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Approximate Linearization of a Class of Underactuated System and its Application in the Ball and Beam System

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Abstract

In this paper, we consider the dynamical model of a class of underactuated systems. By combination of the partial fedback linearization and Yamada's global linearization, we deduced a global approximate linearization method for underactuated systems. By using this method, the dynamical equations can be transformed into an state eqution that is expressed as a pseudolinear term with Brunovsky canonical form plus a high order nonlinear term, where the nonlinear term is high order on the equilibrium manifold of the system. By standard nonlinear feedback method, the system is transformed into the sum of a stable linear term and a high order nonlinear term. Take proper feedforward value as the input to reduce the influence of the nonlinear term to the system and thus the underactuated system can be regulated. This method is applied to the ball and beam system and simulation results show that the proposed approximate linearization method is effective for setpoint control.

Keywords: *underactuated systems, approximate linearization, nonlinear control, setpoint control, ball and beam system*

1. Introduction

The underactuated system is a class of nonlinear system with fewer control inputs than the degrees of freedoms and the control problems are more complex Because of the missing of the input torgue on the underactuated joint. The coupling relationship between degrees of freedom is considered to achieve the control objectives. Many typical underacutated mechanical systems, such as inverted pendulum, Furuta pendulum, Pendubot, Acrobot, have been built up. A good many control methods have been investigated for the control problems of the underactuated models, such as linearization method, sliding mode variable structure control method, based on passive control methods, intelligent control method. In this paper, we study a linearization method about the underactuated system.

For normal nonlinear system, we can control it with control the linear system if we can transform the nonlinear system to the linear system by nonlinear feedback and coordinate transformations. But such accurate feedback linearization method can not apply to the underactuated system, it means we cannot transform the underactuated system to linear system exactly. So the underactuated system cannot be analyzed with linear control theory and need to be controled with Approximate Linearization or nonlinear control method .There were plenty of other linear methods about linearization for nonlinear systems in addition to accurate feedback linearization. Such as Jacobian linear method [1], which is effective for the partial stabilization of systems but cannot work well for the larger state transition. Spong proposed a method which linearized part coordinate variables of the underactuated system and transformed the system to affine nonlinear system. But the system still was a nonlinear system and need further analysis to design its controller [2]. Yamada proposed a global linear method to the nonlinear systems which

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used nonlinear feedback control the system [3].But many conditions proposed in such method were not always right to the actual systems. And there are many other methods such as pseudo-linear method [4-7], approximate feedback linearization [8], extended linearization [9]. But all of them were proposed for partial stabilization of equilibrium points and cannot realize the globe stabilization of system [10-15]. In this paper, a kind of global approximate linearization method which apply to under actuated system will be proposed by combination of the partial feedback linearization and Yamada's global linearization and apply it to the set point control problems of ball and beam system.

2. Global Approximate Linearization Model

2.1. Partial Feedback Linearization

Consider the underactuated system with two joint, without loss of generality, assume the first joint has no motivation, the dynamic equation can be expressed as

$$\begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_{11}(q,\dot{q}) & C_{12}(q,\dot{q}) \\ C_{21}(q,\dot{q}) & C_{22}(q,\dot{q}) \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1(q) \\ G_2(q) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\tau \tag{1}$$

Denote the state variables $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$, then the dynamic equation can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{(M_{12}C_{21} - M_{22}C_{11})x_3 + (M_{12}C_{22} - M_{22}C_{12})x_4 + M_{12}G_2 - M_{22}G_1}{M_{11}M_{22} - M_{12}M_{21}} \\ \frac{(M_{21}C_{11} - M_{11}C_{21})x_3 + (M_{21}C_{12} - M_{11}C_{22})x_4 + M_{21}G_1 - M_{11}G_2}{M_{11}M_{22} - M_{12}M_{21}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{M_{12}}{M_{11}M_{22} - M_{12}M_{21}} \\ -\frac{M_{11}}{M_{11}M_{22} - M_{12}M_{21}} \end{bmatrix}\tau \tag{2}$$

