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LQ-Servo Design with Reduced Order Observer for Two-wheeled Self-balancing Robot based on HIL Simulation

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Abstract

In this paper, an LQ-servo design method based on reduced order observer is proposed in order to control the two-wheeled self balancing robot. We employ reduced order observer, which estimates the states that cannot be accessible by the direct measurements from sensors in order to make its implementation on real time digital processor simpler and cost-effective. The performance index of LQ-servo controller is determined such that overshoot and oscillation of the controlled system in time domain are minimized. The effectiveness and performance of the proposed control system are shown via numerical simulations. It is also shown from hardware-in-loop simulator that the proposed design for self balancing robot reflects the real time behavior quite well.

Keywords: *Two-wheeled self-balancing robot, LQ-servo, Reduced-order observer, HILS*

1. Introduction

Recently, the two-wheeled self-balancing (TWSB) called "Segway" robots have gained much attention as a potential means for a personal transportation and a playing instrument. Since the inverted pendulum dynamics of TWSB exhibit the nonlinearity as well as instability, it is crucial to design a robust controller that can guarantee the outstanding system performance and stability [1-3].

Recently, various classic and modern control methods such as linear quadratic regulator (LQR), PID, and pole-placement controller have been applied [2, 4-6] Especially, LQR optimal controller has the advantages in that it can guarantee high stability margin and achieve the optimality in terms of quadratic performance index. However, it is not appropriate to apply it to the tracking problem due to nontrivial steady state error [5, 6]. While the conventional LQR method guarantees the robust stability and optimal performance, it costs high expenses to measure or estimate all states when employing sensors or full-order observer. Indeed, the design of the full order observer such as Kalman Filter or Luenberger observer would be too complex to realize due to their high order dynamics [7].

In this paper, we propose the LQ-servo optimal controller with the reduced-order observer. The reduced-order observer allows us to decrease the number of high cost sensors, thereby leading to lower implementation cost. We use LQ-servo with an integrator that has not only zero steady state error but also ensure robust stability and optimal performance inherited from LQR. There have been several existing control

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systems employing the LQ-servo. In [8], the integrator state is augmented with the nominal system while the outputs are set as new states. In [9], reduced-order observer is applied in order to estimate the remaining states other than the outputs, assuming that sensors are sufficiently reliable to measure output from noise.

The main goal of this work is to reduce the complexity for acquiring full state observations while maintaining performance of LQR method. In reduced order observer, the states that cannot be easily measured are estimated without using sensors. The LQ-servo control gain is determined in order to meet time domain specifications of TWSB. The weighting factors of the quadratic performance index are obtained by solving optimization problem expressed as algebraic Riccati equation (ARE) which accounts for the overshoot and settling time of controlled output [8].

Two kinds of simulators, PC-based and FPGA-based, for the numerical simulations are implemented to show the effectiveness of the proposed control system [10]. We numerically test our controller using hardware-in loop simulator (HILS) to verify the proposed control method before it is embedded to the real hardware of TWSB robots [11].

The main contributions of the proposed method are summarized as follows:

(i) The proposed method provides the easy optimal tracking control method using the reduced order observer without any complicated filtering of the signals obtained by full sensors.

(ii) We show that the performance index Q and R can be determined by considering the specific time domain performances of transient response such as overshoot, rising time and settling time.

(iii) The proposed LQ-servo controller has no steady state error and stability margin as like the general LQR control.

(iv) HILS test is performed to verify the control method via the real-time discrete simulator.

The rest of the paper is organized as follows. In Section 2, we briefly introduce the dynamics of two wheel balancing robot as nonlinear state space equation and linearized model. The design methodologies for the proposed controller are discussed in sub section, separating the LQ-servo with integrator and reduced-order observer in Section 3. Numerical simulation is performed by PC-based for computer simulator and FPGA-based real-time discrete event simulator in order to show the effectiveness of the proposed method in Section 4. Finally, conclusions of the proposed method are summarized in Section 5.

