5}$$

The longitudinal displacement $\tilde{u}(x,z,t)$ of the point which is z away from neutral layer is analyzed, and it is composed of the following three parts as shown in Figure 3: the longitudinal translation $u(x,t)$ of cross section of cables, the displacement $z\theta(x,t)$ induced by rotation of cross section in plane, u_1 induced by bending of the cable element.

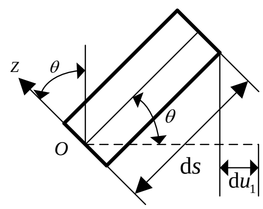

Figure 3. Displacement Caused by Transverse Bending

According to the geometric analysis as Figure 3 above, we can get $\mathrm{d}u_1 = (1 - \cos\theta)\mathrm{d}s$. After Eq.5 substituted into Eq.6, the geometric relationship $\tan\theta = \partial w / \partial x$ is introduced:

$$u_1 = \int_0^x \sqrt{1 + (\partial w / \partial x)^2}\,\mathrm{d}x - x \tag{6}$$

The longitudinal displacement of the point can be performed as follows:

$$\tilde{u}(x, z, t) = u + z\theta + \int_0^x \left[1 + (\partial w / \partial x)^2\right]^{1/2} \mathrm{d}x - x \tag{7}$$

Due to $\varepsilon(x, z, t) = \partial\tilde{u}/\partial s$ and combined withEq.1 and Eq.7, Eq.8 can be obtained, then

$$\varepsilon(x, z, t) = \left[\partial u / \partial x + z(\partial\theta / \partial x) + \sqrt{1 + (\partial w / \partial x)^2} - 1\right]\cos\theta \tag{8}$$

$\left[1 + (\partial w / \partial x)^2\right]^{1/2}$ is expanded with Taylor's formulas, then

$$\varepsilon(x, z, t) \approx \left[\partial u / \partial x + z(\partial\theta / \partial x) + (\partial w / \partial x)^2 / 2\right]\cos\theta \tag{9}$$

Within linear elastic range, the normal stress acting on the cross section of cables is $\sigma(x, z, t) = E\varepsilon(x, z, t)$. $T(x, t)$ and $M(x, t)$ can be obtained:

$$T(x, t) = \iint_A \sigma(x, z, t)\mathrm{d}A = EA\left[\partial u / \partial x + (\partial w / \partial x)^2 / 2\right]\cos\theta \tag{10}$$

$$M(x, t) = \iint_A \sigma(x, z, t) z\mathrm{d}A = EI(\partial\theta / \partial x)\cos\theta \tag{11}$$

where $I = I_z = \iint_A z^2\mathrm{d}A$ is inertia moment of section. According to $\tan\theta = \partial w / \partial x$, then

$$\partial\theta / \partial x = \cos^2\theta\left(\partial^2 w / \partial x^2\right) \tag{12}$$

Substituting Eq.12 in Eq.11, then

$$M(x, t) = EI\left(\partial^2 w / \partial x^2\right)\cos^3\theta \tag{13}$$

With Eq.5, Eq.10 and Eq.13substituted into Eq.4, we can obtain

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$$\left[\rho A\frac{\partial^2 u}{\partial t^2}-EA\frac{\partial^2 u}{\partial x^2}-EI\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right]-EA\left(1-2\frac{\partial u}{\partial x}\right)\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}-EI\frac{\partial^4 w}{\partial x^4}\frac{\partial w}{\partial x}+ 6EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3\frac{\partial w}{\partial x}+\left[\frac{3}{2}EA\frac{\partial^2 u}{\partial x^2}+\frac{25}{2}EI\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right]\left(\frac{\partial w}{\partial x}\right)^2=0 \tag{14}$$

$$\left[\rho A\frac{\partial^2 w}{\partial t^2}-EA\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}+EI\frac{\partial^4 w}{\partial x^4}-3EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3\right]-EA\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}-11EI\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial x} +\left[2EA\left(\frac{\partial u}{\partial x}-\frac{3}{4}\right)\frac{\partial^2 w}{\partial x^2}-3EI\frac{\partial^4 w}{\partial x^4}+\frac{21}{2}EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3\right]\left(\frac{\partial w}{\partial x}\right)^2=0 \tag{15}$$

