International Journal of Control and Automation
Vol.8, No.12 (2015), pp.175-186
http://dx.doi.org/10.14257/ijca.2015.8.12.16

An Efficient Adaptive Monopulse Estimation Algorithm Based on MPASTd Approach

Jia Yu[1*], Mingwei Shen[1], Di Wu[2] and Daiyin Zhu[2]

[1]*College of Computer and Information Engineering, Hohai University, Nanjing, 211100, China*

[2]*Key Laboratory of Radar Imagine and Microwave Photonics (Nanjing University of Aeronautics and Astronautics), Ministry of Education, Nanjing, 210016, China*

[*]*yujiahhu@126.com*

Abstract

In this paper, an improved adaptive monopulse scheme based on modified projection approximation subspace tracking deflation (MPASTd) approach is proposed. Owing to the fast convergence speed of MPASTd for recursively estimating the interference subspace, the proposed algorithm can obtain the formula for adaptive monopulse ratio calculation by a subspace projection. Therefore, it is able to accurately estimate the source location using less training data and reduce the computational complexity significantly. The effectiveness and fast convergence speed of the proposed method is verified by simulation results.

Keywords: *Interference suppression, Adaptive monopulse, Subspace estimation, DOA estimation*

1. Introduction

The monopulse is a well established technique to obtain the target angle for radars high precision tracking, but its performance will be severely degraded in the presence of interference. Adaptive monopulse tracking system can produce a beam according to the target location adaptively while eliminating the external influence and thus becomes an effective angle estimation method. Davis, Brennan and Reed [1] were first to propose the adaptive monopulse technique and derive three different adaptive monopulse formulas based on the maximum likelihood estimation for the case of a linear array. Nickel derived the adaptive monopulse formula based on the radar scanning power function, and generalized it to arbitrary planar or volume arrays of arbitrary structure and then presented the correction formula of monopulse ratio and slope value [2, 3]. Performance analysis of the adaptive monopulse has been described in [4, 5]. However, the performance of adaptive monopulse angle measurement may decrease significantly under the condition of small number of training samples, due to the poor estimation of the interference plus noise covariance matrix which is ultimately estimated by making use of L training samples. According to Reed's rule [6], to achieve good performance, L has to be larger or at least equal to $2N$, where N is the dimension of the input vector. But in an operational situation, this condition is difficult to satisfy. One way to overcome this is to reduce the dimension, which means that the subspace projection methods [7, 8] can be introduced into the adaptive monopulse technique to improve the performance.

In this paper, we will show that implementing a subspace-based approach in conjunction with the adaptive monopulse algorithm can improve the convergence speed. Moreover, we will then present a fast implementation of the subspace-based approach that reduces the computational complexity load.

ISSN: 2005-4297 IJCA

The rest of this paper is organized as follows. Section 2 summarizes the principle of adaptive monopulse algorithm. The proposed MPASTd-based subspace method for adaptive monopulse is presented in Section 3. First, we briefly introduce the eigencanceler, and give the weight vector based on eigen-decomposition (ED). Then the improved adaptive monopulse scheme based on MPASTd is introduced in detail. In Section 4, the computer simulation results are given to demonstrate that the proposed approach outperforms the original adaptive monopulse while reducing the computational complexity and the convergence time. A brief conclusion is given in Section 5.

2. The Principle of the Adaptive Monopulse Estimation

Consider an airborne radar system utilizing an N isotropic elements array with uniform element spacing d . We wish to estimate the target angle from a single data snapshot $\boldsymbol{z}$, which is the complex output of the array. The data $\boldsymbol{z}$ consists of the target $\boldsymbol{a}_u b$ plus the receiver interference and noise $\boldsymbol{n}$ which is assumed to be uncorrelated with the target signal. That is