Denote $\dot{x} = 0$, then the equilibrium point of the system satisfies $G_1(q) = 0$. All Controllable equilibrium points of the system constitute a one-dimensional manifold called equilibrium manifold [4]. In order to apply it to the ball and beam system in the next section, It may be assumed that the system satisfy the partial strong inertia coupling, that is $M_{12} \neq 0$, using the partial feedback linearization method without been Configured [2], denote

$$\tau = C_{21}x_3 + C_{22}x_4 + G_2 - \frac{M_{22}}{M_{12}}(C_{11}x_3 + C_{12}x_4 + G_1) + (M_{21} - \frac{M_{11}M_{22}}{M_{12}})u \tag{3}$$

then the equation (2)can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ 0 \\ -\frac{C_{11}x_3 + C_{12}x_4 + G_1}{M_{12}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{M_{11}}{M_{12}} \end{bmatrix} u \equiv f(x) + g(x)u \tag{4}$$

2.2. Global Approximate Linearization

So we are going to deduce the global approximate linearization of the system (4) using the Yamada's theory. For the $G_1(q)$, we can denote

$G_1(q) = \eta_1(q)q_1 + \eta_2(q)q_2 + \eta_0$,

where $\eta_i(q) \in C^1$ and $\eta_i(0) \neq \infty$.

Without loss of generality, we can assume $x = 0$ is a equilibrium point of the system (4), the $G_1(0) = 0$,

so $G_1(q) = \eta_1(q)q_1 + \eta_2(q)q_2$. And then, in (4),

$f_4(x) = \phi_1(x)x_1 + \phi_2(x)x_2 + \phi_3(x)x_3 + \phi_4(x)x_4$,

where $\phi_i(x) \in C^1$ and $\phi_i(0) \neq \infty$.

Then (4) can be represented as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ \phi_1 & \phi_2 & \phi_3 & \phi_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{M_{11}}{M_{12}} \end{bmatrix} u \equiv A(x)x + b(x)u \tag{5}$$

In accordance with the Linearization method of the paper [5], we can denote $T(x) = T_1(x)T_2(x)$, where

$$T_1(x) = \begin{bmatrix} b & Ab & A^2b & A^3b \end{bmatrix} \tag{6}$$

$$T_2(x) = \begin{bmatrix} a_1 & a_2 & a_3 & 1 \\ a_2 & a_3 & 1 & 0 \\ a_3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\det(sI - A(x)) = s^4 + a_3(x)s^3 + a_2(x)s^2 + a_1(x)s + a_0(x)$$

Substitute $A(x)$ and $b(x)$ into (6) and (7) to obtain

$$T_1(x) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \vartheta_1 & \vartheta_2 & \vartheta_3 \\ 1 & 0 & 0 & 0 \\ \vartheta_1 & \vartheta_2 & \vartheta_3 & \vartheta_4 \end{bmatrix}, \quad T_2(x) = \begin{bmatrix} 0 & -\phi_2 & -\phi_4 & 1 \\ -\phi_2 & -\phi_4 & 1 & 0 \\ -\phi_4 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

where $\vartheta_1 = -\frac{M_{11}}{M_{12}}$, $\vartheta_2 = \phi_3 - \frac{M_{11}}{M_{12}}\phi_4$,

$\vartheta_3 = \phi_1 - \frac{M_{11}}{M_{12}}\phi_2 + \phi_4\vartheta_2$, $\vartheta_4 = \phi_2\vartheta_2 + \phi_4\vartheta_3$, then

$$T(x) = \begin{bmatrix} -\phi_2 & -\phi_4 & 1 & 0 \\ \phi_1 & \phi_3 & -\frac{M_{11}}{M_{12}} & 0 \\ 0 & -\phi_2 & -\phi_4 & 1 \\ 0 & \phi_1 & \phi_3 & -\frac{M_{11}}{M_{12}} \end{bmatrix} \tag{8}$$