3. Nonlinear Vibration Model of Coupling Constraints Cables with One End Under Disturbance

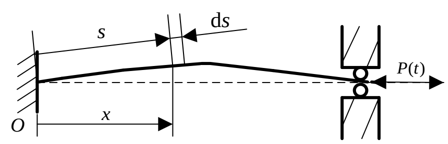

Figure 4. Model of Coupling Constraints Cables with One End Suffering Disturbance

The coupled model retains a small amount of nonlinear terms of a low order[11], or neglects the axial items and nonlinear terms of a higher order to ignore the longitudinal inertia effect[12].

The corresponding boundary conditions should be defined as:

$$u(0,t)=0,\quad EA\left[\partial u(l,t)/\partial x\right]=-P(t),\quad w(0,t)=\left[\partial^2 w(l,t)/\partial x^2\right]=0 \tag{16}$$

1) The situation of $P(t)=P_0\cos(\omega t)$

Eq.15can be transformed into

$$\rho A\frac{\partial^2 w}{\partial t^2}+P_0\cos(\omega t)\frac{\partial^2 w}{\partial x^2}+EI\frac{\partial^4 w}{\partial x^4}-3EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3-\left[11EI\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right]\frac{\partial w}{\partial x} -\left[2P_0\cos(\omega t)+\frac{3}{2}EA\right]\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2-3EI\frac{\partial^4 w}{\partial x^4}\left(\frac{\partial w}{\partial x}\right)^2+\frac{21}{2}EI\left(\frac{\partial^2 w}{\partial x^2}\right)^3\left(\frac{\partial w}{\partial x}\right)^2=0 \tag{17}$$

The dimensionless quantities are introduced[13], then

$$\begin{cases} U=u/l,\ W=w/l,\ X=x/l,\ \tau=\omega_0 t \\ \beta=P_0\cos\omega t/\left(\rho A l^2\omega_0^2\right),\ \lambda_1=E/\left(\rho l^2\omega_0^2\right),\ \lambda_2=EI/\left(\rho A l^4\omega_0^2\right) \end{cases} \tag{18}$$

where is the characteristic frequency. And thenEq.18is substituted into Eq.17, then

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$$\left[\frac{\partial^2 w}{\partial t^2}+\beta\frac{\partial^2 w}{\partial x^2}+\lambda_2\frac{\partial^4 w}{\partial x^4}-3\lambda_2\left(\frac{\partial^2 w}{\partial x^2}\right)^3\right]-\left[11\lambda_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right]\frac{\partial w}{\partial x}$$
$$-\left(2\beta+\frac{3}{2}\lambda_1\right)\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2-3\lambda_2\frac{\partial^4 w}{\partial x^4}\left(\frac{\partial w}{\partial x}\right)^2+\frac{21}{2}\lambda_2\left(\frac{\partial^2 w}{\partial x^2}\right)^3\left(\frac{\partial w}{\partial x}\right)^2=0 \tag{19}$$

After Eq.18 substituted into Eq.16, we obtain

$$u(0,t)=0,\quad \partial u(l,t)/\partial x=-\beta/\lambda_1,\quad w(0,t)=0,\quad \partial^2 w(l,t)/\partial x^2=0 \tag{20}$$

The lateral displacement of cable is used the following expression to describe.