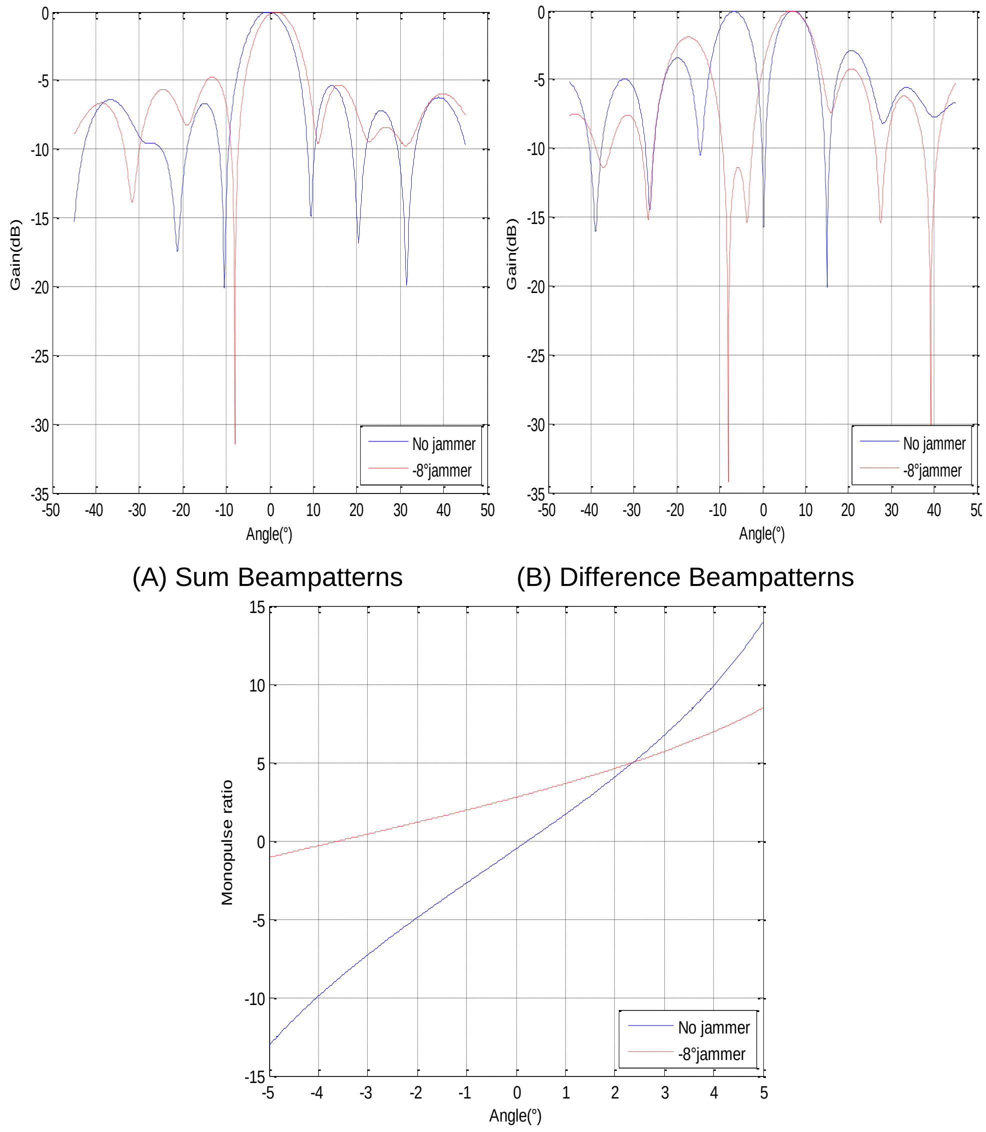
$$\boldsymbol{z} = \boldsymbol{a}_u b + \boldsymbol{n} \tag{1}$$

where u describes the unknown direction of the target and could be the sine of the angle of the incidence, *i.e.*, $u = \sin\theta$. $\boldsymbol{a}_u = [1 \;\; e^{j\frac{2\pi d}{\lambda}u} \cdots \; e^{j\frac{2\pi d}{\lambda}(N-1)u}]^T$ is the output of the array from the direction u , and b denotes the complex amplitude. The superscript ^{T} denotes transposition and λ is the signal wavelength. We assume that the interference and noise is Gaussian distributed with zero mean and covariance matrix $\boldsymbol{Q}$.

The adaptive monopulse processor uses the sample matrix inversion (SMI). To give a quantitative analysis, the outputs of the adaptive sum and difference beams can be defined as $f_\Sigma = \boldsymbol{w}_\Sigma^H \boldsymbol{z}$ and $f_\Delta = \boldsymbol{w}_\Delta^H \boldsymbol{z}$, where $\boldsymbol{w}_\Sigma = \gamma \boldsymbol{Q}^{-1}\boldsymbol{s}_\Sigma$ and $\boldsymbol{w}_\Delta = \gamma \boldsymbol{Q}^{-1}\boldsymbol{s}_\Delta$ represent the adapted sum and difference weight vectors respectively. γ is an arbitrary complex constant and the superscript ^{H} denotes Hermitian transposition. $\boldsymbol{s}_\Sigma$ and $\boldsymbol{s}_\Delta$ are the conventional sum and difference weights, where $\boldsymbol{s}_\Sigma$ is the steering vector in the look direction u_0 , *i.e.*, $\boldsymbol{s}_\Sigma = \boldsymbol{a}_{u_0} = [1 \;\; e^{j\frac{2\pi d}{\lambda}u_0} \cdots \; e^{j\frac{2\pi d}{\lambda}(N-1)u_0}]^T$ and $\boldsymbol{s}_\Delta$ is the derivative of the steering vector, *i.e.*, $\boldsymbol{s}_\Delta = \boldsymbol{a}_{u,0} = \left.\frac{d\boldsymbol{a}_u}{du}\right|_{u=u_0}$. Therefore, the monopulse ratio of the adaptive sum and difference beam outputs is

$$r_u = \mathrm{Re}\left\{\frac{f_\Delta}{f_\Sigma}\right\} = \mathrm{Re}\left\{\frac{\boldsymbol{w}_\Delta^H \boldsymbol{z}}{\boldsymbol{w}_\Sigma^H \boldsymbol{z}}\right\} \tag{2}$$

where $\mathrm{Re}\{\cdot\}$ represents the operation of taking the real part of the ratio. As shown in Figure 1, because of the adaptation, the shape of the sum and difference beam is perturbed, which consequently results in a distorted value of the adaptive monopulse ratio. Therefore, the usual monopulse formula produces errors if applied directly to the adaptive sum and difference beam outputs.