2. Model Description of the Two-Wheeled Self-Balancing Robot

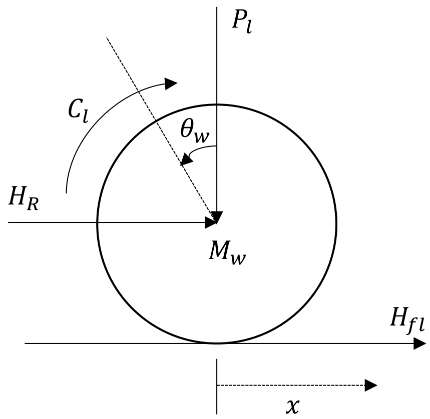

Figure 1. Description of the Coordinate Systems and Angles

In this chapter, we briefly introduce the system description and mathematical model of TWSB robots. Conceptually, the readers can easily understand the dynamics of the system, considering a modern adaption of the inverted pendulum which has an up-side-down mechanism. It consists of two parts, the revolver and the car body force analysis. The wheel dynamics of self-balancing robot are depicted in Figure 1, where the parameters are listed at table 1 [6].

Table 1. Parameters of TWSB Robots

Symbol	Physical Meaning	Units
C_R and C_L	Right and left wheel torque	Nm
H_L and H_R	Z axis forces of the left and right wheels	N
H_{fR} and H_{fL}	Interatomic forces of the right and left wheels	N
θ_w	Angle of wheel around the Z axis direction	rad
θ_p	Angle of car body on Z axis direction	rad
M_w	Weight of the wheel	kg
M_p	Weight of the car body	kg
I_w	Inertia moment of the wheel	$kg \cdot m^2$
I_p	Inertia moment of the car body	$kg \cdot m^2$
R	Radius of the wheel	m
l	the height of the car body	m

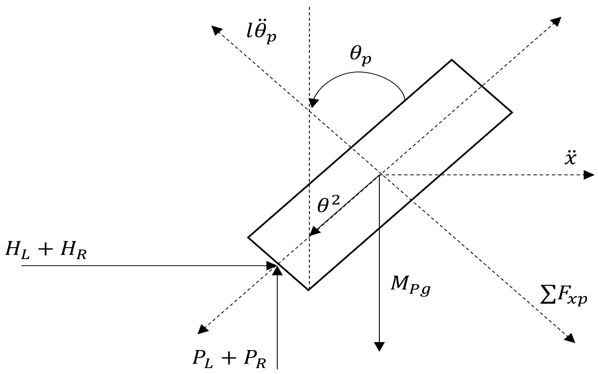

Figure 2. The Force of the Body

According to Newton's law and the rotational torque formula, the two wheel force equations are as follows:

$$M_w\ddot{x} = H_{fR} - H_R$$

$$I_w\ddot{\theta}_w = C_R - H_{fR} \tag{1}$$

The right wheel force equation is as follows:

$$M_w\ddot{x} = H_{fL} - H_L$$

$$I_w\ddot{\theta}_w = C_L - H_{fL}R \qquad (2)$$

Total force equation is summarized as follows :

$$2\left(M_w + \frac{I_w}{R^2}\right)\ddot{x} = \frac{C_R + C_L}{R} - (H_R + H_L) \qquad (3)$$

Figure 2 describes the force dynamics of the balancing body, where the parameters relative to the car body are listed on table 1. Using Newton's second law, the horizontal and vertical direction forces are respectively modeled as follows:

$$psin\theta p{-}Ml\theta pcos\theta p{=}Mp(x{+}lsin\theta p) \qquad (4)$$

$$p)cos\theta p \quad (5)$$

Above nonlinear dynamic model should be linearized in order to implement the optimal control method to the system. In this paper, the angular states θ_p and θ_w are linearized as following equation at the operating point (0,0) such that $\theta_p^2 \approx 0$, $sin\theta_p \approx \theta_p$, and $\cos\theta_p \approx 1$

$$\left(M_p + 2M_w\frac{I_w}{R^2}\right)\ddot{x} = \frac{C_R + C_L}{R} - 2M_p l\theta_p,$$

$$(2M_p l^2 + I_p)\ddot{\theta}_p = M_p Gl\theta_p - M_p l\ddot{x}. \qquad (6)$$

The output torque of the wheel is given by $C_R = C_L = I_R(dw/dt) = (k_m/R)U_a - (-k_m k_e/R)\dot{\theta}_w$;

Based on the dynamics shown above, the linearized state-space formulation is written by

$$\begin{bmatrix} \dot{x} \\ \dot{\theta}_p \\ \ddot{x} \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{M_p^2 g l^2}{A} & \dfrac{2k_m k_e(M_p lr - M_p l^2)}{Rr^2 A} & 0 \\ 0 & \dfrac{M_p g l\beta}{A} & \dfrac{2k_m k_e(rB - M_p l)}{Rr^2 A} & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_p \\ \dot{x} \\ \dot{\theta}_p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{2k_m k_e(I_p + M_p l^2)}{RrA} \\ \dfrac{2k_m(M_p l - rB)}{RrA} \end{bmatrix} U_a \quad (7)$$

where $A = I_p\beta + 2M_p l^2(M_w + (I_w/r^2))$, $B = (2M_w + (2I_w/r^2) + M_p)$.