$$w(x,t)=\varphi_2(x)q_2(t) \tag{21}$$

Substituting Eq.21in Eq.19, numerical integration is carried on within the range of $[0,l]$:

$$c_1\ddot{q}_2+c_2q_2+c_3q_2^3+c_4q_2^5=0 \tag{22}$$

where

$c_1=\int_0^l\varphi_2^2(x)\mathrm{d}x$, $c_2=\int_0^l\left\{\beta\left[\partial^2\varphi_2(x)/\partial x^2\right]+\lambda_2\left[\partial^4\varphi_2(x)/\partial x^4\right]\right\}\varphi_2(x)\mathrm{d}x$

$$c_3=-\int_0^l\left\{3\lambda_2\left[\partial^2\varphi_2(x)/\partial x^2\right]^3+11\lambda_2\left[\partial^2\varphi_2(x)/\partial x^2\right]\left[\partial\varphi_2(x)/\partial x\right]\left[\partial^3\varphi_2(x)/\partial x^3\right]\right.$$
$$\left.+(3\lambda_1/2+2\beta)\left[\partial^2\varphi_2(x)/\partial x^2\right]\left[\partial\varphi_2(x)/\partial x\right]^2+3\lambda_2\left[\partial^4\varphi_2(x)/\partial x^4\right]\left[\partial\varphi_2(x)/\partial x\right]^2\right\}\varphi_2(x)\mathrm{d}x$$

$$c_4=\frac{21}{2}\lambda_2\int_0^l\left[\partial^2\varphi_2(x)/\partial x^2\right]^3\left[\partial\varphi_2(x)/\partial x\right]^2\varphi_2(x)\mathrm{d}x$$

2) The situation of $P(t)=P_0$

Based on coupling motion equations of cables and with simplifying assumptions introduced, the Eq.14 can be simplified to the linear wave equation:

$$\rho A\left(\partial^2 u/\partial t^2\right)-EA\left(\partial^2 u/\partial x^2\right)=0 \tag{23}$$

The condition of constant force acting on the end of cables is taken into account, then

$$P(t)=P_0 \tag{24}$$

Due to invariant numerical values and directions of P_0, the following can be obtained:

$$T(x,t)\approx EA(\partial u/\partial x)=-P_0 \tag{25}$$

Given Eq.23，the following relationship can be shown as:

$$T(x,t)\approx EA(\partial u/\partial x)=-P_0 \tag{26}$$

Based on Eq.15，the mathematical model can be expressed as:

$$\partial T/\partial x=EA\left(\partial^2 u/\partial x^2\right)=0 \tag{27}$$

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With Eq.27dimensionless and Eq.18 substituted into Eq.27, the following expression is given:

$$\left[\frac{\partial^2 w}{\partial t^2}+\beta_0\frac{\partial^2 w}{\partial x^2}+\lambda_2\frac{\partial^4 w}{\partial x^4}-3\lambda_2\left(\frac{\partial^2 w}{\partial x^2}\right)^3\right]-\left[11\lambda_2\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial x^3}\right]\frac{\partial w}{\partial x}- \\ \frac{1}{2}(4\beta_0+3\lambda_1)\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2-3\lambda_2\frac{\partial^4 w}{\partial x^4}\left(\frac{\partial w}{\partial x}\right)^2+\frac{21}{2}\lambda_2\left(\frac{\partial^2 w}{\partial x^2}\right)^3\left(\frac{\partial w}{\partial x}\right)^2=0 \tag{28}$$

where $\beta_0 = P_0 / (\rho A l^2 \omega_0{}^2)$ is constant. Substituting Eq.21 into Eq.28, then

$$d_1 q_2 + d_2 q_2 + d_3 q_2{}^3 + d_4 q_2{}^5 = 0 \tag{29}$$

where

$d_1 = \int_0^l \varphi_2(x)^2 \mathrm{d}x$, $d_2 = \int_0^l \left[-\beta_0\left[\partial^2\varphi_2(x)/\partial x^2\right]+\lambda_2\left[\partial^4\varphi_2(x)/\partial x^4\right]\right]\varphi_2(x)\mathrm{d}x$

$$d_3 = -\int_0^l \Big\{3\lambda_2\left[\partial^2\varphi_2(x)/\partial x^2\right]^3+11\lambda_2\left[\partial^2\varphi_2(x)/\partial x^2\right]\left[\partial^3\varphi_2(x)/\partial x^3\right]\left[\partial\varphi_2(x)/\partial x\right]+ \\ (-4\beta_0+3\lambda_1)\left[\partial^2\varphi_2(x)/\partial x^2\right]\left[\partial\varphi_2(x)/\partial x\right]^2/2+3\lambda_2\left[\partial^4\varphi_2(x)/\partial x^4\right]\left[\partial\varphi_2(x)/\partial x\right]^2\Big\}\varphi_2(x)\mathrm{d}x$$

$$d_4 = 21\lambda_2\int_0^l\left[\partial^2\varphi_2(x)/\partial x^2\right]^3\left[\partial\varphi_2(x)/\partial x\right]^2\varphi_2(x)\mathrm{d}x/2$$