(A) Sum Beampatterns (B) Difference Beampatterns

(C) Monopulse Ratio

Figure 1. Sum and Difference Beampatterns and the Resulting Monopulse Ratio

A general corrected adaptive monopulse formula is proposed in reference [2]. It can be summarized as follows. Since a null is produced in the direction of the interference and the output signal to noise ratio (SNR) value is enhanced to be very high, the monopulse ratio can be replaced by the function of the target direction vector

$$r_u = \mathrm{Re}\left\{\frac{\boldsymbol{w}_\Delta^H \boldsymbol{z}}{\boldsymbol{w}_\Sigma^H \boldsymbol{z}}\right\} = \mathrm{Re}\left\{\frac{\boldsymbol{w}_\Delta^H \boldsymbol{a}_u}{\boldsymbol{w}_\Sigma^H \boldsymbol{a}_u}\right\} \tag{3}$$

Consider that the location of the target is close to the look direction u_0, we evaluate the monopulse ratio at the look direction by a first-order Taylor series [2]

$$r_u \approx r_{u_0} + \left.\frac{dr_u}{du}\right|_{u=u_0} (u - u_0) \tag{4}$$

where $r_{u_0} = r_u \big|_{u=u_0}$ represents the monopulse ratio at u_0. Therefore, the original adaptive monopulse should be modified as

$$u = u_0 + (r_{u,0})^{-1}(r_u - r_{u_0}) \tag{5}$$

where $r_{u,0} = \left.\dfrac{dr_u}{du}\right|_{u=u_0}$ denotes the slope of the monopulse ratio, and r_{u_0} is the bias of the monopulse ratio. However, this adaptive monopulse processor may be limited in real-time application for a high degrees of freedom (DOF) system with large number of elements, due to the heavy computation load and slow convergence speed, which motivates the following investigation of implementing a subspace-based approach [7, 8] in conjunction with the adaptive monopulse algorithm.

3. MPASTD-Based Subspace Method for Adaptive Monopulse

In this section, we will introduce the subspace methods into the adaptive monopulse for weights calculation. In reference [9], it is pointed out that the dominant eigenvectors of the covariance matrix contain all the information about the distribution of the interference. Therefore, if the number of the interference signals is P and all training data is independent and identically distributed, choosing $L = 2P$ training samples yields an average performance loss of roughly 3dB [10- 12] compared to the $L = 2N$ samples needed with full rank sample matrix inversion methods. This reduced-rank subspace method, also called eigencanceller [9], can improve the convergence speed effectively, and this technique is much more robust to a bad estimation of the covariance matrix than the SMI method.

The eigencanceller is taking advantages of the low-rank nature of the interference subspace. Hence, the ED of $\boldsymbol{Q}$ can be written as:

$$\boldsymbol{Q} = \boldsymbol{V}_J \boldsymbol{\Lambda}_J \boldsymbol{V}_J^H + \boldsymbol{V}_N \boldsymbol{\Lambda}_N \boldsymbol{V}_N^H \tag{6}$$

where $\boldsymbol{V}_J$ contains the P dominant eigenvectors of $\boldsymbol{Q}$ spanning the interference subspace, and $\boldsymbol{V}_N$ contains the remaining $N-P-1$ eigenvectors spanning the noise subspace. The diagonal matrix $\boldsymbol{\Lambda}_J$ consists of the P principle eigenvalues of $\boldsymbol{Q}$, and $\boldsymbol{\Lambda}_N$ consists of the eigenvalues corresponding to $\boldsymbol{V}_N$. Then the subspace-based method accelerates the convergence by only incorporating the P principle eigencomponents, in which the weight vector is

$$\boldsymbol{w}_{ED} = \gamma\left(\boldsymbol{I} - \boldsymbol{V}_J \boldsymbol{V}_J^H\right)\boldsymbol{s} \tag{7}$$

where $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{s}$ denotes the signal steering vector. The eigencanceller can reduce the convergence time but its operation corresponding to ED is almost the same as the SMI method. To avoid ED procedure, the interference subspace can be estimated by subspace tracking techniques. Owing to the efficiency and robustness, the MPASTd approach [10, 13, 14] which can avoid sequential tracking of the multiple dominant eigenvectors and accelerate the convergence speed, then becomes a good candidate.

To apply the MPASTd-based eigencanceller with adaptive monopulse, we firstly estimates the most dominant eigenvector $\boldsymbol{e}_1$ (the eigenvector corresponding to the maximum eigenvalue) using all sample data while calculating the eigenvalue so as to estimate the rank of the interference signal subspace. For $i = 1, 2, \cdots, L$

$$y_1(i) = \boldsymbol{e}_1^H(i-1)\,\boldsymbol{x}_1(i) \tag{8}$$

$$\lambda_1(i) = \lambda_1(i-1) + \left|y_1(i)\right|^2 \tag{9}$$

$$\boldsymbol{e}_1(i) = \boldsymbol{e}_1(i-1) + \frac{\left[\boldsymbol{x}_1(i) - \boldsymbol{e}_1(i-1)\, y_1(i)\right] \cdot y_1^*(i)}{\lambda_1(i)} \tag{10}$$

