

Sensor Array Processing for High Resolution of DOA Estimation of Spatial Subspace using Smoothing Technique of Beam-space MUSIC in Coherent Channels

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Abstract

MUSIC algorithm has often used to solve the direction of arrival estimation problem to take advantage of the benefits of beam-space operations, such as reduced computation complexity, reduced sensitivity to system errors, good resolution, reduced bias in the estimation. In this paper, a variant of the beam-space MUSIC algorithm is developed, taking advantage of unitary transformations. In general, the beam-space MUSIC algorithm improves direction of arrival estimation by using reduced order characteristic polynomial rooting through Gaussian column reduction. The unitary beam-space MUSIC algorithm reduces the computational complexity of MUSIC algorithm by using real valued forward and backward, while maintaining the original precision. We use forward backward spatial smoothing technique as the preprocessor of beam-space MUSIC algorithm to recover the reduced rank of the covariance matrix due to the coherence of source signals. Through computer simulations show that the combination of spatial smoothing a beam-space MUSIC can achieve good resolution performance.

Keywords: *Beamformer, subspace, Eigen Value, MUSIC Algorithm*

1. Introduction

In recently application of antennas, point to point communication is of interest. A highly directive antenna beam can be used to advantage [1]. The direction beam can be realized by forming an array with a number of elements radiators. Beamforming is based on capturing the desired signals the antenna elements and converting them into two streams of binary baseband, which jointly represent the amplitudes and phases of signals received at the elements of the array. Beamforming is to signal receive or transmission for specific direction. It is called beamforming spatial filtering. Spatial filter is made using array antenna [2, 3].

Beamformed adaptive system allow the antenna to steer the beam to any direction of interest while simultaneously nulling interfering signals. Adaptive array antennas is called smart antenna [4]. The smart antenna concept is opposed to the fixed beam dumb antenna, which does not attempt to adapt its radiation pattern to an ever changing electromagnetic environment. In the past, smart antennas have alternatively been labeled adaptive arrays or digital beamforming arrays [5]. This new terminology reflects our penchant for smart technologies and more accurately identifies an adaptive array that is controlled by sophisticated signal processing. Fixed beam array where the mainlobe can be steered, by defining the fixed array weights. However, this configuration is neither smart nor adaptive. Smart antennas have numerous important benefits in wireless applications as well as in sensors such as radar. In the realm of mobile wireless application, smart antennas can provide higher system capacities by directing narrow beams toward the users of interest, while nulling other users not of interest. This allows for higher signal to interference

rations, lower power levels, and permits greater frequency reuse within the same cell. Another benefit of smart antenna is that deleterious effects of multipath can be mitigated [7].

Smart antennas can be used to enhance direction of arrival techniques by more accurately finding angles of arrival [8]. Smart antennas can direct the array main beam toward signals of interest even when no reference signal or training sequence is available. This capability is called blind adaptive beamforming [9, 10].

One of the most popular algorithms for performing direction of arrival estimation is the MUSIC(Multiple Signal Classification) algorithm. To further reduce the computation complexity in several references. Except for the reduction of computation, there are several other advantages of beamspace MUSIC algorithm compared with element space MUSIC algorithm, such as reduced sensitivity to system errors, reduced resolution threshold, reduced bias in the estimate. Beamspace MUSIC algorithm has limitation that it performs poorly in the presence of highly correlated and coherent source signals. Unfortunately, strongly correlated source signals are often encountered in sonar or radar environment. Where signal correlate occurs in multipath or some other scenarios. In beamspace MUSIC algorithm, forward and backward spatial smoothing techniques have been used before direction of arrival estimation to overcome this problem when the receiving array is a uniform linear array.

In this paper, a variant of the beamspace MUSIC algorithm is developed, taking advantage of unitary transformations. In general, the beamspace MUSIC algorithm improves direction of arrival estimation by using reduced order characteristic polynomial rooting though Gaussian column reduction. The unitary beamspace MUSIC algorithm reduces the computational complexity of MUSIC algorithm by using real valued forward and backward, while maintaining the original precision. We use forward backward spatial smoothing technique as the preprocessor of beamspace MUSIC algorithm to recover the reduced rank of the covariance matrix due to the coherence of source signals. Through computer simulations show that the combination of spatial smoothing a beamspace MUSIC can achieve good resolution performance. The rest of the paper is organized as follows: section 2 discussed the array MUSIC signal model of array processing. Sub MUSIC signal method is proposed in section 3. Section 4 are presented computer simulation comparing with other methods. Finally, Section 5 is conclusion.