4. Analysis of Nonlinear Vibration of Cables

The specific parameters of cables refer to cables of American Raychem company, and the geometrical and physical parameters are shown in Table 1.

Table 1. The Cable Parameters

Geometrical and physical parameters	Parameter symbol	Numerical value
Cross-sectional area	A	$6\mathrm{mm}^2$
Material	-	silver-plated copper
Density	ρ	$8.9\mathrm{g/cm}^3$
Elastic modulus	E	$1.1\times10^5\mathrm{Mpa}$
Poisson's ratio	ν	0.31
Polar moment of inertia relative to z	I	$5.730\mathrm{mm}^4$

The vibration example of cables with local constraints during $P(t) = P_0\cos(\omega t)$:

The length of the cable between adjacent constraints is l. During $\varphi_1(x)=\sin(\pi x/l)$ and $P_0 = 4\mathrm{N}$, the time history response of laying cables is considered with $\omega=\omega_0$.Based on Eq.22, the following parameters can be obtained, then

$$\begin{cases} c_1 = l/2 \\ c_2 = l\left[\lambda_2 (\pi/l)^4 - \beta (\pi/l)^2\right]/2 \\ c_3 = \left[(3\lambda_1/2 + 2\beta)\, l\, (\pi/l)^4\right]/8 - \left[5\lambda_2 l (\pi/l)^6\right]/8, \\ c_4 = \left[357\lambda_2 (\pi/l)^8 l\right]/16 \end{cases} \tag{30}$$

Eq.18 andEq.30 are combined with parameters as shown in Table 1, then

$$\begin{cases} c_1 = 0.5l \\ c_2 = 6\pi^4/\left(\omega_0^2 l^7\right) - 37.5\pi^2 \cos(n\omega_0 t)/\left(\omega_0^2 l^3\right) \\ c_3 = 2.3\times 10^6 \pi^4/\left(\omega_0^2 l^5\right) + 18.7\pi^4 \cos(n\omega_0 t)/\left(\omega_0^2 l^5\right) - 7.3\pi^6/\left(\omega_0^2 l^9\right), \\ c_4 = 263\pi^8/\left(\omega_0^2 l^{11}\right) \end{cases} \tag{31}$$

Above all, $\omega_0 = 1.86\pi/l^4$ is obtained, and ω_0 is substituted into Eq.31 and Eq.22:

$$\begin{aligned} &0.5l\,\ddot{q}_2 + \left[1.73\pi^2 l - 1.08 l^5 \cos(1.86 n\pi t)\right] q_2 + \\ &\left[6.6\times 10^5 \pi^2 l^3 + 0.54\pi^2 l^3 \cos(1.86 n\pi t) - 2.1\pi^4/l\right] q_2^3 + 76\pi^6 q_2^5/l^3 = 0 \end{aligned} \tag{32}$$

During $n=1$, Eq.25 is solved with the method of Runge-Kutta of four or five order. The Figure 5 reflecting the vibration condition of cables with the end under perturbation can be obtained.