where L represents the number of the training samples $\boldsymbol{x}(i)$, $\lambda_1(i)$ represents the eigenvalue corresponding to $\boldsymbol{e}_1(i)$ and * denotes the complex conjugation. In the last iteration, it is necessary to normalize the $\boldsymbol{e}_1(L)$ in order to guarantee the orthonormality between the estimated eigenvectors

$$\boldsymbol{e}_1 = \frac{\boldsymbol{e}_1(L)}{\left\|\boldsymbol{e}_1(L)\right\|_2} \tag{11}$$

where $\|\cdot\|$ is the Euclidean norm. Since the first eigenvector $\boldsymbol{e}_1$ has been determined, we can remove the projection of each training sample $\boldsymbol{x}_1(i)$ onto $\boldsymbol{e}_1$, *i.e.*, $\boldsymbol{e}_1 \boldsymbol{y}_1(i)$, from $\boldsymbol{x}_1(i)$ itself, and then estimate the second eigenvector in the same way, which becomes the dominant one in the process of the update. Applying this procedure repeatedly and all the eigenvectors corresponding to the interference can be sequentially determined [10].

In practice, it is highly desirable to automatically determine the rank of the interference subspace. In this case, we can estimate the residual average projection power by subtracting successively larger sums of the estimated eigenvalues from the total sample matrix power, which is defined as $E_0 = \frac{1}{L}\sum_i^L \boldsymbol{x}^H(i)\,\boldsymbol{x}(i)$ until the difference lies below the chosen threshold defined in [15] (usually chosen to be twice of the noise power of system). That is

$$E_j = \frac{E_0 - \sum_{k=1}^{j} \lambda_k}{N - j} \quad j = 1, 2, \cdots, P \tag{12}$$

where $\lambda_j = \frac{1}{L}\sum_i^L y_j(i) y_j^*(i)$ is the eigenvalue of $\boldsymbol{e}_j$. Compare each E_j to the threshold. The index number of E_j exceeding the threshold is the rank of the interference subspace, which means that only P principal eigencomponents required to be estimated. Neglecting the comparison with the threshold, the additional computation requirement for adaptive rank estimation is only $O(LN)$ complex multiplications.

Therefore, both the adaptive sum and difference beam weights using the MPASTd-based eigencanceller can be written as follows

$$\boldsymbol{w}_{\Delta\mathrm{MPASTd}} = \gamma\left(\boldsymbol{I} - \boldsymbol{U}_J \boldsymbol{U}_J^H\right)\boldsymbol{s}_\Delta \tag{13}$$

$$\boldsymbol{w}_{\Sigma\mathrm{MPSATd}} = \gamma\left(\boldsymbol{I} - \boldsymbol{U}_J \boldsymbol{U}_J^H\right)\boldsymbol{s}_\Sigma \tag{14}$$

where $\boldsymbol{U}_J$ is the estimated interference subspace which comprises eigenvectors of $\boldsymbol{e}_1\ \boldsymbol{e}_2 \cdots\ \boldsymbol{e}_P$. We insert the newly obtained adaptive sum and difference beam weights into the adaptive monopulse algorithm, then the slope and the bias of monopulse ratio in equation (5) can be also corrected as

$$r_{u,0} = \frac{\mathrm{Re}\left\{ \mathbf{w}^H_{\Delta MPASTd} \mathbf{a}_{u,0} \mathbf{a}^H_{u_0} \mathbf{w}_{\Sigma MPASTd} + \mathbf{w}^H_{\Delta MPASTd} \mathbf{a}_{u_0} \mathbf{a}^H_{u,0} \mathbf{w}_{\Sigma MPASTd} \right\}}{\left| \mathbf{a}^H_{u_0} \mathbf{w}_{\Sigma MPASTd} \right|^2} - 2 r_{u_0} \mathrm{Re}\left\{ \frac{\mathbf{w}^H_{\Sigma MPASTd} \mathbf{a}_{u,0}}{\mathbf{w}^H_{\Sigma MPASTd} \mathbf{a}_{u_0}} \right\} \tag{15}$$

$$r_{u_0} = \mathrm{Re}\left\{ \frac{\mathbf{w}^H_{\Delta MPASTd} \mathbf{a}_{u_0}}{\mathbf{w}^H_{\Sigma MPASTd} \mathbf{a}_{u_0}} \right\}. \tag{16}$$

From equations (13)-(16), it is clear that all variables have been determined to calculate the direction of the target in according to the equation (5). The detailed performance assessments will be given in the next section.

4. Simulation Results and Analysis

The Monte Carlo simulation is performed to investigate the performance of the proposed adaptive monopulse scheme via MPASTd-based eigencanceller. In the simulation, a uniform linear array with 32 half-wavelength spaced elements is used, and the beam direction is -3°. There are three interference signals located at the -30°、-18° and 20°, respectively, and the interference to noise ratio (INR) on the element level is 30 dB. A target is injected, and its direction is -4.5°, which has a -1.5° offset angle from the look direction. The SNR of the target is 30 dB.

