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An Approach based on TOPSIS for Interval Type-2 Fuzzy Multiple Attributes Decision-making

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Abstract

The type-2 fuzzy sets (T2FSs) provide us additional degrees of freedom to represent the uncertainty and the fuzziness of the real world. As a special kind of T2FSs, interval type-2 fuzzy sets (IT2FSs), whose operations are much simpler than the ones of general T2FSs. So the research of IT2FSs has become a hot topic in decision-making field in recent years. In this paper, firstly, we introduce the operational laws of IT2FSs, and propose a distance measure of IT2FSs. Secondly, we present a novel approach for handling multiple attributes decision-making problems with IT2FSs, on base of the idea of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The method proposed is more practical and flexible because it not only consider the importance weights, but also reflect the decision strategies that both are essential to different decision makers for choosing different decision tactics. Finally, a numerical example of service quality assessment is used to verify the effectiveness of the presented approach.

Keywords: *Interval type-2 fuzzy sets, multiple attributes decision-making, Distance measure, TOPSIS*

1. Introduction

The techniques of Multiple attributes decision-making (Abbr. MADM) are used mainly to find a desirable solution from a set of alternatives that evaluated by multiple attributes. In recent years, various methods have been proposed for handling fuzzy decision-making problems. Among those methods, the TOPSIS is the most popular one because it gives us an alternative from comparing the relative closeness to the ideal solution. Chen [1] presented a method to assess the rate of aggregative risk in software development using the fuzzy set theory under the fuzzy GDM environment. However, the above methods are just based on type-1 fuzzy sets (T1FSs) with one approach. If we can use T2FSs to solve MADM problems, then the flexible and space will be enhanced greatly. Because the membership functions (Abbr. MFs) of T2FSs are characterized by more parameters than the ones for T1FSs. In T1FSs, MFs are totally certain, whereas in T2FSs, MFs are fuzzy themselves. In other words, a T2FS can be visualized as a three dimensional, primary and secondary MF. The primary membership is any subset in [0, 1] and there is a secondary membership value corresponding to each primary membership value that defines the possibility for primary membership. Hence, T2FSs provide us with additional degrees of freedom and using T2FSs has the potential to outperform using T1FSs, especially when we are in deeply uncertain environments.

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For an IT2FS that third-dimension value is the same, and only the footprint of uncertainty is used to describe it. As a special kind of T2FSs, IT2FSs have received the most attention and have applied in main fields because their operations are much simpler than the ones of general T2FSs. That is to say that IT2FS is the most widely used in T2FS today because it is computationally simple to use. Mendel [2] introduced the advances about type-2 fuzzy sets and systems. Mendel and Wu [3] found new results about the centroid of an IT2FS including the centroid of a fuzzy granule. Wu and Mendel [4] designed an uncertainty measures for IT2FSs. Wu and Mendel [5] developed a vector similarity measure for linguistic approximation IT2FSs and T1FSs. As for similarity measures and uncertainty measures of IT2FSs, Wu and Mendel [6] presented a comparative study of ranking methods. Based on arithmetic operations and fuzzy preference relations of IT2FSs, Chen and Lee [7] proposed a new method for handling fuzzy multiple criteria hierarchical GDM problems. Zhai and Mendel [8] designed an uncertainty measures for general T2FSs. Under IT2F environment, Wang et al [9] developed multi-attribute group decision making models. Takac [10] proposed an inclusion and subset-hood measure for interval valued fuzzy sets and for continuous T2FSs. Using feedback error learning approaches, Sabahi et al [11] presented an application of type 2 fuzzy logic system for load frequency control.

Since T2FSs have the superiority that they provide us additional degrees of freedom to represent the uncertainty and the fuzziness of the real world rather than traditional T1FSs to assess the evaluating values. In this paper, we firstly introduce the basic laws of T2FSs and IT2FSs and then propose a score function to compare with multiple IT2FSs. Next, a novel distance measure of IT2FSs is presented to measure the difference of them. Furthermore based on which, we construct a MADM method under the TOPSIS methodology in IT2FSs. Finally, an example is applied to testify the validity of the method and to compare to other approaches in order to reflect the advantage of it. The results indicate that the proposed method provides us a useful way to handle fuzzy MADM problems in a more reasonable and more flexible manner.

2. Interval Type-2 Fuzzy Sets

In this section, we briefly review some basic concepts of IT2FSs [12] and r-polygonal interval type-2 fuzzy sets (RIT2FSs), and then we define a score function to compare with multiple IT2FSs.

Definition 1. A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $u_{\tilde{\tilde{A}}}$, shown as follows:

$$\tilde{\tilde{A}} = \{((x,u), u_{\tilde{\tilde{A}}}(x,u)) \mid \forall x \in X, \forall u \in J_X \subseteq [0,1], 0 \le u_{\tilde{\tilde{A}}}(x,u) \le 1)\} \tag{1}$$

where J_X denotes an interval in $[0,1]$.