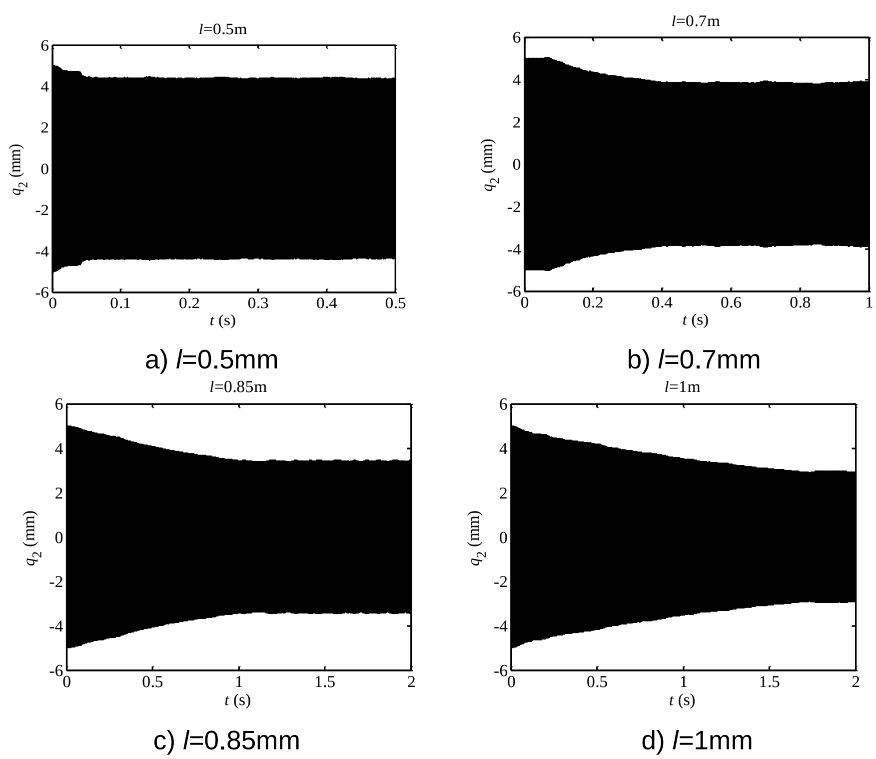

Figure 5. Response of Cables with the End Under Perturbation

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The Figure 5 reflects the vibration condition of cables with the end under perturbation. As the lengths of cables are $l = 0.5\text{m}, 0.7m, 0.85\text{m}, 1\text{m}$, the time of cables into the stable vibration are about $t = 0.05\text{s}, 0.4\text{s}, 1\text{s}, 1.75\text{s}$, and the amplitudes of vibration of cables are $4.1\text{mm}, 3.8\text{mm}, 3.4\text{mm}, 3\text{mm}$ respectively. In the case of the same interference, the amplitude of the system vibration decreases with the increasing length of the cable.

The longitudinal vibration example of cables with local constraints during $P(t) = P_0$:

Eq.14 is simplified, then

$$\partial^2 u \big/ \partial t^2 - a^2 \left(\partial^2 u \big/ \partial x^2\right) = 0, \quad a = \left(E \big/ \rho\right)^{1/2} \tag{33}$$

Numerical method is used to obtain the approximate solution. Interval values of time and space are Δt and Δx respectively, and then node vibration position can be shown as:

$$u_j^i = u\left(j\Delta x, i\Delta t\right) \tag{34}$$

By analogy to Eq.34, the following values of vibration position can be expressed as

$$\begin{cases} u_j^{i-1} = u\left[j\Delta x, (i-1)\Delta t\right], & u_j^{i+1} = u\left[j\Delta x, (i+1)\Delta t\right] \\ u_{j-1}^i = u\left[(j-1)\Delta x, i\Delta t\right], & u_{j+1}^i = u\left[(j+1)\Delta x, i\Delta t\right] \end{cases} \tag{35}$$

Eq.33 is discredited with the central difference method, then

$$\partial^2 u \big/ \partial t^2 \approx \left(u_j^{i+1} - 2u_j^i + u_j^{i-1}\right) \Big/ \left(\Delta t\right)^2, \quad \partial^2 u \big/ \partial x^2 \approx \left(u_{j+1}^i - 2u_j^i + u_{j-1}^i\right) \Big/ \left(\Delta x\right)^2 \tag{36}$$

Eq.36 is substituted into Eq.33, then

$$\left(u_j^{i+1} - 2u_j^i + u_j^{i-1}\right) \Big/ \left(\Delta t\right)^2 - a^2\left(u_{j+1}^i - 2u_j^i + u_{j-1}^i\right) \Big/ \left(\Delta x\right)^2 = 0 \tag{37}$$

Based on Eq.37, Eq.38 can be obtained:

$$u_j^{i+1} = c(u_{j+1}^i + u_{j-1}^i) + 2(1-c)u_j^i - u_j^{i-1}, \quad c = a^2\left[(\Delta t)^2 \big/ (\Delta x)^2\right] \tag{38}$$

The difference format of three layers and five points is shown as Figure 6.The longitudinal vibration behavior is analyzed with the finite difference method of five points.