The output equation is given by

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_p \\ \dot{x} \\ \dot{\theta}_p \end{bmatrix} \quad (8)$$

The outputs, (x, θ_p), are measured by position and encoder sensors, respectively.

3. LQ-Servo Design with Reduced-Order Observer

In this chapter, we explain the LQ-servo design method based on the information of all states obtained from the reduced-order observer. LQR optimal control method is very popular in control engineers due to their robust stability and optimal performance. However, it is not appropriate to tracking problem since LQR doesn't guarantee zero steady state error. We develop the LQ-servo design with integrator in order to have no steady state error with respect to step reference. LQ-servo requires full information about states as LQR controllers such that it needs to implement full-order observer or many sensors. In this paper, reduced-order observer is applied in order to estimate only stats not to be sensed under the assumption that the obtained outputs are reliable from noise. We present the design method and structure of LQ-servo controller and reduced-order observer.

3.1. LQ-Servo Controller Design

The structure of the LQ-servo with integrator is described in Figure 3. As shown in Figure 3, Equation (9), the augmented system consists of two parts, plant and integral elements of dynamics

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) \\ y_p(t) = C_p x_p(t) \end{cases} \tag{9}$$

where $x_p(t) = \begin{cases} y_p(t) \\ x_r(t) \end{cases}$, $C_p = [I \quad 0]$, $\dot{z}_p(t) = I y_p(t)$, and $z_p(s) = \frac{I}{s} y_p(s)$.

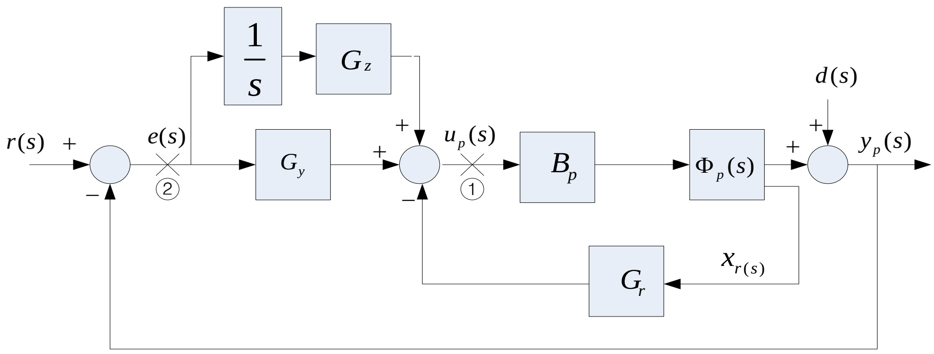

Figure 3. Block Diagram of the LQ-Servo Controller

$z_p(t)$ and $y_p(t)$ mean the plant output and integral state. The augmented state vector $x(t)$ is defined as follows:

$$x(t) = \left[z_p(t) \quad y_p(t) \quad x_r(t) \right]^T \tag{10}$$

The augmented state equation of the system is expressed as a new state-space equation as follows:

$$\dot{x}(t) = Ax(t) + Bu_p(t) \tag{11}$$

where $A = \begin{bmatrix} 0 & C_p \\ 0 & A_p \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ B_p \end{bmatrix}$.

If the controllability matrix $[A_p \quad B_p]$ is controllable, then$[A \quad B]$is also controllable. In the first place, control input $u_p(t)$ is obtained from LQ regulator and LQ servo to incorporate integral control elements.

The linear feedback control law to regulate the state, $x(t)$, is obtained as follows:

$$u_p(t) = -Gx(t), \tag{12}$$

$$G = \rho^{-1}B^T K, \tag{13}$$

where the optimal gain matrix, $K = K^T$, is obtained by solving the ARE:

$$KA + A^T K + Q - \frac{1}{\rho} KBB^T K = 0. \tag{14}$$

The input $u_p(t)$ is defined as follows :

$$u_p(t) = -G_z z_p(t) - G_y y_p(t) - G_r x_p(t) \tag{15}$$

The control gain K is determined in order to minimize the quadratic optimal performance,

$$J = \frac{1}{2}\int_0^\infty \left(x(t)^T Q x(t) + \rho u(t)^T u(t) \right) dt \tag{16}$$

The prime goal of LQ-servo controller is to set the angle to be zero and minimize the overshoot, settling time of the angle at relative time elapsed, and the distance traveled for the two-wheeled self-balancing robot. Note that Q and R matrices and the location of the poles are determined in LQ-servo control system to meet these time domain specifications.

