

## Electromagnetic Radial Force of Rounded Square PMDC Motor Affected By Harmonic Injection Method

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### **Abstract**

*According to the study of traditional cylindrical PM motor, it is generally acknowledged that the electromagnetic radial force is the root cause of electromagnetic vibration and noise. For round square PM motor, the radial force is the main cause of electromagnetic vibration and noise. , for the first time, use the voltage harmonic injection method, study the motor noise affected by the electromagnetic radial force. Analyze the electromagnetic radial force affected by the frequency, phase angle and amplitude of injection harmonic voltage. For the 4 poles 14 slots rounded square PMDC motor, at first, build the rounded square PMDC motor electromagnetic radial force theoretical model, find the main order of electromagnetic radial force, build the harmonic injection theoretical model, build the harmonic injection simulation model. analyze the electro- magnetic radial force affected by the frequency, phase angle and amplitude of injection harmonic voltage, compare the electromagnetic radial force before and after harmonic injection, at last, compare the motor noise before and after harmonic injection through the noise test, provide the support for the motor noise reduction with harmonic injection method..*

**Keywords:** *Rounded Square PMDC Motor, Harmonic Injection Method, Radial Force, Motor Noise*

### **1. Introduction**

PMDC motor is widely applied for Automotive Systems, with the advantages of mature technology, low production cost, etc. It is the main noise source of vehicle, so noise suppression of PMDC motor is particularly important, because noise is an important evaluation indicator of vehicle's quality. PMDC motor has multiple electromagnetic harmonic in air gap field, which related to slot pole numbers. The multiple electromagnetic harmonic lead to electromagnetic noise in related orders. This is an urgent problem to be solved.

The existing Electromagnetic force of Rounded square PMDC motor affected by harmonic injection method is focus on the PMSM and Flux-switching Permanent Magnet Motor. As reference [1], the vibration of fractional slot PMSM motor is bigger than the integer slot PMSM motor, because there is low modulus electromagnetic harmonic force in fractional slot PMSM motor, the harmonic injection method is effectively to suppress the vibration and noise of fractional slot PMSM motor. As reference [2], there is high air gap flux density, cogging torque of Flux-switching permanent magnet motor is high, the harmonic injection method is effectively to suppress the cogging torque of Flux-switching permanent magnet motor. As reference [3], because the distortion of air gap magnetic field and nonlinear of inverter, there is high order harmonic in the current of PMSM motor, it deduce the motor electromagnetic radial force ripple., the harmonic injection method is effectively to suppress the electromagnetic radial force ripple of PMSM motor.

## 2. The Theoretical Study of Electromagnetic Radial Force of Rounded Square PMDC Motor Affected by Harmonic Injection Method

### 2.1. Rounded Square PMDC Motor Structure

Research is on the rounded square PMDC motor with 14 slots 4 poles. Figure 1 shows cross-section of the rounded square PMDC motor with 14 slots 4 poles. Table 1 shows the contrast of the structure parameters.

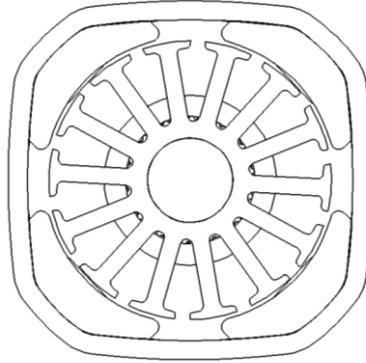


Figure 1. Cross-Section of Motors

Table 1. Structure Parameter of Motor

Para	Value	Para	Value
Pole	4	Armature length/mm	34
Slot	14	Tooth width/mm	4.5
Housing outside dim/mm	33	Magnet thickness/mm	3.9
	X33	Magnet width /mm	13.6
Rotor OD/mm	26.4	Remanence/K	35
		A/m	0
Air gap/mm	0.4		

### 2.2. The Theoretical Model of Electromagnetic Radial Force of Rounded Square PMDC Motor Affected by Harmonic Injection Method