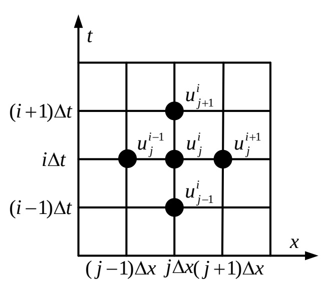
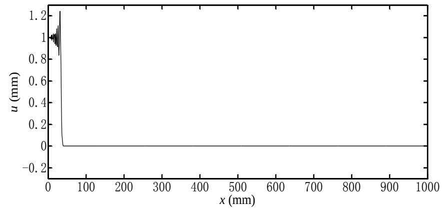
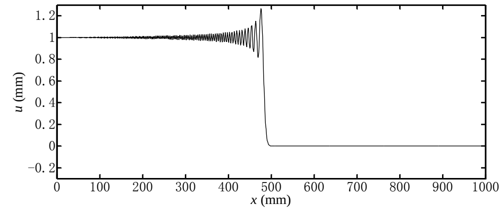


Figure 6. The Difference Format of Three Layers and Five Points

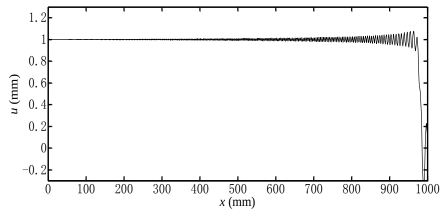
The problem of the zero initial condition of cables is studied. The length of cables between the fixed end and the clamp is 1000mm , and then Eq.23 is solved with the numerical calculation. Select four moments:



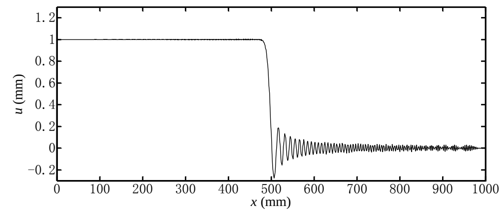
a) The First Moment



b) The Second Moment



c) The Third Moment



d) The Fourth Moment

Figure 7. The Longitudinal Vibration Response of Cables

From Figure 7, we can see that the response value with the end of the cable under a constant force is 1, and this value is passed to the direction of fixed end with the medium of the cable point by point. When the vibration spreads to the fixed end of the cable, the original vibration is eliminated by the fixed constraint, and the effect of the opposite direction is produced to lead to reciprocating motion for some time.

In conclusion, in order to improve the working reliability of cables, behavior rules of non-linear vibration of laying cables must be analyzed with considering two kinds of situations of the longitudinal disturbing force as the variable force and a constant. By means of numerical simulation, with different positions of hoop constraints and different lengths of cables, vibration condition so cables with the end under the disturbance are different.

5. Conclusion

(1) Aiming at the nonlinear vibration of cables, the inertial coordinate systemic established to describe the cable shape. The cable model whose wiring path passes through the clamp constraints and fixed at both ends is built with the method of elasticity, and it represents the coupling effect of the transverse vibration and the longitudinal vibration of cables.

(2) Based on the vibration model of the cable with local constraints and under longitudinal disturbance, the boundary conditions are introduced to obtain the vibration model of cables in this condition. The differential vibration model is obtained with the Galerkin method, and then time and space are handled discretely by using the finite difference method.

(3) The simulation results show that the vibration of cables occurs under the outer longitudinal disturbance, and the vibration condition of cables is eventually stable over time. The conclusions can provide more complete dynamic models for other studies on the object similar to cables.

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