3.2. Reduced-Order Observer Design

Generally, one argument against state-feedback and observer controllers is that they have high order. Since the full-order observer includes a system model, the observer has the same order of the open-loop system at least. In this paper, we implement reduced-order observer in order to decrease the order of the controller. The key observation is that the outputs obtained by sensors include the information about states.

Let the system equations be

$$\begin{bmatrix} \dot{y}_p(t) \\ \dot{x}_r(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_p(t) \\ x_r(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_p(t) \tag{17}$$

where the first states mean the output from the sensors. Because we measure the outputs, the observer needs to estimate only the remaining state, $x_r(t)$.

The reduced order observer has the structure of Figure 4.

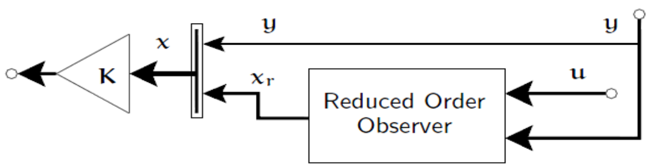

Figure 4. Block Diagram of the Reduced-Order Observer

The state, $y_r(t)$, can be considered a "virtual" output of a reduced order state equation as follows :

$$y_r(t) \triangleq A_{12}x_r(t) = \dot{y}(t) - A_{11}y(t) - B_1 u_p(t) \tag{18}$$

Then the observer required to estimate the remaining state, $x_r(t)$, can be represented as

$$\dot{\hat{x}}_r(t) = A_{22}\hat{x}_r(t) + A_{21}y(t) + B_2u_p(t) + L_r\big(y_r(t) - A_{12}x_r(t)\big) \qquad (19)$$

It can be guaranteed that the estimated state, $\hat{x}_r(t)$, is asymptotically converged to $x_r(t)$ if observer gain L_r makes $(A_{22} - L_rA_{12})$ Hurwitz. The detail structure of the reduced-order observer is shown in Figure 5.

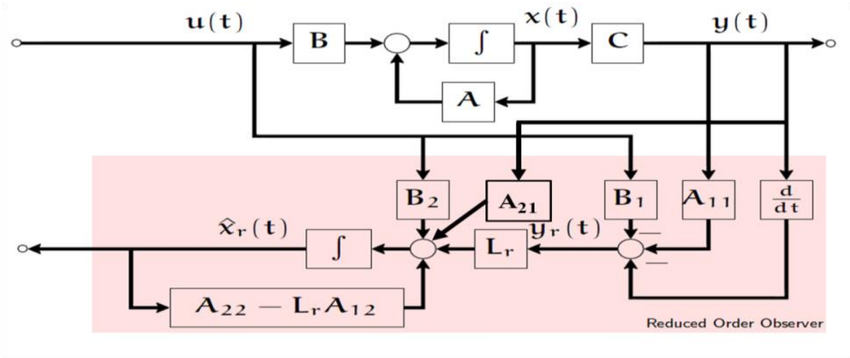

Figure 5. Delicate Structure of the Reduced-Order Observer

It can generally be a problem to differentiate the output signal in practice since it is very sensitive to noise. Therefore, we eliminate the need to differentiate the output by block diagram algebra, shown in Figure 6.

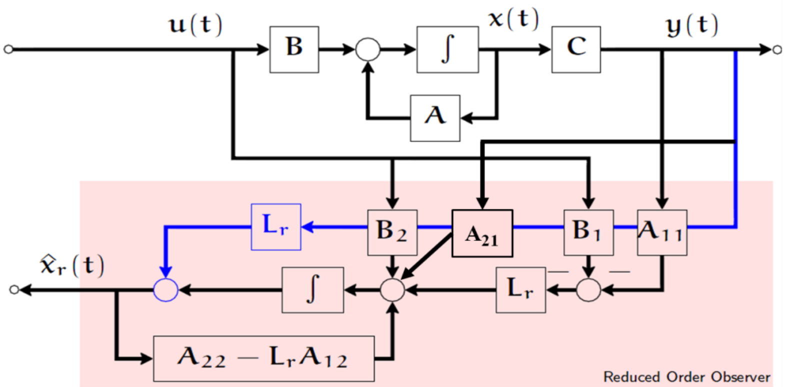

Figure 6. Structure of the Reduced-Order Observer without Differential Output

We design the observer gain by pole-placement method to locate all poles of the observer on the desired place in order to minimize the effect from sensor noise.