For another simple form in integral, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} u_{\tilde{\tilde{A}}}(x,u)/(x,u) \tag{2}$$

where $J_X \subseteq [0,1]$ and $\iint$ denotes union over all admissible x and u. If all

$u_{\tilde{\tilde{A}}}(x,u)=1$, then $\tilde{\tilde{A}}$ is called an IT2FS [12].

IT2FSs have used widely in most applications because their operations are much simpler than the ones of general T2FSs. Moreover for interval functions, they always vibrate in a fixed field as an uncertain number. So, in this part, we import a RIT2FS to present the uncertain number, shown as follow:

Definition 2 . The r-polygonal interval type-2 fuzzy set $\tilde{\tilde{A}}$ is defined as $\tilde{\tilde{A}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L)$

$$= ((\tilde{a}_1^U, \tilde{a}_2^U, ..., \tilde{a}_r^U; H_1(\tilde{\tilde{A}}^U), ..., H_{r-2}(\tilde{\tilde{A}}^U)), (\tilde{a}_1^L, \tilde{a}_2^L, ..., \tilde{a}_r^L; H_1(\tilde{\tilde{A}}^L), ..., H_{r-2}(\tilde{\tilde{A}}^L))) \quad (3)$$

where r denotes the number of edges in the upper r-polygonal membership function $\tilde{\tilde{A}}^U$ and the lower r-polygonal membership function $\tilde{\tilde{A}}^L$, respectively. $H_j(\tilde{\tilde{A}}^U)$ denotes the membership value of the element a_{j+1}^U in the upper r-polygonal membership function $\tilde{\tilde{A}}^U$, $H_j(\tilde{\tilde{A}}^L)$ denotes the membership value of the element a_{j+1}^L in the lower r-polygonal membership function $\tilde{\tilde{A}}^L$,and $1 \le j \le r-2, H_1(\tilde{\tilde{A}}^U) \in [0,1], ... H_{r-2}(\tilde{\tilde{A}}^U) \in [0,1]$, $H_1(\tilde{\tilde{A}}^L) \in [0,1]$, $... H_{r-2}(\tilde{\tilde{A}}^L) \in [0,1], r \ge 3$.

Some basic operational laws of RIT2FS are shown as follow:

Definition 3 . For any two r-polygonal interval type-2 fuzzy sets, let $\tilde{\tilde{A}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L) = ((\tilde{a}_1^U, \tilde{a}_2^U, ..., \tilde{a}_r^U; H_1(\tilde{\tilde{A}}^U), ..., H_{r-2}(\tilde{\tilde{A}}^U)), (\tilde{a}_1^L, \tilde{a}_2^L, ..., \tilde{a}_r^L; H_1(\tilde{\tilde{A}}^L), ..., H_{r-2}(\tilde{\tilde{A}}^L)))$ and $\tilde{\tilde{B}} = (\tilde{\tilde{B}}^U, \tilde{\tilde{B}}^L) = ((\tilde{\tilde{b}}_1^U, \tilde{\tilde{b}}_2^U, ..., \tilde{\tilde{b}}_r^U; H_1(\tilde{\tilde{B}}^U), ..., H_{r-2}(\tilde{\tilde{B}}^U)), (\tilde{\tilde{b}}_1^L, \tilde{\tilde{b}}_2^L, ..., \tilde{\tilde{b}}_r^L; H_1(\tilde{\tilde{B}}^L), ..., H_{r-2}(\tilde{\tilde{B}}^L)))$, then;

(1)The addition operation $\oplus$: $\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L) \oplus (\tilde{\tilde{B}}^U, \tilde{\tilde{B}}^L) =$

$$((\tilde{a}_1^U + \tilde{\tilde{b}}_1^U, \tilde{a}_2^U + \tilde{\tilde{b}}_2^U, ..., \tilde{a}_r^U + \tilde{\tilde{b}}_r^U; \min(H_1(\tilde{\tilde{A}}^U), H_1(\tilde{\tilde{B}}^U))..., \min(H_{r-2}(\tilde{\tilde{A}}^U), H_{r-2}(\tilde{\tilde{B}}^U)), \quad (4)$$

$$(\tilde{a}_1^L + \tilde{\tilde{b}}_1^L, \tilde{a}_2^L + \tilde{\tilde{b}}_2^L, ..., \tilde{a}_r^L + \tilde{\tilde{b}}_r^L; \min(H_1(\tilde{\tilde{A}}^L), H_1(\tilde{\tilde{B}}^L)), ..., \min(H_{r-2}(\tilde{\tilde{A}}^L), H_{r-2}(\tilde{\tilde{B}}^L)))$$