2. Beamforming Signal Model

Let's consider a uniform linear array composed of L omnidirectional element antennas with element antenna spacing d receiving two coherent sinusoidal sources and noise signal. Assume that narrow band signal receiving upon vector of the array, which is corrupted by additive white noise. Receiving signal on array antenna can be written [11].

$$x(t) = \sum_{i=1}^P a(\theta_i) s_i(t) + n_i(t) \quad (1)$$

Where $a(\theta_i) = [1, e^{-j2\pi\frac{d}{\lambda}\sin\theta_i}, \dots, e^{-j2\pi(L-1)\frac{d}{\lambda}\sin\theta_i}]$ is a steering vector. d and λ are inter element spacing and the wavelength, respectively. The noise vector is additive white noise which is spatially uncorrelated at different antennas, independent to the incident waveform and Gaussian distributed with zero mean and variance σ^2 . In matrix form, Equation(1) can be written

$$x(t) = A s(t) + n(t) \quad (2)$$

$$A(\theta) = [a(\theta_1), a(\theta_2), \dots, a(\theta_p)] \quad (3)$$

$$s(t) = [s_1(t), s_2(t), \dots, s_p(t)]^T \quad (4)$$

where $A(\theta)$ and $s(t)$ are the $M \times P$ steering matrix and the $P \times 1$ complex signal vector, respectively. $n(t)$ is additive noise vector to be a zero mean Gaussian noise vector with

covariance $\sigma^2 I$, where I denotes an $M \times M$ identity matrix. The covariance matrix can be written [12].

$$R = E[x(t)x^H(t)] \quad (5)$$

$$= A S A^H + \sigma^2 I \quad (6)$$

Where S is the signal correlation matrix. Output signal of the system can be written

$$y(t) = W R W^H \quad (7)$$

Where W is weight vector, $()^H$ is Hermit matrix. Weight vector can be written

$$W = [w_1, w_2, \dots, w_P] \quad (8)$$

The wiener filter can be used to estimate the desired signal $d(t)$ form the received signal in the minimum mean square error sense. If the desired signal $s(t)$ is known, one may choose to minimize the error between the beamformer output an d the desired signal. Knowledge of the desired signal eliminates the need for beamforming. However, for many applications, characteristics of the desired signal may be known with sufficient detail to generate a signal $d(t)$ that closely represents it, or at least correlates with the desired signal to a certain extent. This signal is called a reference signal. It should be noted that we express the reference signal as a complex conjugate only for mathematical convenience. It would not make any difference in the final result. The weights are chosen to minimize the mean square error between the beamformer output and the reference signal as follow

$$\epsilon^2(t) = E[d(t) - W^H x(t)]^2 \quad (9)$$

Taking the expected values of both sides of equation(9) and carrying out some basic algebraic manipulation, we can be written[13, 14]

$$E[\epsilon^2(t)] = E[d^2(t)] - 2W^H r + W R W^H \quad (10)$$

Where $r = E[d(t)x(t)]$. The minimum is given by setting the gradient vector of equation(10) with respect to w equal to zero:

$$\nabla_W E[\epsilon^2(t)] = -2r + 2R W = 0 \quad (11)$$

It follow that the solution is

$$W_{wf} = \frac{E[x(t) d^*(t)]}{R} \quad (12)$$

Which is referred to as the Wiener-Hopf equation or the optimum Winer solution. If $s(t) = d(t)$, $r = E[d^2(t)]$. Let us further express

$$R = E[d^2 A A^H] + R_n \quad (13)$$

Where $R_n = E[n n^H]$ and apply Woodbury's Identity to $\frac{1}{R}$ and we have

$$R^{-1} = \left[\frac{1}{1+E[d^2(t)]A^H R_n^{-1} A} \right] R_n^{-1} \quad (14)$$