The estimation of the rank of the interference subspace is vital important to the whole signal processing scheme. In order to further study the merits of the proposed MPASTd-based subspace algorithm, we firstly give the rank estimation result. From Figure 2, it is obviously that the residual average projection power is nearly zero after three components have been removed from the total sample matrix power which means that there are no more dominant signals remained. Therefore, the rank of the interference can be correctly determined to be $P = 3$.

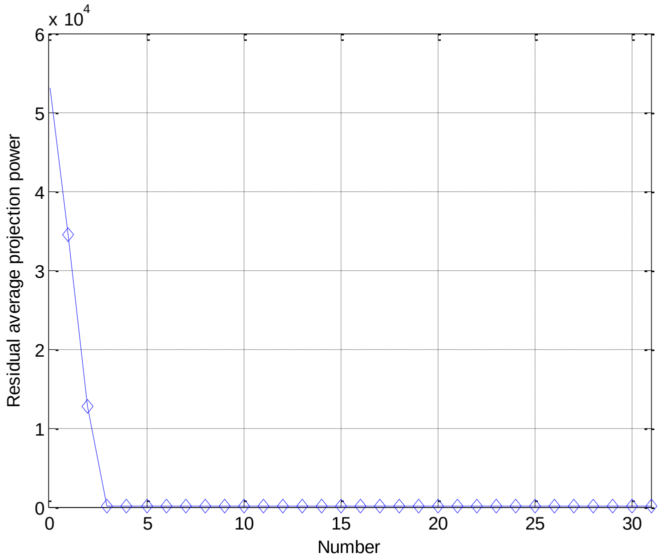
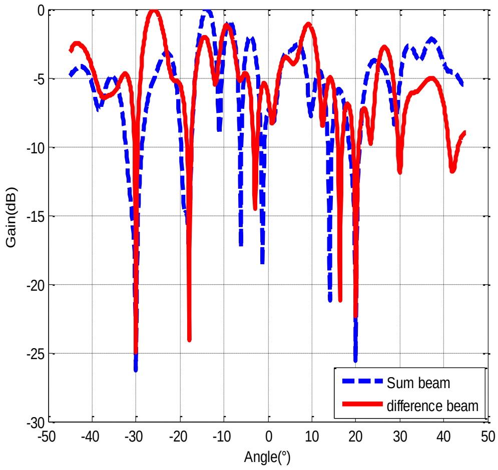
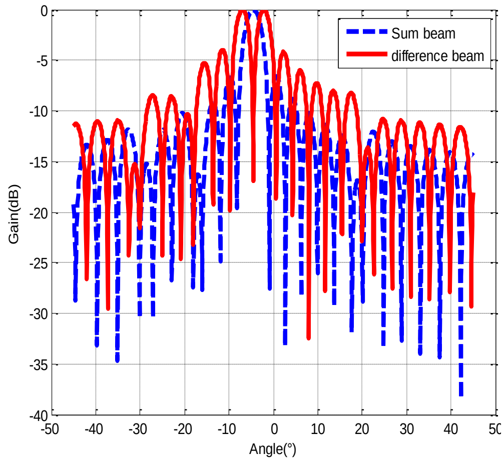


Figure 2. The Residual Average Projection Power

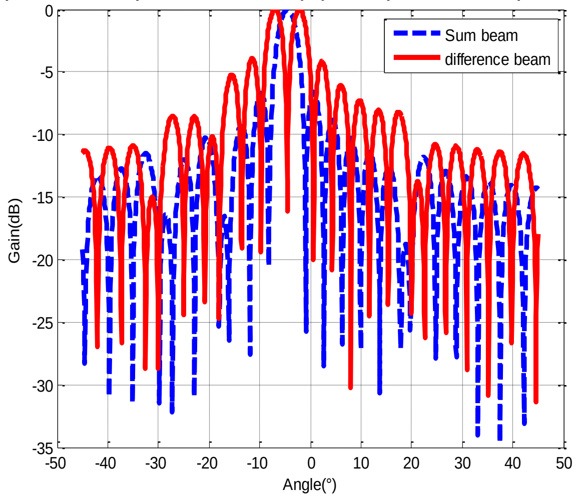
Typically, the number of the samples L is selected to be the twice the dimension of the interference subspace, that is $L = 6$, so that performance is within 3dB loss. Figure 3 shows the resulting sum and difference beampatterns for the three approaches used in adaptive monopulse, which are SMI, ED and MPASTd, respectively. It is interesting to note that the shape of sum and difference beams obtained by our methods are roughly equal to the quiescent beampatterns, while the beampatterns for the adaptive monopulse which uses SMI are severely distorted. This highlights the fact that restricting the sample data severely effects the covariance matrix estimation and the following weights calculation. Therefore, in this situation, the original adaptive monopulse detector can hardly give accurate angle orientation estimation under low sample counts. A note about this is that the SMI weights are obtained by means of the pseudo-inverse matrix technique, since the actual matrix is not full-rank and can't be allowed to take the inversion.



(A) Adaptive Monopulse.