The controlled system with reduced-order observer is represented in Figure 7, which in the control gain, *G*, of LQ-server is determined by ARE to meet the time domain specification and the observer gain, L_r, of reduced-order observer is obtained by pole-placement method.

4. Simulation

The computer simulations have been performed by the off-line simulator Matlab/Simulink and the real-time RT-Lab simulator considering the effect of sensor noise. The mathematical model of the TWSB systems has the parameters shown on the table 2.

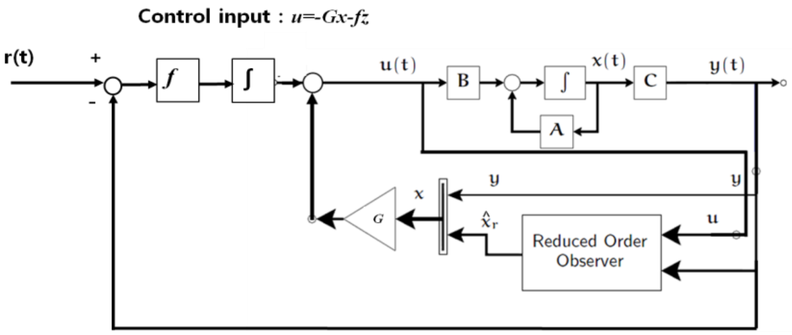

Figure 7. Structure of the Controlled System by LQ-Servo and Reduced-Order Observer

Table 2. The Symbol and Actual Values of the Parameters

Symbol	Actual value
k_m	0..0136 Nm / A
k_e	0.01375 $V / (rad / s)$
M_p	0.52 kg
M_w	0.02 kg
L	0.16 m
I_p	0.038 $kg \cdot m^2$
G	9.8 m / s^2
I_w	0.0032 $kg \cdot m^2$
R	0.025 m

According to Equation (7), the state-space equation is obtained as follows :

$$\begin{bmatrix} \dot{x} \\ \dot{\theta}_p \\ \ddot{x} \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.4848 & -0.0402 & 0 \\ 0 & 4.7469 & 0.4992 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_p \\ \dot{x} \\ \dot{\theta}_p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.0730 \\ -0.9077 \end{bmatrix} U_a \quad (20)$$

The initial condition of the system is set to $\begin{bmatrix} x & \theta_p & \dot{x} & \dot{\theta}_p \end{bmatrix} = [20 \quad 20 \quad 20 \quad 20]$. x and θ_p are the measured outputs from the sensors, while $\dot{x}$ and $\dot{\theta}_p$ are estimated by the reduced order estimator. The reduced-order observer gain L_r is determined as $\begin{bmatrix} 0.196 & 0 \\ 0.499 & 3 \end{bmatrix}$ in order to locate the poles of the observer on $(-3, -0.2366)$ for Hurwitz. After the reduced order observer is initialized, LQ-servo is designed from ARE to minimize the quadratic performance index with Q and R.

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}, \quad R = 1.7682 \tag{21}$$

We found Q and R matrix that generated the time domain performance satisfying the time specifications including - low overshoot and minimum settling time. In addition, we optimize low oscillation and prompt evaluation such that the controlled system has the system poles on $(-6.99, -2.21, -0.346 \pm 0.44j)$ in the left half plane.

Figure 8 shows that the settling time of the controlled angle and position states set at 8 seconds, satisfying the specification.

Figure 9 and 10 illustrate velocity and angle rate at relative time elapsed. Note that Figure 9 shows the estimated velocity and angle rate to follow the real states in Figure 10. Based on the outputs and estimated states, the system results as well-controlled response in time domain.