There, the Wiener solution can be generalized as follow

$$W_{wf} = \beta R_n^{-1} A \quad (15)$$

Where β is a scalar coefficient. In the case of minimum mean square error can be written

$$\beta = \frac{E[d^2(t)]}{1+E[d^2(t)]A R_n^{-1} A^H} \quad (16)$$

The representation of R in terms of its eigenvalues $u_1 \geq u_2 \geq \dots u_Q$ and corresponding eigenvectors t_1, t_2, \dots, t_Q is as follows

$$R = \sum_{j=1}^Q u_j t_j t_j^H \quad (17)$$

Where t_1, t_2, \dots, t_P corresponds to the signal subspace and $t_{P+1}, t_{P+2}, \dots, t_Q$ corresponds to the noise subspace. The steering vector of the signal are orthogonal to the noise subspace.

3. Estimation of Subspace

Signal correlation can occur when they are arriving from different direction of arrival. It is well known that the performance of subspace techniques degraded as the incident signals become highly correlated. MUSIC algorithm has several advantages to the element MUSIC such as lower computation and more robust. The MUSIC algorithm performs however poorly in the presence of highly correlated and coherent. For coherent signals, the spatial smoothing method, also known as the subarray averaging method is proved that in can decorrelate the signals. Direction of arrival is estimated by using adaptive algorithm to a reception matrix x. Space moving average processing is performed in order to suppress the correlation between signals. The receiving matrix y can be written

$$x = \begin{bmatrix} x_{1,1} & \dots & x_{P,1} \\ \vdots & \ddots & \vdots \\ x_{1,M} & \dots & x_{P,M} \end{bmatrix} \quad (18)$$

Subarray number n_{sub} is given as follows when subarray distance is set to M

$$n_{sub} = P - M + 1 \quad (19)$$

The receiving signal matrix y_j of the j^{th} subarray can be written

$$x_j = [x_j, x_{j+1}, \dots, x_{j+M-1}]^T \quad (20)$$

By applying the forward backward spatial smoothing method before beamforming, rank of the source signal autocorrelation matrix can be equal to the number of direction of arrival. To remove the signal coherence, the P element antenna array is divided into M subarray which each subarray has $K = P - M + 1$ elements. The output of forward subarray can be written

$$x_f(t) = [x_k, x_{k+1}, \dots, x_{k-M-1}] \quad (21)$$

and the output of backward subarray can be written

$$x_b(t) = [x_{L-k+1}, x_{L-1}, \dots, x_{L-M-1}] \quad (22)$$

Where $k = 1, 2, \dots, K$. An forward backward spatially smoothed covariance matrix R_{fb} can be as follow

$$R_{fb} = \frac{1}{KN} \sum_{k=1}^K [x_{fk}(t)x_{fk}^H(t) + x_{bk}(t)x_{bk}^H(t)] \quad (23)$$

Where N is snapshots. The adaptive algorithm can be used to estimate direction of arrival. The covariance matrix is as follow

$$R = W R_{fb} W^H \quad (24)$$

Using the estimated angle value obtained by adaptive algorithm, we design beamformer as follow. A matrix is defined as follows

$$W = [w_1(\theta), w_2(\theta), \dots, w_P(\theta)] \quad (25)$$

$$w_m = \exp \left[j 2\pi (m - 1) f d \frac{\sin(\theta)}{c} \right] \quad (26)$$

$$W = E[X(t)d(t)^H] + R a(\theta) \frac{1 - \{p(\theta|X) a(\theta) R E[X(t)d^H(t)]\}}{p(\theta|X) a(\theta)^H p(\theta|X)^H R a(\theta)} \quad (27)$$

Where $m = 1, 2, \dots, P$ To apply beamforming matrix for the receiving matrix can be written

$$\bar{x} = W x \quad (28)$$

This \bar{x} is the receiving matrix of the outputs of beamformer with its main beam directed to the estimated direction. The steering vector of the signal are orthogonal the noise subspace as follow

$$a^H(\theta_j) W t_j = 0, \quad j = P + 1, \dots, Q \quad (29)$$

We can form the MUSIC spatial spectrum can be written

$$P(\theta) = \frac{a(\theta)W^H W a^H(\theta)}{a(\theta)W^H N N^H W a^H(\theta)} \quad (30)$$

Where $N = [N_{P+1}, \dots, N_Q]$ is the beamspace noise subspace eigenvector matrix. We find the locations of the peaks of $P(\theta)$ as the direction of arrival of the sources.