(2)The subtraction operation $\ominus$: $\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L) \ominus (\tilde{\tilde{B}}^U, \tilde{\tilde{B}}^L) =$

$$((\tilde{a}_1^U - \tilde{\tilde{b}}_1^U, \tilde{a}_2^U - \tilde{\tilde{b}}_2^U, ..., \tilde{a}_r^U - \tilde{\tilde{b}}_r^U; \min(H_1(\tilde{\tilde{A}}^U), H_1(\tilde{\tilde{B}}^U)), ..., \min(H_{r-2}(\tilde{\tilde{A}}^U), H_{r-2}(\tilde{\tilde{B}}^U)), \quad (5)$$

$$(\tilde{a}_1^L - \tilde{\tilde{b}}_1^L, \tilde{a}_2^L - \tilde{\tilde{b}}_2^L, ..., \tilde{a}_r^L - \tilde{\tilde{b}}_r^L; \min(H_1(\tilde{A}^L), H_1(\tilde{B}^L)), ..., \min(H_{r-2}(\tilde{A}^L), H_{r-2}(\tilde{B}^L)))$$

(3)The multiplication operation $\otimes$: $\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} = (\tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L) \otimes (\tilde{\tilde{B}}^U, \tilde{\tilde{B}}^L) =$

$$((\tilde{a}_1^U \times \tilde{\tilde{b}}_1^U, \tilde{a}_2^U \times \tilde{\tilde{b}}_2^U, ..., \tilde{a}_r^U \times \tilde{\tilde{b}}_r^U; \min(H_1(\tilde{A}^U), H_1(\tilde{B}^U)), ..., \min(H_{r-2}(\tilde{A}^U), H_{r-2}(\tilde{B}^U)), \quad (6)$$

$$(\tilde{a}_1^L \times \tilde{a}_1^L, \tilde{a}_2^L \times \tilde{a}_2^L, ..., \tilde{a}_r^L \times \tilde{a}_r^L; \min(H_1(\tilde{A}^L), H_1(\tilde{B}^L)), ..., \min(H_{r-2}(\tilde{A}^L), H_{r-2}(\tilde{B}^L)))$$

In order to compare with the IT2FS, in what following we define a score function of the RIT2FS.

Definition 4. Let $\tilde{\tilde{a}} = (\tilde{a}^U, \tilde{a}^L) = ((a_1^U, a_2^U ..., a_r^U; H_1(\tilde{a}^U), ..., H_{r-2}(\tilde{a}^U)), (a_1^L, a_2^L, ..., a_r^L;$

$H_1(\tilde{\tilde{a}}^L),...,H_{r-2}(\tilde{\tilde{a}}^L)))$, then the score function of IT2FS $\tilde{\tilde{a}}$ is defined as follows:

$$score(\tilde{\tilde{a}})=\frac{1}{r}\left(\sum_{j=1}^{r-2}\left(\frac{H_j(\tilde{\tilde{a}}^L)\tilde{\tilde{a}}_j^L+H_j(\tilde{\tilde{a}}^U)\tilde{\tilde{a}}_j^U}{H_j(\tilde{\tilde{a}}^L)+H_j(\tilde{\tilde{a}}^U)}\right)+\frac{1}{2}\left(\tilde{\tilde{a}}_1^L+\tilde{\tilde{a}}_1^U+\tilde{\tilde{a}}_r^L+\tilde{\tilde{a}}_r^U\right)\right) \quad (7)$$

Obviously, $score(\tilde{\tilde{a}})\in[0,1]$.

Example 1: Assume two simple IT2FSs $\tilde{\tilde{a}}=((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,$ $0.9))$ and $\tilde{\tilde{b}}=((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))$, from the formula above, we can get: $score(\tilde{\tilde{a}})=0.1, score(\tilde{\tilde{b}})=0.26$. Since $score(\tilde{\tilde{a}})<score(\tilde{\tilde{b}})$, so we can know $\tilde{\tilde{a}}$ is more stable than $\tilde{\tilde{b}}$, and $\tilde{\tilde{a}}$ is a better choice.

3. Distance Measure for IT2FS

Distance measure is an important tool for distinguishing the difference between two objects, and has become important due to the significant applications in diverse fields like decision making, pattern recognition, market prediction and so on. In this section, we propose a distance measure for two IT2FSs.

Definition 3.1 Let $\tilde{\tilde{a}}=(\tilde{\tilde{a}}^U,\tilde{\tilde{a}}^L)=((a_1^U,a_2^U...,a_r^U;H_1(\tilde{\tilde{a}}^U),...,H_{r-2}(\tilde{\tilde{a}}^U)),(a_1^L,a_2^L,...,a_r^L;$ $H_1(\tilde{\tilde{a}}^L),...,H_{r-2}(\tilde{\tilde{a}}^L)))$, be two IT2FSs, then the distance measure of $\tilde{\tilde{a}}$ and $\tilde{\tilde{b}}$ is defined as:

$$d(\tilde{\tilde{a}},\tilde{\tilde{b}})=\frac{1}{r}\left(\sum_{j=1}^{r-2}\frac{\min(H_j(\tilde{\tilde{a}}^L),H_j(\tilde{\tilde{b}}^L))\,|\,\tilde{\tilde{a}}_j^L-\tilde{\tilde{b}}_j^L\,|+\min(H_j(\tilde{\tilde{a}}^U),H_j(\tilde{\tilde{b}}^U))\,|\,\tilde{\tilde{a}}_j^U-\tilde{\tilde{b}}_j^U\,|}{\min(H_j(\tilde{\tilde{a}}^L),H_j(\tilde{\tilde{b}}^L))+\min(H_j(\tilde{\tilde{a}}^U),H_j(\tilde{\tilde{b}}^U))}\right. \quad (8)$$

$$\left.+\frac{1}{2}\left(|\,\tilde{\tilde{a}}_1^L-\tilde{\tilde{b}}_1^L\,|+|\,\tilde{\tilde{a}}_1^U-\tilde{\tilde{b}}_1^U\,|+|\,\tilde{\tilde{a}}_r^L-\tilde{\tilde{b}}_r^L\,|+|\,\tilde{\tilde{a}}_r^U-\tilde{\tilde{b}}_r^U\,|\right)\right)$$

It is easy to prove that the defined distance measure satisfies the following properties:

1. Non-negativity: $0\le d(\tilde{\tilde{a}},\tilde{\tilde{b}})\le 1$;
2. Commutability: $d(\tilde{\tilde{a}},\tilde{\tilde{b}})=d(\tilde{\tilde{b}},\tilde{\tilde{a}})$;
3. Reflexivity: $d(\tilde{\tilde{a}},\tilde{\tilde{b}})=0$ if and only if $\tilde{\tilde{a}}=\tilde{\tilde{b}}$.

Example 2. Assuming two IT2FSs $\tilde{\tilde{a}}=((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))$ and $\tilde{\tilde{b}}=((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))$, based on Eq. (8), then we can draw that

$$d(\tilde{\tilde{a}},\tilde{\tilde{b}})=\frac{1}{4}[\frac{0.9\times|0.05-0.2|+1\times|0-0.1|}{0.9+1}+\frac{0.9\times|0.1-0.3|+1\times|0.1-0.3|}{0.9+1}$$

$$+\frac{1}{2}\left(|0.05\text{-}0.2|+|0\text{-}0.1|+|0.2\text{-}0.4|+|0.3\text{-}0.5|\right)]$$

$$=\frac{1}{4}[\frac{0.13+0.1+0.18+0.2}{0.9+1}+\frac{1}{2}\left(0.15+0.1+0.2+0.2\right)]=0.16\,.$$

From the above, we can know that the distance measure is only used for two IT2FSs, but in solving practical problems, there always have two or more group IT2FSs. So, in the following we adhere to deal the group IT2F problems.

What's more, in dealing with the actual MADM problems, there always have some weighting factors to different attributes. Among various distance measures, the ones most commonly employed decision-making are the weighted distance measure (such as the weighted hamming distance (WHD) and the weighted Euclidean distance (WED)). Recently, motivated by the idea of the ordered weighted averaging (OWA) operator, some researchers are interested in the OWD measure and have a further investigation. However, the weighted distance (WD) measure focuses solely on the weight of the individual distance itself, and ignores the position weight with respect to the individual distance value, while the OWD measure focuses only on the position weight with respect to the individual distance value, and ignores the weight of the individual distance value itself. Therefore, weights represent different aspects in the two measures. But both of the measures consider only one weight from one aspect. For the sake of solving the drawbacks, Peng, et al [13] presented a synergetic weighted distance (SWD) measure based on the synergetic weighted ideal, in this part, we propose a SWD measure between two collections of IT2FS.

Definition 3.2. Let $\tilde{\tilde{A}}=(\tilde{\tilde{a}}_1,\tilde{\tilde{a}}_2,...,\tilde{\tilde{a}}_n)$ and $\tilde{\tilde{B}}=(\tilde{\tilde{b}}_1,\tilde{\tilde{b}}_2,...,\tilde{\tilde{b}}_n)$ be two IT2FSs of a finite universe of discourse $X=\{x_1,x_2,...,x_n\}$, then a synergetic weighted distance measure of $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ is defined as:

$$D(\tilde{\tilde{A}},\tilde{\tilde{B}})=\sum_{i=1}^{n}\frac{w_{\rho(i)}d(\tilde{\tilde{a}}_i,\tilde{\tilde{b}}_i)\omega_i}{\sum_{i=1}^{n}w_{\rho(i)}\omega_i} \tag{9}$$

where $\rho:\{1,2,...,n\}\to\{1,2,...,n\}$ is a permutation function such that $d(\tilde{\tilde{a}}_i,\tilde{\tilde{b}}_i)$ is the $\rho(i)th$ largest element of the collection of individual distances $d(\tilde{\tilde{a}}_i,\tilde{\tilde{b}}_i)(i=1,2,...,n)$, and $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the relative weighting vector of the $d(\tilde{\tilde{a}}_i,\tilde{\tilde{b}}_i)(i=1,2,...,n)$, with $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$.