For 4 poles 14 slot rounded square motor, the electromagnetic radial force can express as [5]

$$\Phi = \int B_{r\_load} ds \quad (1)$$

$$f_r = \frac{B_{r\_load}^2}{\mu_0} \quad (2)$$

$$B_{r\_load} = B_{sr}(r, \theta) + B_{sr\_arm}(r, \theta) \quad (3)$$

$$B_{sr\_arm}(r, \theta) = B_{r\_arm}(r, \theta)\lambda_a + B_{\theta\_arm}(r, \theta)\lambda_b \quad (4)$$

$$B_{sr}(r, \theta) = B_r(r, \theta)\lambda_a + B_{\theta}(r, \theta)\lambda_b \quad (5)$$

$$\lambda_a = \lambda_0 + \sum_{i=1}^{\infty} \lambda_{ai} \cos(iN_s \theta) \quad (6)$$

$$\lambda_b = \sum_{i=1}^{\infty} \lambda_{bi} \sin(iN_s \theta) \quad (7)$$

$$B_{r\_arm}(r, \theta) = \mu_0 \sum_{m=1}^{\infty} \sum_{j=1}^{\infty} J_m F_m(r) I_j \times \sin[m\theta \pm (jp \omega_r t + \theta_j)] \quad (8)$$

$$B_{\theta\_arm}(r, \theta) = \mu_0 \sum_{m=1}^{\infty} \sum_{j=1}^{\infty} J_m G_m(r) I_j \times \sin[m\theta \pm (jp \omega_r t + \theta_j)] \quad (9)$$

Where

Where  $N_s$  is armature slots number,  $f_{\theta}$  is magnet tangential force,  $B_{r\_load}$  is radial magnetic density on load,  $B_{sr}$  is radial magnetic density with free load,  $B_{s\theta}$  is tangential magnetic density with free load, the  $\mu_0$  is magnetic permeability in the air,  $B_r(r, \theta)$  is radial magnetic radial density without slot effect,  $B_{\theta}(r, \theta)$  is tangential magnetic density with slot effect,  $\lambda_a$  and  $\lambda_b$  are the real and imaginary components of the relative complex air-gap permeance.  $B_{sr\_arm}(r, \theta)$  is armature reaction tangential magnetic density,  $J_m, G_m, F_m$  is constant related with motor structure.  $I_j$  is branch current.

From equation (2)-(5), radial force and harmonic is 14 order and harmonics, the radial force can be expressed as

$$f_r = f_{r0} + f_{r14} + f_{r28} + f_{r42} + f_{r56} \dots \quad (10)$$

Where  $f_{r0}$  is radial steady constant,  $f_{\theta42}$  is 42 order harmonics component, can be expressed as

$$f_{r42} = f_{Ar42} \sin(42\theta_r + \varphi_{42}) \quad (11)$$

Where  $f_{Ar42}$  is amplitude of 42order harmonic tangential force,  $\varphi_{T42}$  is initial phase angle of 42 order harmonic tangential force,  $\theta_r$  is rotor position angle.

### 3. Harmonic Injection Theoretical

In the actual situation, most power source is voltage power source for the PMDC motor, so the research target is inject voltage harmonic to reduce the tangential force ripple, not considered the inductance, the voltage balance equation can be expressed as

$$U = E + IR \quad (12)$$

According to the Fourier transform method, the voltage can be expressed as

$$U = U_0 + U_{14} + U_{28} + U_{42} + U_{42} \dots \quad (13)$$

Where  $U_{42}$  is 42 order voltage ripple, can be expressed as

$$U_{42} = U_{A42} \sin(42\theta_r + \varphi_{U42}) \quad (14)$$

Where  $U$  is power source voltage,  $E$  is EMF of motor,  $I$  is branch current,  $R$  is branch resistance,  $\varphi_{U42}$  is 42 order voltage ripple phase angle.

Flux can be express as

$$\Phi = \Phi_0 + \Phi_{14} + \Phi_{28} + \Phi_{42} + \Phi_{56} + \dots \quad (15)$$

$$\Phi_{42} = \Phi_{A42} \sin(42\theta_r + \varphi_{\Phi42}) \quad (16)$$

$\varphi_{U42}$  is 42 order flux ripple 42 order phase angle.