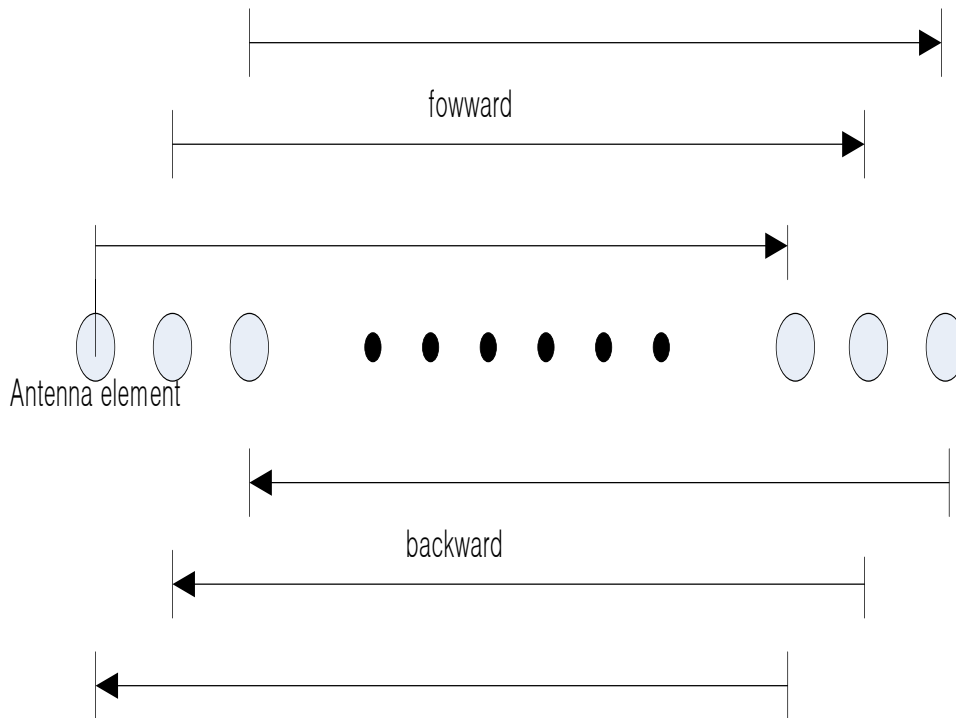


Figure 1. Subarray System

4. Simulation

The sensor spacing is half wavelength. We are showed an improvement that proposed algorithm in the estimation of the desired signal better than the general algorithm. We used array element 6 elements separated by half wavelength and SNR is 20dB, Snapshots are 100 numbers. Figure 2 is showed about 3numbers direction of arrival among $[-5^\circ, 0^\circ$,

5°]. Figure 3 is showed estimation direction of arrival with proposed algorithm in this paper at $[-5^\circ, 0^\circ, 5^\circ]$. Figure 3 is correctly estimation direction of arrival of 3 numbers signals in $[-5^\circ, 0^\circ, 5^\circ]$. In Figure 4 , we are showed that proposed method is to be superior probability estimation than classical method in desired signal estimation. When signal to ration ratio is 5dB, proposed method is showed probability estimation 0.9 and classical method is showed probability estimation 0.4.

Table 1. Simulation Parameters

Parameter	Value
Frequency	10GHz
Frequency step	1MHz
Element antenna	10
Antenna distance	0.5
Subarray number	5
Snapshot number	500
S/N	20dB

5. Conclusion

Array antenna of spectral estimation techniques are currently used to estimate the angles of arrival in order to decipher the present emitters and their possible angular locations. Accurate estimation of the direction of arrival is of a paramount importance. This paper presented comparison of the resolution of proposed algorithm which estimated the direction of arrival of narrow band sources of the same central frequency in the far field considering an element antenna array. The direction of arrival of four targets have been estimated by the proposed algorithm. In additions, it turns out that high precision has been realized especially with respect direction of arrival accuracy than general method. Moreover, the proposed method shows that matching of direction of arrival realizable by using subarray beamforming.