The predominated advantages of the distance measure are that it not only consider the importance weights, but also reflect the decision strategies that both are extremely vital in the practical decision making situations.

4. TOPSIS Method

TOPSIS method is proposed by Hwang and Yoon [14]. The essential principle is that the alternative that been chosen should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution.

Suppose a multiple attributes decision making problem, $G=\{g_1,g_2,...,g_m\}$ be the set of attributes, (Generally, the attribute can be divided to benefit and cost types. For benefit attribute, the larger the better, and for cost attribute, the smaller the better). $A=\{a_1,a_2,...,a_n\}$ be the alternatives, then constructing a decision matrix $A=(a_{ij})_{n\times m}$, where $1\le i\le n, 1\le j\le m$. The steps of TOPSIS method consist of the following [15]:

Step 1: Calculate the normalized decision matrix$(k_{ij})_{n\times m}$;

$$k_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} a_{ij}^2}}, 1 \le i \le n, 1 \le j \le m \quad (10)$$

Step 2: Calculate the weighted normalized decision matrix x_{ij};

$$x_{ij} = w_j k_{ij}, 1 \le i \le n, 1 \le j \le m \quad (11)$$

Where w_j is the weight of the attribute g_j, with $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

Step 3: Determine the ideal and negative ideal solution;

$$I^* = \{x_1^*, ..., x_m^*\} = \{(Max_j x_{ij} \mid j \in \Omega_b), (Min_j x_{ij} \mid j \in \Omega_c)\} \quad (12)$$

$$I^- = \{x_1^-, ..., x_m^-\} = \{(Min_j x_{ij} \mid j \in \Omega_b), (Max_j x_{ij} \mid j \in \Omega_c)\} \quad (13)$$

where Ω_b and Ω_c are the sets of benefit and cost criteria respectively.

Step 4: Calculate the separation measures, using the n dimensional Euclidean distance. The separation of each alternative from the positive ideal solution is given as;

$$D_i^* = \sqrt{\sum_{j=1}^{m} (x_{ij} - x_j^*)^2}, 1 \le i \le n, 1 \le j \le m \quad (14)$$

Similarly, the separation from the negative ideal solution is given as;

$$D_i^- = \sqrt{\sum_{j=i}^{m} (x_{ij} - x_j^-)^2}, 1 \le i \le n, 1 \le j \le m \quad (15)$$

Step 5: Calculate the relative closeness to the ideal solution. The relative closeness is defined as;

$$RC_i = \frac{D_i^-}{D_i^- + D_i^*}, 1 \le i \le n \quad (16)$$

Step 6: Rank and prioritize the alternatives according to their relative closeness.

5. An IT2F-TOPSIS Method for Handing MADM Problems

In this section, based on the TOPSIS method, we propose a method for handling MADM problem under IT2FSs environment. Assume that there are n alternatives $x_1, x_2, ..., x_n$ and m attributes $f_1, f_2, ..., f_m$. The decision values take form as an IT2FSs. The proposed method is now presented as follows:

Step 1: Construct the decision matrix X:

$$X = (\tilde{\tilde{x}}_{ij})_{n\times m} = \begin{array}{c c} & \begin{array}{cccc} f_1 & f_2 & \cdots & f_m \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_n \end{array} & \begin{bmatrix} \tilde{\tilde{x}}_{11} & \tilde{\tilde{x}}_{12} & \cdots & \tilde{\tilde{x}}_{1m} \\ \tilde{\tilde{x}}_{21} & \tilde{\tilde{x}}_{22} & \cdots & \tilde{\tilde{x}}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{x}}_{n1} & \tilde{\tilde{x}}_{n2} & \cdots & \tilde{\tilde{x}}_{nm} \end{bmatrix} \end{array} \quad (17)$$

where $\tilde{\tilde{x}}_{ij}$ is an IT2FS, $1 \le i \le n, 1 \le j \le m$.

Step 2: Find the positive ideal solution $\tilde{\tilde{x}}^+$ (whose score is the highest) and the negative ideal solution $\tilde{\tilde{x}}^-$ (whose score is the lowest), shown as follows:

$$\tilde{\tilde{x}}^+ = (\tilde{\tilde{x}}_1^+, \tilde{\tilde{x}}_2^+, ..., \tilde{\tilde{x}}_m^+) = (\tilde{\tilde{x}}_{ij} \mid \max_{1\le i\le n}\{Score(\tilde{\tilde{x}}_{ij})\}, 1\le j\le m) \tag{18}$$

$$\tilde{\tilde{x}}^- = (\tilde{\tilde{x}}_1^-, \tilde{\tilde{x}}_2^-, \cdots, \tilde{\tilde{x}}_m^-) = (\tilde{\tilde{x}}_{ij} \mid \min_{1\le i\le n}\{Score(\tilde{\tilde{x}}_{ij})\}, 1\le j\le m) \tag{19}$$