(B) Adaptive Monopulse Based on ED



(C) Adaptive Monopulse Based on Mpastd

Figure 3. Sum and Difference Beam Patterns

The root mean square error (RMSE) is used to quantitatively analyze the angular accuracy of the proposed adaptive monopulse processor, which is defined as

$$\theta_{RMSE} = \sqrt{\frac{1}{M}\sum_{m=1}^{M}(\hat{\theta}_m - \theta)^2} \qquad (17)$$

where M is the number of the Monte Carlo trials, $\hat{\theta}_m$ denotes the estimated target angle and θ is the real target angle. Figure 4 shows the DOA estimation RMSE as a function of the number of training samples for the three approaches, *i.e.*, SMI, ED and MPASTd, used in adaptive monopulse. The results were average of 200 independent Monte Carlo experiments. It can be observed form Figure 4 that both the original adaptive monopulse and subspace reduced-rank adaptive monopulse have good performance for large numbers of training samples, *i.e.*, $L > 20$ in this simulation. It can also be noted that the ED and MPASTd methods can achieve comparable performance and have a faster convergence speed in contrast to the original adaptive monopulse. Apparently, in the case of low availability of samples, the performance of the proposed algorithm is far superior to that of the original method. For instance, when 6 training samples are used, the RMSE for the improved application is only 0.03°, which is great smaller than that of the original adaptive monopulse. However, if the training data is sufficient, the adaptive monopulse can attain slightly better accuracy than our method, due to its precise estimation of the interference and noise covariance and the increased performance of interference cancellation.

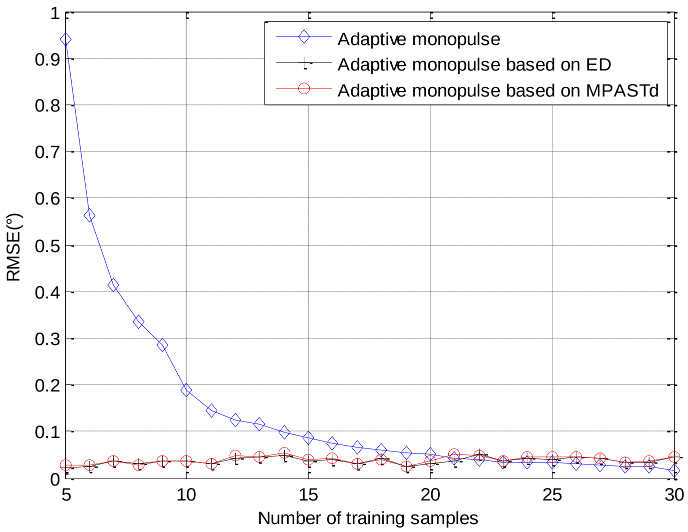


Figure 4. DOA Estimation RMSE versus Number of Training Samples

Figure 5 exhibits the RMSE of the DOA estimates as a function of SNR for the aforementioned three methods, where 6 training samples are used for calculating the adaptive weights. As expected, the ED and MPASTd methods can achieve comparable performance and converge much more rapidly than the adaptive monopulse.

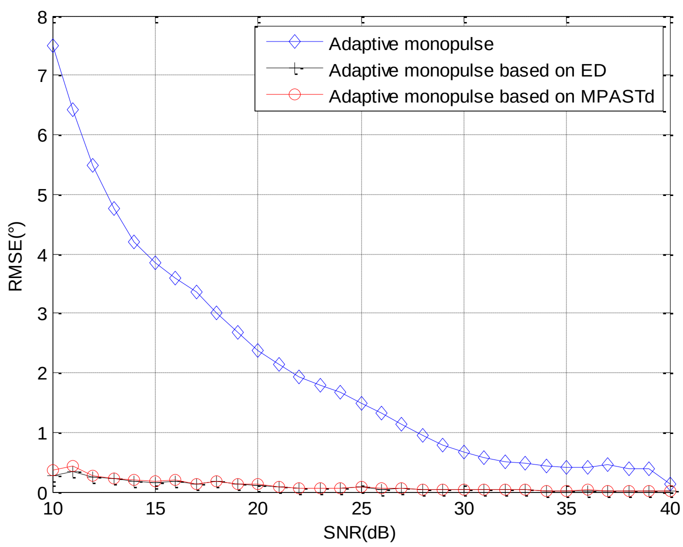


Figure 5. DOA Estimation RMSE versus SNR

As shown in references [2, 16], a multistep monopulse procedure can further reduce the bias between the estimated target angle and the real one. The estimated target angle as a function of iteration index for the subspace-based algorithms are given in Figure 6, similarly, $L = 6$ is employed. It can be noted that by a second step, with the previous estimate as the initial value, the bias introduced by the adapted pattern can be almost completely removed for both subspace-based methods.