EMF can be expressed as

$$E = E_0 + E_{14} + E_{28} + E_{42} + E_{56} \dots \quad (17)$$

Where  $E_0$  is voltage steady constant,  $E_{42}$  is 42 order EMF ripple, can be expressed as

$$E_{56} = E_{A56} \sin(56\theta_r + \varphi_{E56}) \quad (18)$$

$\varphi_{E56}$  is EMF ripple 42 order initial phase angle, the branch current can be express as

$$I = I_0 + I_{14} + I_{28} + I_{42} + I_{56} \dots \quad (19)$$

$$I_{56} = I_{A56} \sin(56\theta_r + \varphi_{I56}) \quad (20)$$

$\varphi_{I56}$  is current ripple 42 order initial phase angle.

$$\Phi = \int E dt \quad (21)$$

From equation (1),(2), the electromagnetic radial force can be express as

$$f_r = \frac{B_r^2}{\mu_0} = \frac{(\frac{d\Phi}{ds})^2}{\mu_0} \quad (22)$$

From equation (12) and (21) can be build the relationship between electromagnetic radial force and injection voltage.

According above study, build the relationship between electromagnetic radial force and injection voltage. Proof the electromagnetic radial force can affected by the voltage harmonic injection, but the last criteria is decided by the comparison between MMF of magnet and MMF O of armature, the MMF of magnet can be express as

$$F_B = HL \quad (23)$$

Where  $F_B$  MMF of magnet,  $H$  is magnet coercivity,  $L$  is armature length. The MMF of armature can be express as

$$F_I = NI \quad (24)$$

Where  $F_I$  is MMF of armature , N is winding turns , I is armature current.  
For this paper, the motor parameter as below.

Magnet romance H(KA/m)	Armatu e length L(mm)	Winding turns N	Winding current (A)
350	35	32	5

the MMF of magnet as below

$$F_B = HL = 12250A \quad (25)$$

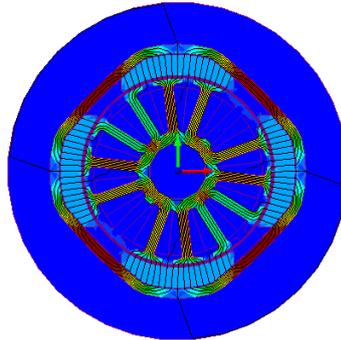
The MMF of armature as below

$$F_I = NI = 160A \quad (26)$$

The MMF of armature is far less than the MMF of magnet, from the theory, the injection harmonic voltage can not affect the motor electromagnetic radial force.

#### 4. Simulation Model of Harmonic Injection

In order to proof the theory study conclusion, build the simulation model of harmonic injection, focus on the 42 order electromagnetic radial force, rounded square PMDC simulation model as below



**Figure 2. Rounded Square PMDC Motor Simulation Model**

From finite element analysis, motor radial force harmonics includes mainly slot 14 and the multiples of 14.

Perform noise test on above sample motor, testing voltage is 12V, rotation rate is 4590RPM.

The injection harmonic frequency as below

$$F_{req} = \frac{n}{60} \times N \quad (27)$$

Where

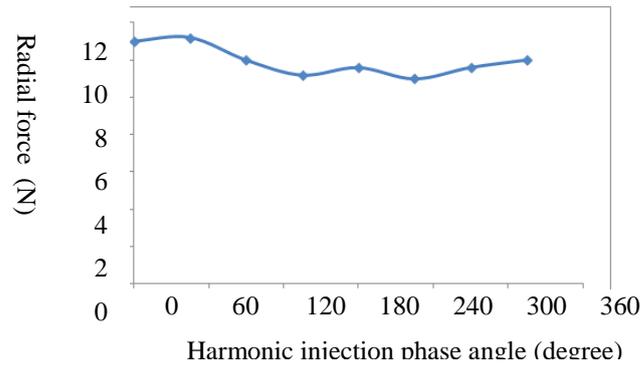
$F_{req}$  motor noise frequency

n rotation rate, unit is RPM

N order of exciting force

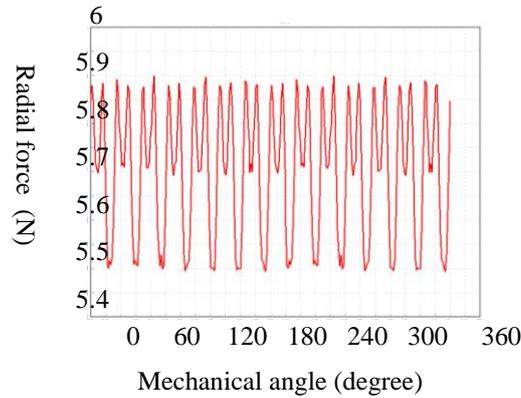
From equation (1) on 100mNm load condition, the speed is 3214 RPM, the 42 order motor electromagnetic radial force frequency is 2250Hz, the harmonic injection frequency should be 2250Hz.