Step 3: Calculate the distance $d^+\left(\tilde{\tilde{x}}_i\right)$ between each alternative $\tilde{\tilde{x}}_i$ and the positive ideal solution $\tilde{\tilde{x}}^+$, shown as follows:

$$d(\tilde{\tilde{x}}_{ij}, \tilde{\tilde{x}}_j^+) = \frac{1}{r}\left(\sum_{k=1}^{r-2}\frac{\min(H_k(\tilde{\tilde{x}}_{ijk}^L), H_k(\tilde{\tilde{x}}_{jk}^{+L}))\,|\,\tilde{\tilde{x}}_{ijk}^L - \tilde{\tilde{x}}_{jk}^{+L}\,| + \min(H_k(\tilde{\tilde{x}}_{ijk}^U), H_k(\tilde{\tilde{x}}_{jk}^{+U}))\,|\,\tilde{\tilde{x}}_{ijk}^U - \tilde{\tilde{x}}_{jk}^{+U}\,|}{\min(H_k(\tilde{\tilde{x}}_{ijk}^L), H_k(\tilde{\tilde{x}}_{jk}^{+L})) + \min(H_k(\tilde{\tilde{x}}_{ijk}^U), H_k(\tilde{\tilde{x}}_{jk}^{+U}))}\right.$$
$$\left. + \frac{1}{2}\left(|\,\tilde{\tilde{x}}_{ij1}^L - \tilde{\tilde{x}}_{j1}^{+L}\,| + |\,\tilde{\tilde{x}}_{ij1}^U - \tilde{\tilde{x}}_{j1}^{+U}\,| + |\,\tilde{\tilde{x}}_{ijr}^L - \tilde{\tilde{x}}_{jr}^{+L}\,| + |\,\tilde{\tilde{x}}_{ij1}^U - \tilde{\tilde{x}}_{jr}^{+U}\,|\right)\right) \tag{20}$$

$$1\le i\le n, 1\le j\le m$$

Calculate the distance $d^-\left(\tilde{\tilde{x}}_i\right)$ between each alternative $\tilde{\tilde{x}}_i$ and the negative ideal solution $\tilde{\tilde{x}}^-$, shown as follows:

$$d(\tilde{\tilde{x}}_{ij}, \tilde{\tilde{x}}_j^-) = \frac{1}{r}\left(\sum_{k=1}^{r-2}\frac{\min(H_k(\tilde{\tilde{x}}_{ijk}^L), H_k(\tilde{\tilde{x}}_{jk}^{-L}))\,|\,\tilde{\tilde{x}}_{ijk}^L - \tilde{\tilde{x}}_{jk}^{-L}\,| + \min(H_k(\tilde{\tilde{x}}_{ijk}^U), H_k(\tilde{\tilde{x}}_{jk}^{-U}))\,|\,\tilde{\tilde{x}}_{ijk}^U - \tilde{\tilde{x}}_{jk}^{-U}\,|}{\min(H_k(\tilde{\tilde{x}}_{ijk}^L), H_k(\tilde{\tilde{x}}_{jk}^{-L})) + \min(H_k(\tilde{\tilde{x}}_{ijk}^U), H_k(\tilde{\tilde{x}}_{jk}^{-U}))}\right.$$
$$\left. + \frac{1}{2}\left(|\,\tilde{\tilde{x}}_{ij1}^L - \tilde{\tilde{x}}_{j1}^{-L}\,| + |\,\tilde{\tilde{x}}_{ij1}^U - \tilde{\tilde{x}}_{j1}^{-U}\,| + |\,\tilde{\tilde{x}}_{ijr}^L - \tilde{\tilde{x}}_{jr}^{-L}\,| + |\,\tilde{\tilde{x}}_{ij1}^U - \tilde{\tilde{x}}_{jr}^{-U}\,|\right)\right) \tag{21}$$

$$1\le i\le n, 1\le j\le m$$

Step 4: Calculate the generalized weight distance $D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^+)$ and $D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^-)$, shown as follows:

$$D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^+) = \sum_{j=1}^{m}\frac{w_{\rho(j)} d(\tilde{\tilde{x}}_{ij}, \tilde{\tilde{x}}_j^+)\omega_j}{\sum_{j=1}^{m} w_{\rho(j)}\omega_j},\ 1\le i\le n \tag{22}$$

$$D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^-) = \sum_{j=1}^{m}\frac{w_{\rho(j)} d(\tilde{\tilde{x}}_{ij}, \tilde{\tilde{x}}_j^-)\omega_j}{\sum_{j=1}^{m} w_{\rho(j)}\omega_j},\ 1\le i\le n \tag{23}$$

Step 5: Calculate the relative degree of closeness $C(\tilde{\tilde{x}}_i)$, shown as follows:

$$C(\tilde{\tilde{x}}_i) = \frac{D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^+)}{D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^+) + D(\tilde{\tilde{x}}_i, \tilde{\tilde{x}}^-)},\ 1\le i\le n \tag{24}$$

Step 6: Rank the alternatives according to the values of $C(\tilde{\tilde{x}}_i)$ in a descending sequence. The larger the value of $C(\tilde{\tilde{x}}_i)$, the more the preference of the alternate $x_i (i = 1, 2, ..., n)$.