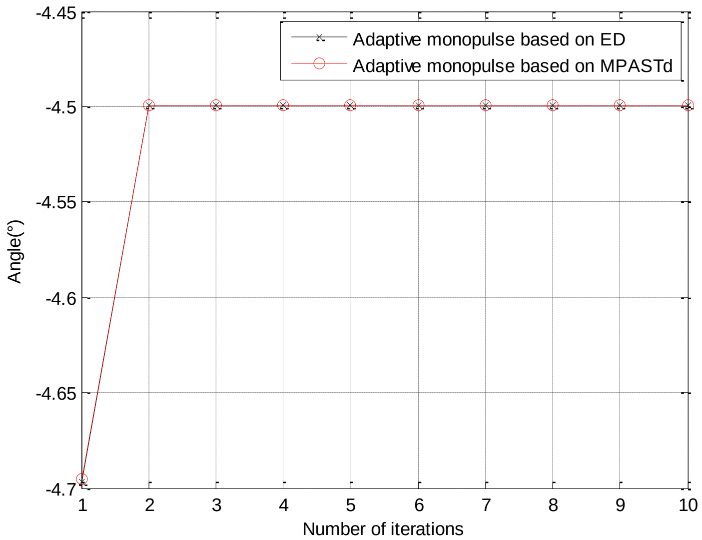


Figure 6. DOA Estimation versus Iteration Index

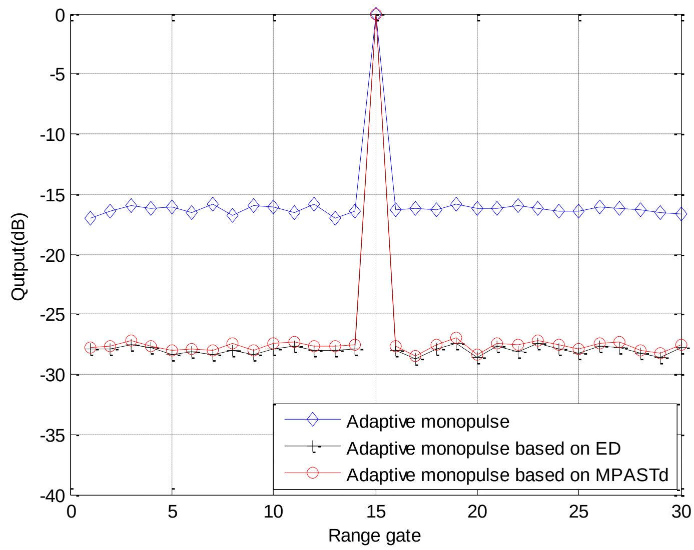
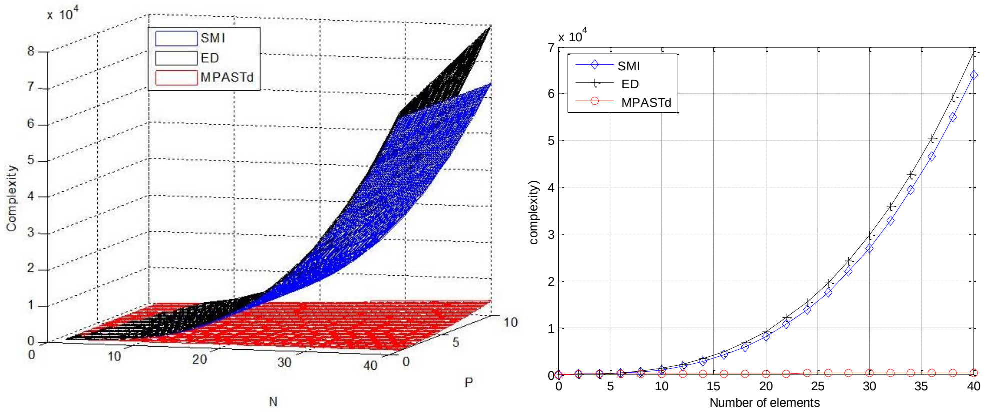


Figure 7. The Outputs of Adaptive Monopulse Processors

The primary objective of adaptive monopulse processing is to cancel interference signals while keeping the response of the array in the desired signal direction [17]. Assume that $L = 6$, we compare the outputs of 30 range gate for both the classic and subspace-based adaptive monopulse approaches. A target is added into the 15th range cell, and its SNR is 30dB. In Figure 7, we can see that MPASTd based method provides an interference suppression capability quite comparable with ED method and yields approximately 10 dB improvement over the classic adaptive monopulse based on SMI approach.



(A) Complexity (B) $P = 3$

Figure 8. The Complexity of the Three Methods

In the end, we compare the proposed method's computational complexity with the original adaptive monopulse technique. The complexity associated with estimating the sample covariance matrix and its inverse value are considered. SMI method needs $L = 2N$ samples, and its complexity can be expressed as $O(N^3)$. For the two subspace-based methods, given a typical scenario of $L = 2P$, the total complex multiplications for ED implementation is $O(N^3 + PN^2)$ when compared with MPASTd, which requires $O(P^2 N)$ operations. Since $N \gg P$, it is evident that the MPASTd algorithm is much more efficient. The computational complexities of the three methods are shown in Figure 8(a), respectively. If the number of interference signals P is selected to be 3, see Figure 8(b), the computational cost of MPASTd will be further reduced compared to ED when we choose to increase the number of elements, *i.e.*, N. Thus, using the MPASTd algorithm to calculate the weight vectors of the adaptive monopulse can significantly reduce the computational cost relative to the ED approach. That is, in fact, considering the sufficiently good angle measuring accuracy for MPASTd, the advantage to use MPASTd in the adaptive monopulse is evident for low sample counts.