Assume $T(x)$ meet the regularity condition, that is to say:

$$\det(T(x)) = \vartheta_2\vartheta_4 - \vartheta_3^2 \neq 0 \tag{9}$$

We introduce the coordinate transformation $z = T^{-1}(x)x$, then the state equation of the system (5) can be represented as:

$$\dot{z} = (T^{-1}(x)A(x)T(x) - T^{-1}(x)\dot{T}(x))z + T^{-1}(x)b(x)u$$

By the further reduction, The above equation can be expressed as

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \phi_2 & \phi_4 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u - T^{-1}(x)\dot{T}(x)z \tag{10}$$

The dynamical equations can be transformed into an state eqution that is expressed as a pseudolinear term with Brunovsky canonical form plus a high order nonlinear term. In the paper [4], Murray Gives the decomposition form of input-output linearization of general nonlinear system

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1} + \psi_i(x) + \theta_i(x)v \\ \dot{\xi}_n &= a(x) + b(x)v + \psi_n(x) + \theta_n(x)v \\ y &= \xi_1 + \psi_0(x) \end{aligned} \tag{11}$$

and point out that if $\psi_i(x)$ and $\theta_i(x)$ is advanced on the equilibrium manifold, then the control law designed by the linear approximation system for trajectory tracking of near equilibrium manifold is stable and bounded.Specially, if denote the output of the sysytem $y = z_1$ in (10), then if the nonlinear term $-T^{-1}(x)\dot{T}(x)z$ is high order on the equilibrium manifold of the system, the control law designed by the pseudo linear part for the setpoint control on the equilibrium manifold is stable, and the system can achieve the stabilization on the equilibrium manifold by making the pseudo linear partial stability.

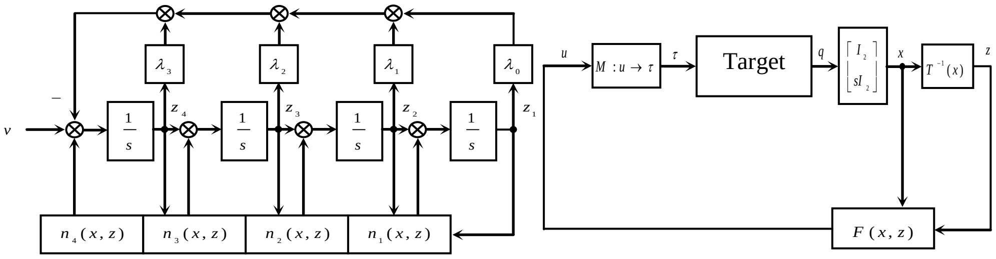
According to (8) and (10), using Yamada's linearization after partial feedback linearization, we can get the result more simple than the paper[5] about the underactuated systems, reduce the computational complexity in the process of linearization, and make them easier to analyze the results of linearization.

2.3. Feedback Control Law

In (10), in order to stabilize the pseudo linear part, we can denote the input as $u = -[\lambda_0 \quad \lambda_1 \quad \phi_2 + \lambda_2 \quad \phi_4 + \lambda_3]z + v$, then the system can be represented as

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \end{bmatrix} z - T^{-1}(x)\dot{T}(x)z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v \tag{12}$$

where λ_i is the real coefficient making $s^4 + \lambda_3 s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0$ become the Hurwitz polynomial. Denote $n(x, z) = -T^{-1}(x)\dot{T}(x)z$, then the system (12) can be represented as a series of integral chain. As is shown in Figure 1(a), each nonlinear term can influence the change of system state with the integrator of different locations, so it is difficult to find the suitable input v to eliminate the influence of all nonlinear terms to the system, and can only set $v = -n_4(x, z)$ to eliminate the effect of the nonlinear term ,but for the effect to the other nonlinear term, we can verify the effectiveness of the control law with the result of the control.



(a) (b

Figure 1. Control Structure of the Underactuated System

So with the Global approximate linearization,the control structure of the c can be represented as Figure 1(b),where the control law is

$$F(x,z) = -[\lambda_0 \quad \lambda_1 \quad \phi_2 + \lambda_2 \quad \phi_4 + \lambda_3]z - n_4(x,z) \tag{13}$$

The control procedure of the system can be expressed as: Firstly, we can get the system state x with the differential of the output q of the system, and acorrding to (8) we can get the state transformation matrix $T(x)$, then solve the auxiliary status z of the system. Nextly we can solve the variable u of the feedback control with (13) and the input torque τ of the dynamical system with (3), so we can achieve the control of the underactuated mechanical systems. In the process of the control ,we require the state transformation matrix $T(x)$ to be regular expression, that is satisfied with (9).