6. Numerical Example

The development and prosperity of a company lie in it has some great teams, so the personnel's recruiting is extremely important for a company. Due to the importance of the personnel, a real example of the personnel selection problem is adopted in this section. Suppose that a Telecommunication Company intends to choose a manager for Research and Development (R&D) department from three volunteers named x_1, x_2, x_3 . The decision making committee assesses the four concerned volunteers based on four criteria which follow: a) proficiency in identifying research areas (f_1), b) proficiency in administration (f_2), c) past experience (f_3), d) self-confidence (f_4), and their weights are $\omega = (0.2, 0.3, 0.4, 0.1)$. The ordered weights are $w^+ = (0.44, 0.31, 0.19, 0.06)$ and $w^- = (0.06, 0.19, 0.31, 0.44)$ .The alternatives are assessed by seven linguistic terms, the linguistic term and their corresponding IT2FSs are listed in table 1. Evaluating values of alternatives with respect to different attributes decided by the decision-makers are shown in table 2.

Table 1. Linguistic Terms and their Corresponding Interval Type-2 Fuzzy Sets

Linguistic Terms	Interval type-2 Fuzzy sets
Very Poor(VP)	((0, 0, 0, 0.1; 1, 1),(0, 0, 0, 0.05; 0.9, 0.9))
Poor(P)	((0, 0.1, 0.1, 0.3; 1, 1),(0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Moderately Poor(MP)	((0.1, 0.3, 0.3, 0.5; 1, 1),(0.2, 0.3, 0.3, 0.4; 0.9, 0.9))
Fair(F)	((0.3, 0.5, 0.5, 0.7; 1, 1),(0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Moderately Good(MG)	((0.5, 0.7, 0.7, 0.9; 1, 1),(0.6, 0.7, 0.7, 0.8; 0.9, 0.9))
Good(G)	((0.7, 0.9, 0.9, 1; 1, 1),(0.8, 0.9, 0.9, 0.95; 0.9, 0.9))
Very Good(VG)	((0.9, 1, 1, 1; 1, 1),(0.95, 1, 1, 1; 0.9, 0.9))

Table 2. Evaluating Value of Alternatives of Decision-Makers with Respect to Different Attributes

	f_1	f_2	f_3	f_4
x_1	MG	F	VG	VG
x_2	MG	MG	MG	G
x_3	VG	VG	VG	F

Step 1: Construct the decision matrices X of alternatives x_1, x_2, x_3 respectively according to Table 2 and Eq. (17), shown as follows：

$$X = \left(\tilde{\tilde{x}}_{ij}\right)_{n\times m} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} f_1 & f_2 & f_3 & f_4 \end{array} \\ \begin{bmatrix} MG & F & VG & VG \\ MG & MG & MG & G \\ VG & VG & VG & F \end{bmatrix} \end{array}$$

Step 2: Get positive ideal solution $\tilde{\tilde{x}}^+$ and the negative ideal solution $\tilde{\tilde{x}}^-$ respectively according to Equations (18) and (19), shown as follows.

$$\tilde{\tilde{x}}^+ = (\tilde{\tilde{x}}_1^+, \tilde{\tilde{x}}_2^+, \tilde{\tilde{x}}_3^+, \tilde{\tilde{x}}_4^+) = (VG, VG, VG, VG)\ ,\ \tilde{\tilde{x}}^- = (\tilde{\tilde{x}}_1^-, \tilde{\tilde{x}}_2^-, \tilde{\tilde{x}}_3^-, \tilde{\tilde{x}}_4^-) = (MG, F, MG, F)$$

Step 3: Calculate the distance $d^+\left(\tilde{\tilde{x}}_i\right)$ between each alternative $\tilde{\tilde{x}}_i$ and the positive ideal solution $\tilde{\tilde{x}}^+$ according to Eq. (20), shown as follows:

$$d(\tilde{\tilde{x}}_{11},\tilde{\tilde{x}}_1^+)=0.28,\ d(\tilde{\tilde{x}}_{21},\tilde{\tilde{x}}_1^+)=0.28,\ d(\tilde{\tilde{x}}_{31},\tilde{\tilde{x}}_1^+)=0,\ d(\tilde{\tilde{x}}_{12},\tilde{\tilde{x}}_2^+)=0.42,$$

$$d(\tilde{\tilde{x}}_{22},\tilde{\tilde{x}}_2^+)=0.28,\ d(\tilde{\tilde{x}}_{32},\tilde{\tilde{x}}_2^+)=0,\ d(\tilde{\tilde{x}}_{13},\tilde{\tilde{x}}_3^+)=0,\ d(\tilde{\tilde{x}}_{23},\tilde{\tilde{x}}_3^+)=0.28,$$

$$d(\tilde{\tilde{x}}_{33},\tilde{\tilde{x}}_3^+)=0,\ d(\tilde{\tilde{x}}_{14},\tilde{\tilde{x}}_4^+)=0,\ d(\tilde{\tilde{x}}_{24},\tilde{\tilde{x}}_4^+)=0.1,\ d(\tilde{\tilde{x}}_{34},\tilde{\tilde{x}}_4^+)=0.42.$$