5. Conclusion

In practice, the traditional adaptive monopulse method usually lacks training data and the performance is impacted negatively. Therefore, a novel, low-complexity, iterative adaptive monopulse estimation scheme based on MPASTd eigencanceller is proposed in this paper. Firstly, the proposed scheme recursively estimates the interference subspace. And then we recalculate the adaptive sum and difference beam weights. Finally, we apply the newly obtained weights into the original adaptive monopulse algorithm to estimate signal source orientation. By simulation, the proposed method is shown to provide better DOA estimates than the adaptive monopulse, while being capable of attaining lower complexity under certain condition of low sample counts. In addition, the method proposed here is also suitable for the applications of plane array and sub-array antennas.

Acknowledgments

This work was supported in part by National Natural Science Foundation of China (No.61201459, No.61301212), Natural Science Foundation for Young Scholars of Jiangsu Province of China (No.BK2012408), Open Project of Key Laboratory of Radar Imaging and Microwave Photonics (NUAA), Ministry of Education (No.RIMP-2013002), and Chinese Universities Scientific Foundation (No.2012B06014). The authors also thank the referees and the editor for thorough and useful comments, which have helped to improve the presentation of this paper.

References

[1] R. C. Davies, L. E. Brennan and I. S. Reed, "Angle estimation with adaptive arrays in external noise field", IEEE Trans. On Aerospace and Electronics Systems, vol. 12, no. 2, 179-186, **(1976)**
[2] U. Nickel, "Monopulse estimation with adaptive arrays", IEE Radar, Sonar and Navigation, vol. 140, no. 5, 303-308, **(1993)**
[3] U. Nickel, "Monopulse estimation with subarray-adaptive arrays and arbitrary sum and difference beams", IEE Radar, Sonar and Navigation, vol. 143, no. 4, 232-238, **(1996)**
[4] U. Nickel, "Performance analysis of corrected adaptive monopulse estimation", IEE Radar. Sonar and Navigation, vol.146, no. 1, 17-24, **(1999)**
[5] U. Nickel, E. Haumette and P. Larzabal, "Statistical performance prediction of generalized monopulse estimation", IEEE Trans. On Aerospace and Electronics Systems, vol. 47, no. 1, 381-404, **(2011)**
[6] L. E. Brennan, I. S. Reed, "Theory of adaptive radar", IEEE Trans. On Aerospace and Electronics Systems, vol. 9, no. 2, 237-252, **(1973)**
[7] J. Degurse, L. Savy and S. Marcos, "Reduced-rank STAP for target detection in heterogeneous environment", IEEE. Aerospace and Electronics Systems, vol. 50, no. 2, 1153-1162, **(2014)**
[8] Y. Kong, "A new reduced-dimension GSC for target tracking and interference suppression," Proceedings of the International Conference of Radar CIE, **(2011)** October 24-27; Chengdu, China
[9] A. Haimovich, "The eigencanceler: adaptive radar by eigenanalysis methods", IEEE Trans. On Aerospace and Electronics Systems, vol. 32, no. 2, 532-542, **(1996)**
[10] M. Shen, "Reduced-rank space-time adaptive processing using a modified projection approximation subspace tracking deflation approach", IET Radar Sonar Navig, vol. 3, no. 1, 93-100, **(2009)**
[11] I. S. Reed, J. D. Mallet and L. E. Brennan, "Rapid convergence rate in adaptive arrays", IEEE Trans. On Aerospace and Electronics Systems, vol. 10, no. 6, 853-863, **(1974)**
[12] Y. Zhang, "A novel monopulse estimation algorithm using projection method", Proceedings of the International Conference of Intelligent Computation Technology and Automation, **(2011)** March 28-29; Shenzhen, China
[13] B. Yang, "Projection approximation subspace tracking", IEEE Trans. On Signal Process, vol. 43, no. 1, 95-107, **(1995)**
[14] B. Yang, "An extension of the PASTd algorithm to both rank and subspace tracking", IEEE Trans. On Signal Process, vol. 2 no. 9, 179-182, **(1995)**
[15] E. C. Real, D. W. Tufts and J. W. Cooley, "Two algorithms for fast approximated subspace tracking", IEEE Trans. Signal Process, vol. 47, no. 7, 1936-1945, **(1999)**
[16] U. Nickel, "Overview of generalized monopulse estimation", IEEE. Aerospace and Electronics Systems, vol. 21, no. 6, 27-56, **(2006)**
[17] R. Li, "Combining sum-difference and auxiliary beams for adaptive monopulse in jamming", IEEE. Engineering and Electronics Systems, vol. 24, no. 3, 372-381, **(2013)**

Authors

Jia Yu was born in 1991, she is currently a M.S. student in the college of computer and information engineering, Hohai University. Her research focuses on array signal processing and space-time adaptive processing (STAP) with applications in radar. She received B.S. degree in communication engineering in 2009 from Hohai University, China.

Mingwei Shen was born in 1981. He received the B.S. and Ph.D. degrees in electronics engineering from Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, in 2003 and 2008, respectively. Now he is an associate professor with the college of computer and information engineering, Hohai University. His current research interests include space-time adaptive processing (STAP), SAR ground moving indication (SAR/GMTI), and array processing.