3. Application of Global Linearization to the Ball and Beam System

3.1. The Setpoint Control of the Ball and Beam System

The ball and beam system is a typical nonlinear system [3]. It consists of a rail turning around the fixed hanging point and a ball rolling along the rail, as it is shown in Figure 2. Some scholars have researched the setpoint control and the tracking control of the ball in the system. But most of the research work is onsidered the rolling joint without driver and the rotary joint with driver. They rarely research on the underactuated rotary joint and the rolling joint with driver. Discussion of this article is the latter case.

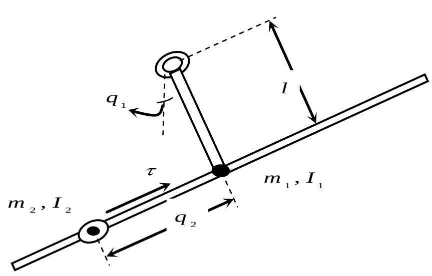


Figure 2. Simplified Model of the B-B System

Assume that the ball is sliding on the rails, then we can get the dynamical equation of the system shown in Figure 2 with Lagrange method:

$$M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + H_1 + G_1 = 0 \tag{14}$$

$$M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + H_2 + G_2 = \tau \tag{15}$$

where

$$M_{11} = (m_1 + m_2)l^2 + m_2 q_2^2 + I_1 + I_2,$$

$$M_{12} = M_{21} = -m_2 l,\ M_{22} = m_2,$$

$$H_1 = 2m_2 q_2 \dot{q}_1 \dot{q}_2,\ H_2 = -m_2 q_2 \dot{q}_1^2,$$

$$G_1 = (m_1 + m_2)gl \sin q_1 - m_2 g q_2 \cos q_1,$$

$$G_2 = -m_2 g \sin q_1$$

Denote $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$,then the state equation of the system can be represented as:

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ \dfrac{m_2^2 (lx_3 - 2x_4) x_2 x_3 + m_2 g (m_2 x_2 \cos x_1 - m_1 l \sin x_1)}{m_1 m_2 l^2 + m_2^2 x_2^2 + m_2 (I_1 + I_2)} \\ \dfrac{d_{11} m_2 x_2 x_3^2 + (I_1 + I_2 + m_2 x_2^2) m_2 g \sin x_1 + (g \cos x_1 - 2x_3 x_4) m_2^2 l x_2}{m_1 m_2 l^2 + m_2^2 x_2^2 + m_2 (I_1 + I_2)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{m_2 l}{m_1 m_2 l^2 + m_2^2 x_2^2 + m_2 (I_1 + I_2)} \\ -\dfrac{(m_1 + m_2) l^2 + m_2 x_2^2 + I_1 + I_2}{m_1 m_2 l^2 + m_2^2 x_2^2 + m_2 (I_1 + I_2)} \end{bmatrix} \tau \equiv f(x) + g(x)\tau \tag{16}$$

The equilibrium point x_e of the system should be satisfied with $f(x_e) + g(x_e)\tau_e = 0$, where τ_e is a given input. The contollable set of the equilibrium points constitutes one-dimensional equilibrium manifold[4]. If the system can be stable, it will be converging to a point on the equilibrium manifold. The substance of the setpoint control is the design of controller making the system to move from one point on the manifold to another, the system can not move on equilibrium manifold, but it can move near the quilibrium manifold. If the nonlinear term is high order on the equilibrium manifold of the approximate linearization system, then the system can be regulated with the method of approximate linearization[4].

Substitute $f(x_e) + g(x_e)\tau_e = 0$ into (16), we can conclude that the equilibrium point of the system is satisfied with

$$x_{1e} = \arctan \frac{m_2 x_{2e}}{(m_1 + m_2)l} \tag{17}$$

So the equilibrium manifold of the system is a ARC-tangent curve, as is shown on Figure 3, the final stable point of the system is one point on the curve.