Calculate the distance $d^-\left(\tilde{\tilde{x}}_i\right)$ between each alternative $\tilde{\tilde{x}}_i$ and the negative ideal solution $\tilde{\tilde{x}}^-$ according to Eq. (21), shown as follows:

$$d(\tilde{\tilde{x}}_{11},\tilde{\tilde{x}}_1^-)=0,\ d(\tilde{\tilde{x}}_{21},\tilde{\tilde{x}}_1^-)=0,\ d(\tilde{\tilde{x}}_{31},\tilde{\tilde{x}}_1^-)=0.28,\ d(\tilde{\tilde{x}}_{12},\tilde{\tilde{x}}_2^-)=0,$$

$$d(\tilde{\tilde{x}}_{22},\tilde{\tilde{x}}_2^-)=0.2,\ d(\tilde{\tilde{x}}_{32},\tilde{\tilde{x}}_2^-)=0.42,\ d(\tilde{\tilde{x}}_{13},\tilde{\tilde{x}}_3^-)=0.28,\ d(\tilde{\tilde{x}}_{23},\tilde{\tilde{x}}_3^-)=0,$$

$$d(\tilde{\tilde{x}}_{33},\tilde{\tilde{x}}_3^-)=0.28,\ d(\tilde{\tilde{x}}_{14},\tilde{\tilde{x}}_4^-)=0.42,\ d(\tilde{\tilde{x}}_{24},\tilde{\tilde{x}}_4^-)=0.38,\ d(\tilde{\tilde{x}}_{34},\tilde{\tilde{x}}_4^-)=0$$

Step 4: Calculate the weight distance $D(\tilde{\tilde{x}}_i,\tilde{\tilde{x}}^+)$ and $D(\tilde{\tilde{x}}_i,\tilde{\tilde{x}}^-)$ respectively according to Equations (22) and (23), shown as follows:

$$D(\tilde{\tilde{x}}_1,\tilde{\tilde{x}}^+)=0.28,\ D(\tilde{\tilde{x}}_2,\tilde{\tilde{x}}^+)=0.27,\ D(\tilde{\tilde{x}}_3,\tilde{\tilde{x}}^+)=0.16$$

$$D(\tilde{\tilde{x}}_1,\tilde{\tilde{x}}^-)=0.09,\ D(\tilde{\tilde{x}}_2,\tilde{\tilde{x}}^-)=0.05,\ D(\tilde{\tilde{x}}_3,\tilde{\tilde{x}}^-)=0.23$$

Step 5: Calculate the relative degree of closeness $C(\tilde{\tilde{x}}_i)$ according to Eq. (24), shown as follows: $C(\tilde{\tilde{x}}_1)=0.76$, $C(\tilde{\tilde{x}}_2)=0.84$, $C(\tilde{\tilde{x}}_3)=0.39$.

Step 6: Rank the preference orders of the alternatives x_1,x_2,x_3, shown as follows: $x_2 \succ x_1 \succ x_3$ So, the best alternative is x_2.

Furthermore, we use the method proposed in this paper to dear with the problems in [16], we find that the results derived by our method are consistent with the one by Chen and Lee [5], which implies that the method we proposed is more effective and simplificative. Meanwhile, comparing with those methods [16], our method is based on interval type-2 fuzzy information and reflects the practical decision strategies with synergetic weighted distance. So, it is more reasonable and flexible.

7. Conclusions

In this paper, we presented an IT2F-TOPSIS method to deal with MADM problems under interval IT2F environment. Firstly, we proposed a score function with respect to the IT2FSs and presented a new distance measure for two IT2FSs. Then, on the basis of the score function and the distance measure, we proposed a novel approach for handling MADM problems under IT2F-TOPSIS method. The novelty of this method is that it enables decision makers to include a wide range of decision strategies with the help of synergetic weighted distance, and then provide decision makers with the flexibility to making decision based on the real world situation. Finally, a numerical example is provided to illustrate the method that is suitable for solving MADM problems with

IT2FSs.

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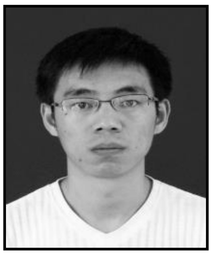

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