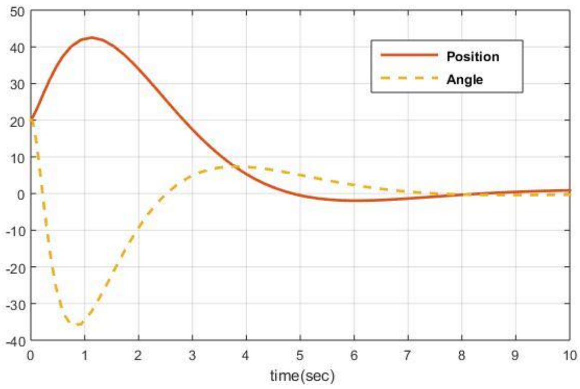

Figure 8. Outputs of the Simulation

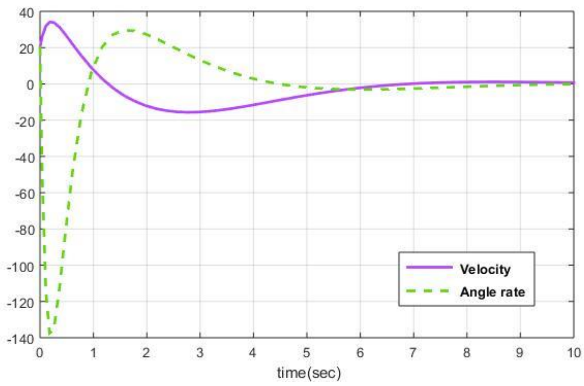

Figure 9. Estimation of the Velocity and Angle Rate

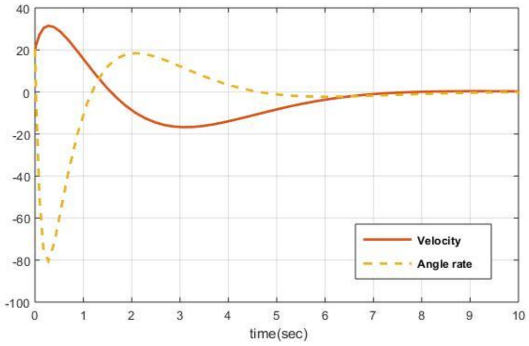

Figure 10. Real Signal of the Velocity and Angle Rate

PC-based simulator with windows flat form may cause the problem such as overrun, time-consuming, precision in the process to discretize the TWSB robot model. In this paper, we apply the real-time discrete simulator, RT-LAB in order to verify the proposed control method by FPGA-based HILS. The real TWSB robot and control logic are realized on FPGA-based flat form and the main PC has user console to show the result to user. The general structure of HILS is illustrated in Figure 11 [10, 11].

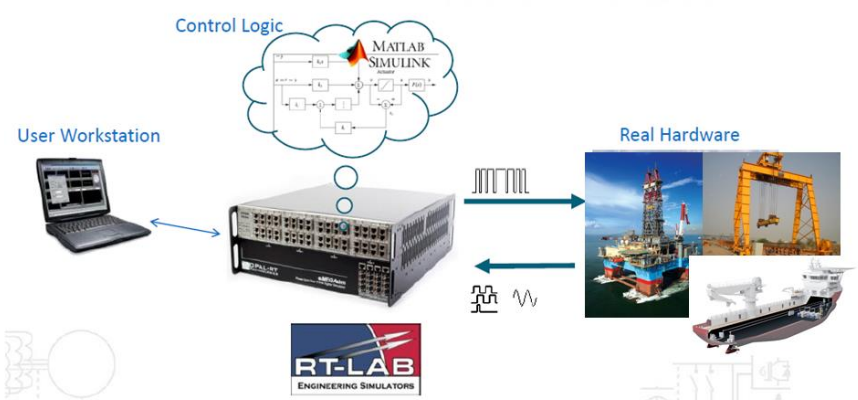

Figure 11. General Structure of HILS Systems

We applied the LQ-servo control gain and reduced-order observer gain to the system plant with the same parameters. In HILS test, the output noise and input disturbance are considered in order to verify the effectiveness of the proposed controller and estimator against from exogenous inputs.

Figure 12 shows the time responses of the position and angle by HILS test. This result has a difference response time, in which position settling time is slower than PC-based simulation without noises. But angle state has the stable aspect in spite of the effect of noise after 8sec.