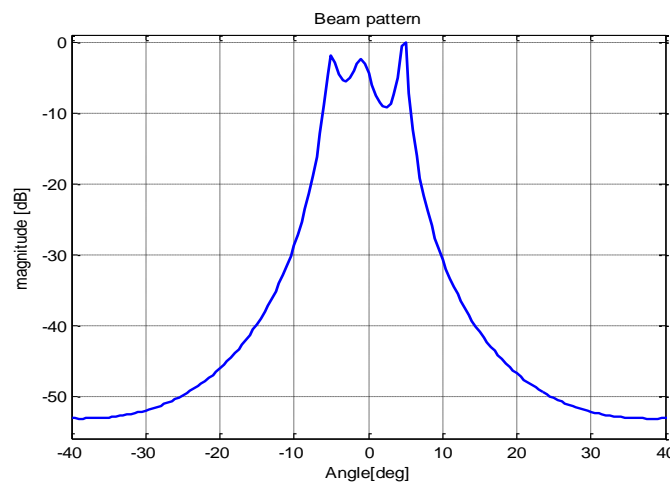


Figure 2. Estimation of General MUSIC

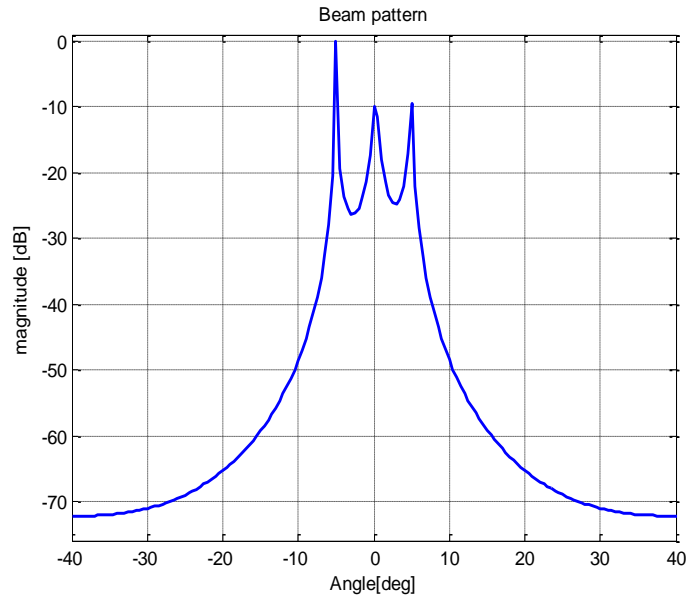


Figure 3. Estimation of Proposed Algorithm

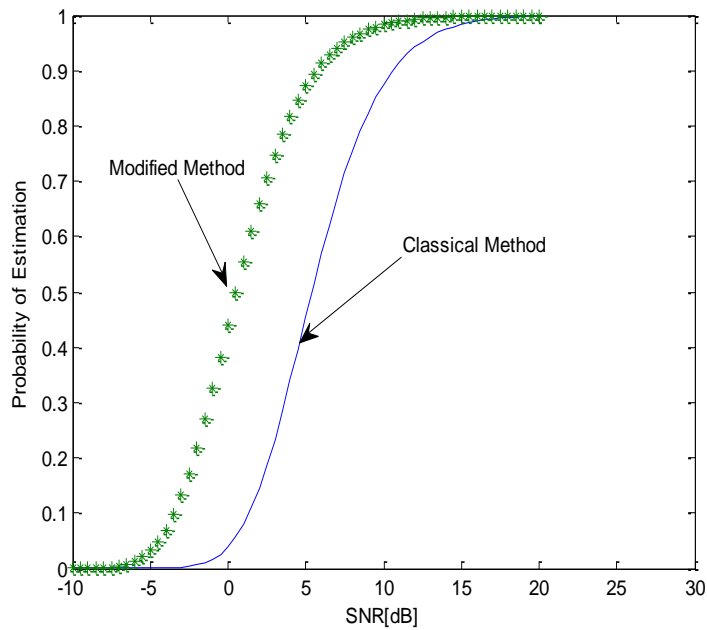


Figure 4. Estimation of General MUSIC

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