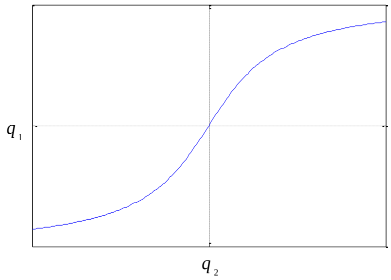


Figure 3. Equilibrium Manifold of B-B System

3.2. The Application of Global Approximate Linearization

According to (3), the state equation of the system becomes to the form of Affine Nonlinear:

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ 0 \\ \frac{2m_2x_2x_3x_4 + (m_1+m_2)gl\sin x_1 - m_2gx_2\cos x_1}{m_2 l} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{(m_1+m_2)l^2 + m_2x_2^{\ 2} + I_1 + I_2}{m_2 l} \end{bmatrix} u \qquad (18)$$

Decompose f_4,

$f_4(x) = -\frac{H_1+G_1}{M_{12}} = \frac{(m_1+m_2)g\sin x_1}{m_2x_1}x_1 + \frac{2x_3x_4 - g\cos x_1}{l}x_2$, that is

$\phi_1 = \frac{(m_1+m_2)g\sin x_1}{m_2x_1}$, $\phi_2 = \frac{2x_3x_4 - g\cos x_1}{l}$, $\phi_3 = \phi_4 = 0$. Obviously for any x of the universe of discourse, $\phi_i \in C^1$ and $\phi_i \neq \infty$. The regularity condition of $T(x)$ turns to $\phi_1 - \phi_2\frac{M_{11}}{M_{12}} \neq 0$, Simplifying it gives

$$\begin{cases} 2M_{11}x_3x_4 - m_2gx_2^{\ 2} - (I_1+I_2)g \neq 0 & if\ \ x_1 = 0 \\ (m_1+m_2)l^2g\sin x_1 + M_{11}(2x_3x_4 - g\cos x_1)x_1 \neq 0 & if\ \ x_1 \neq 0 \end{cases} \qquad (19)$$

For any x of the universe of discourse, the above equation is not always right, and it is difficult to find the method avoiding the singular of $T(x)$ with theoretical analysis, but we can verify that the above equation on equilibrium manifold with constant non-zero. Simulation results show, if the initial state is close to equilibrium manifold, the regularity condition of $T(x)$ can be guaranteed.

In (12),the nonlinear Term is

$$n(x,z) = \frac{1}{M_{11}\phi_2 - M_{12}\phi_1}\begin{bmatrix} (M_{11}\dot{\phi}_2 - M_{12}\dot{\phi}_1)z_1 + \dot{M}_{11}z_3 \\ (M_{11}\dot{\phi}_2 - M_{12}\dot{\phi}_1)z_2 + \dot{M}_{11}z_4 \\ M_{12}(\phi_1\dot{\phi}_2 - \dot{\phi}_1\phi_2)z_1 + \phi_2\dot{M}_{11}z_3 \\ M_{12}(\phi_1\dot{\phi}_2 - \dot{\phi}_1\phi_2)z_2 + \phi_2\dot{M}_{11}z_4 \end{bmatrix} \qquad (20)$$

where $\dot{M}_{11} = 2m_2 x_2 x_4$, $\dot{\phi}_1 = \frac{(m_1 + m_2) g (x_1 \cos x_1 - \sin x_1) x_3}{m_2 x_1^2}$,

$$\dot{\phi}_2 = \frac{2\dot{x}_3 x_4 + 2x_3\dot{x}_4 + g x_3 \sin x_1}{l}$$

Thus the feedback control law of the ball and beam system should be

$$\begin{aligned} u &= -\lambda_1(z_1 - z_{1d}) - (\lambda_2 + \phi_2)(z_2 - z_{2d}) - \lambda_3(z_3 - z_{3d}) \\ &- \lambda_4(z_4 - z_{4d}) - \frac{M_{12}(\phi_1\dot{\phi}_2 - \dot{\phi}_1\phi_2)(z_2 - z_{2d}) + \phi_2\dot{M}_{11}(z_4 - z_{4d})}{M_{12}\phi_1 - M_{11}\phi_2} \end{aligned} \tag{21}$$

Where $\lambda_1 \sim \lambda_4$ is Hurwitz's constant coefficient, $z_d = [z_{1d} \quad z_{2d} \quad z_{3d} \quad z_{4d}]^T = T^{-1}(x_d) x_d$ is the expectation state of the system in new state space.