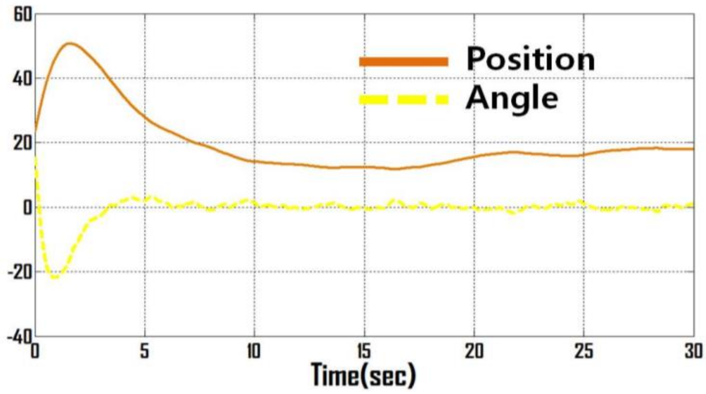

Figure 12. Time Responses of Position and Angle by HILS Test

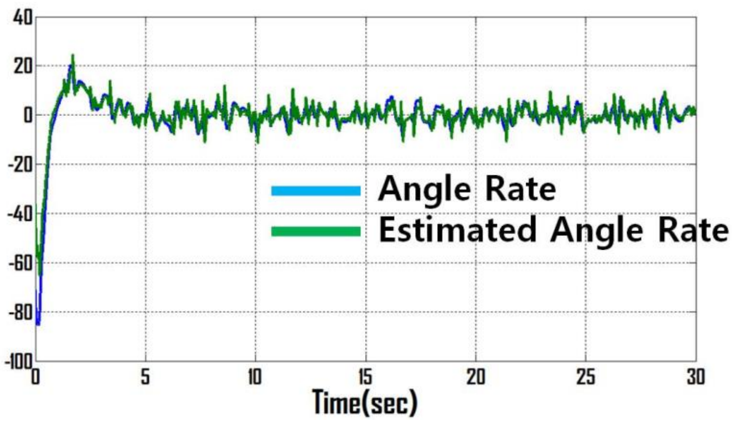

Figure 13. Comparison between Real Velocity and Estimated Velocity

The performance of the design observer can be analyzed through comparison between the characteristics of the real states and estimated states with respect to the position and angle velocities in fig. 13 and 14.

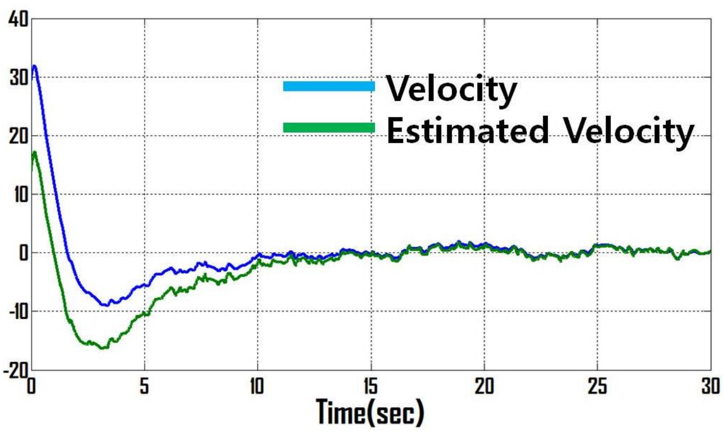

Figure 14. Comparison between Real Angle Rate and Estimated Angle Rate

The estimated velocity has the good state following after 15sec such that the estimator reflects well the real plant. The estimated angle velocity state immediately follows the real signal, shown in Figure 14.

5. Conclusion

We have developed the LQ-servo control method with reduced order observer for the two-wheeled self-balancing robot. The control gain of LQ-servo with integrator has the advantages to minimize the quadratic performance index with Q and R and have no steady-state error with respect to step response different from general LQ-servo and LQR optimal control method. In this paper, the performance weight factors Q and R are determined in order that the position angles of the robot meet the time domain specifications. The proposed method obtains the full state information from reduced order observer since the reduced order observer estimates only the immeasurable states except the outputs obtained by sensors, assuming that the outputs are reliable from noises. Consequently, it is expected by the proposed control method that the cost for the sensor installation or high order filter design constituting great proportion of the two-wheeled self-balancing robot can be reduced without the loss of performance compared to the traditional LQR technique.

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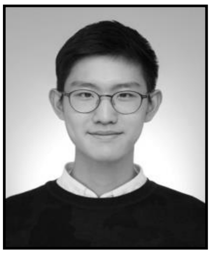

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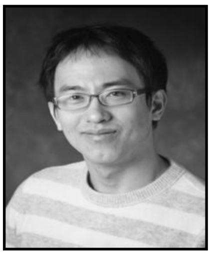

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