We can verify that $n(x, z)$ and its first derivative on equilibrium manifold is zero, that said , $n(x, z)$ is high order on equilibrium manifold, it means that the global approximate linearization method is effective for local stabilization on equilibrium manifold in the ball and beam system.

3.3. Simulation and Analysis

Simulations are given for the position control of the ball and beam system with the above control law. Parameters of the ball and beam system is as shown in Tab.1.And Hurwitz's coefficients are: $\lambda_0 = 16$, $\lambda_1 = 32$, $\lambda_2 = 24$, $\lambda_3 = 8$ 。

Table 1. Parameters of the ball and beam system

m1	m2	l	I1	I2	g
4.8 kg	2.5 kg	0.5 m	1.667 gm2	0	9.8 m/s2

Experiment 1: The control about from one balance setpoint to another on equilibrium manifold to verify the validity of the controller for fixed-point control. Assuming that the initial state of the system is $x_0 = [0 \quad 0 \quad 0 \quad 0]^T$, and the expectation state is $x_d = [0.5 \quad 0.8 \quad 0 \quad 0]^T$. The situation of change of joint variables is expressed in Figure 4(a), the required joint input τ is expressed in Figure 4(b), the numerical change of nonlinear components is expressed in Figure 4(c), the relations between the two joints of variables and equilibrium manifold is expressed in Figure 4(d). From Figure 4, we can discover the joints can convergence to the expectation equilibrium point with a smooth input torque in short time, and there is no larger overshoot and show the similar characteristics to the overdamp linear system. It is because in this case, the values of nonlinear component are minor, and they are the high order smalls of the pseudo –linear items. And the lead section of the system is the pseudo-linear section of the overdamp system by pole placement. At the same time, because the nonlinear term is high order on equilibrium manifold and the pseudo –linear section could control the system to move along the equilibrium manifold, the system will move near the equilibrium manifold, so it shows the good linearity. The whole control process can be divided into the following four stages: At the beginning, it needs exert a positive smaller force to corrupt the equilibrium state of the system, and the system deviates from the equilibrium manifold gradually; Secondly, impose a smaller force in positive and negative directions to the system, suppress the deviation of the system state, pull the system back near the

equilibrium manifold; Nextly, impose a increase force gradually, make the state of the system move along the equilibrium manifold; Finally, decrease the increasing slope of the force gradually, suppress the increase of the joint speed, ultimately achieve a constant force, make the system stability at the specified equilibrium point.

Experiment 2: From non-equilibrium point outside of equilibrium manifold to a specific balance of control.This experiment is mainly test the anti-sway stability of the controller to the system. Assuming that the initial state of the system is $x_0 = [-0.05 \quad 0.1 \quad 0.2 \quad 0.3]^T$, and the expectation state is $x_d = [0 \quad 0 \quad 0 \quad 0]^T$. The response curve of joint variables is expressed in Figure 5(a). The relationship between Two-joint variable and balancing manifolds is expressed in Figure 5(b). Since the initial state of the system is not an equilibrium, and further away from equilibrium manifold, that is, at the time of the initial state of the system with redundancy "energy", So it needs to move the system to a point with "energy" near equilibrium manifold, then use the control programme about balance to balance to move it to specify balance point. In the response curve of joint variable, there is a big overshoot due to "redundancy". In the control process, nonlinear high-order small quantity can no longer be ignored, sometimes equal to the value of pseudo linear. So the controller designed in this paper is effective to the case which the initial state is near equilibrium manifold and control effect on equilibrium manifold of the more far away the worse, could not even calm.

Experiment 3: Trajectory tracking experiment. While we discussed the set-point control of the system, but the controller is also effective to a certain extent for trajectory tracking. Assuming that the initial state of the system is $x_0 = [0 \quad 0.1 \quad 0 \quad 0]^T$, the expected path of the system is given by moving joints, path equation is $q_{2d}(t) = 0.2\sin(\frac{\pi}{5}t) + 0.1$. Since control objective is not eventually reach an equilibrium point, but tracking a trail, so the expectations path of the rotary joint cannot be obtained in accordance with equilibrium manifold, but should be calculated by type (14), as the dashes shown in Figure 6(a). The trajectory tracking response curve of two joint is expressed in Figure 6(a) and Figure 6(b), the tracking error of two joint is expressed in Figure 6(c) and Figure 6(d). Thus, the controller for the trajectory tracking has some effect, but there is still more obvious dynamic errors. This is because the track is no longer the problems on the balanced manifold, but it needs to consider the dynamics of the system. And in linearization method, the nonlinear terms reflected the dynamic characteristics is considered as high-order small quantities, so the approximate linearization method has some limitations on the trajectory tracking control.

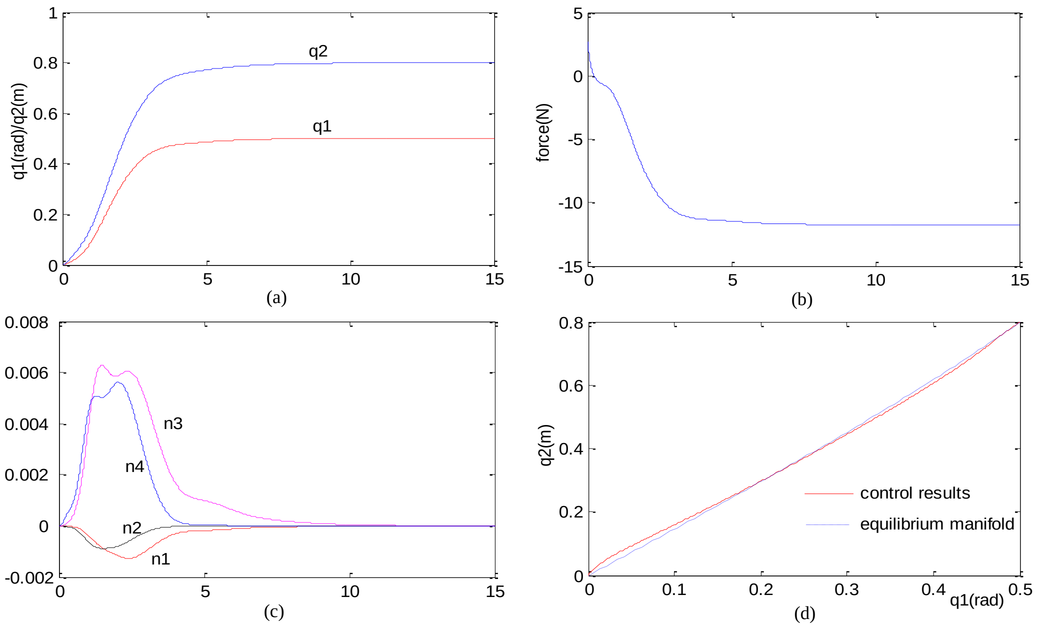


Figure 4. Simulation Results of Set-Point Control Problem

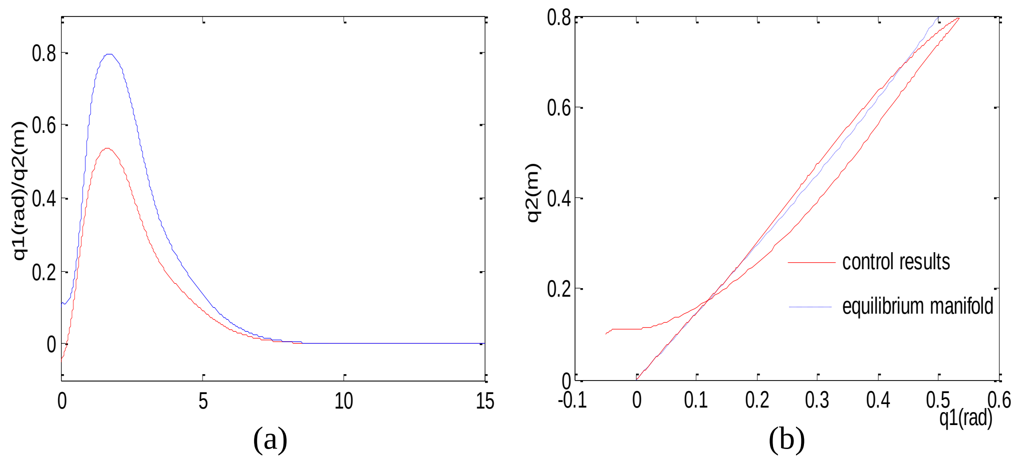


Figure 5. Simulation Results for Case of Start Point

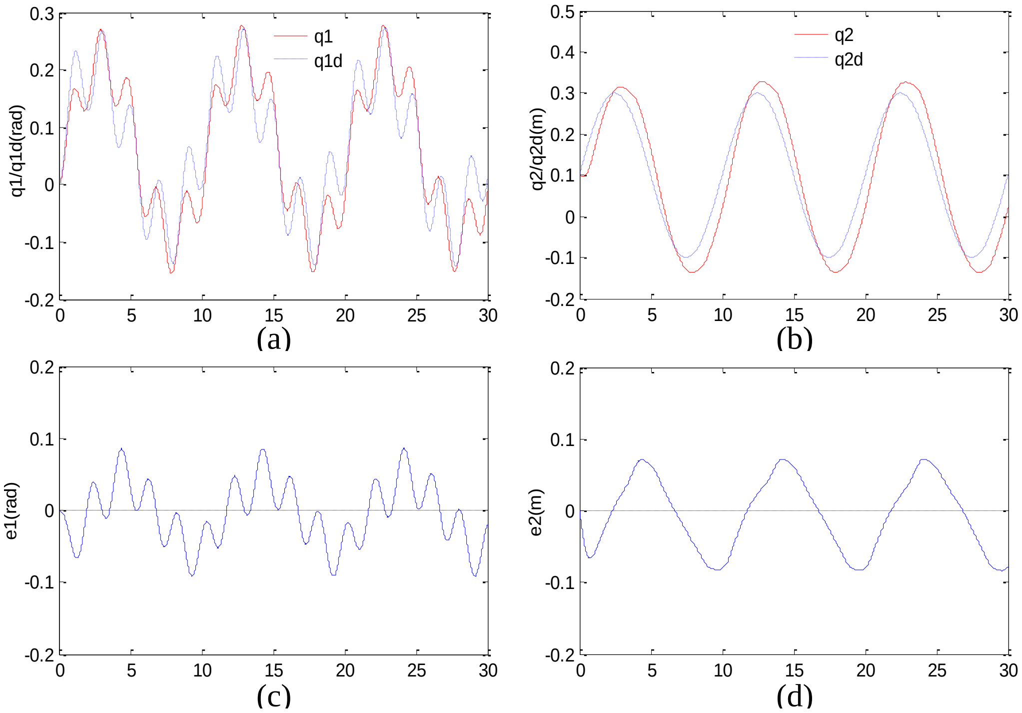


Figure 6. Simulation Results of Trajectory Tracking Problem

4. Discussion and Conclusion

In this paper, we studied the dynamical model of a class of underactuated systems. By combination of the partial feedback linearization and Yamada's approximate linearization method for underactuated systems. By using this method, the dynamical global linearization, we deduced a global equations can be transformed into an state eqution that is expressed as a pseudolinear term with Brunovsky canonical form plus a high order nonlinear term, where the nonlinear term is high order on the equilibrium manifold of the system. By standard nonlinear feedback method, the system is transformed into the sum of a stable linear term and a high order nonlinear term. Take proper feedforward value as the input to reduce the influence of the nonlinear term to the system and thus the underactuated system can be regulated. This method is applied to the ball and beam system and simulation results show that the proposed approximate linearization method is effective for setpoint

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