Analyze the electromagnetic radial force with different phase angle.

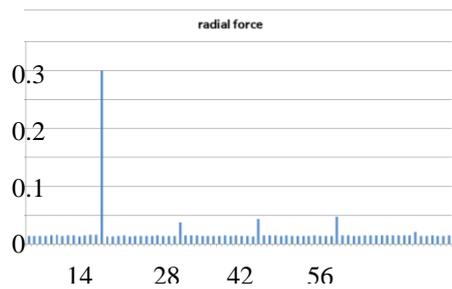


**Figure 3. Relationship between Electromagnetic Radial Force and Injection Harmonic Initial Phase Angle**

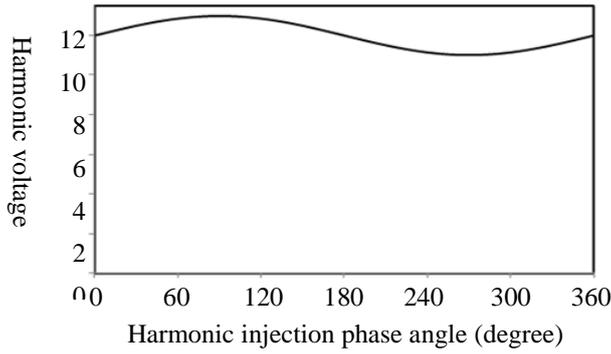
From figure 3, at different phase angle, the variation of electromagnetic radial force is very small, so the electromagnetic radial force affected by the harmonic injection can be ignored.



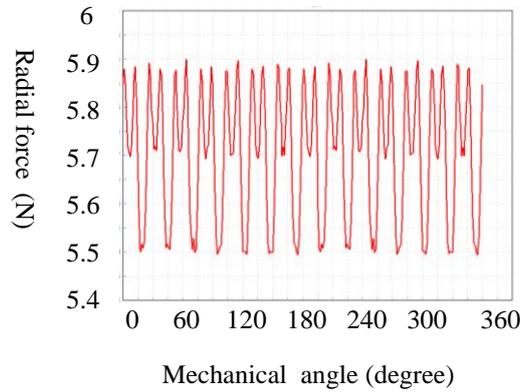
**Figure 4. Electromagnetic Radial Force Curve Before Harmonic Injection**



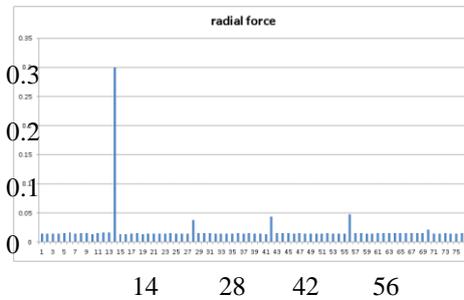
**Figure 5. Electromagnetic Radial Force Order before Harmonic Injection**



**Figure 6. Injection Harmonic Voltage**



**Figure 7. Electromagnetic Radial Force Curve after Harmonic Injection**

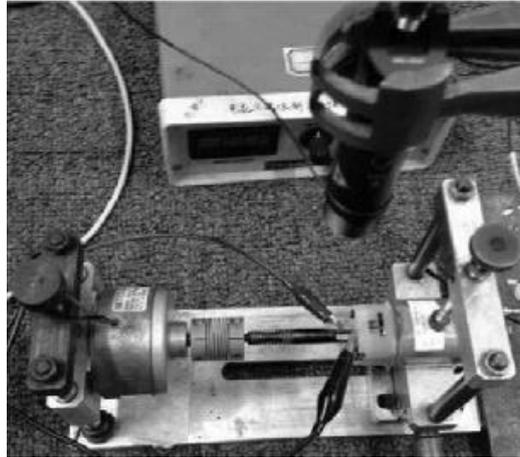


**Figure 8. Electromagnetic Radial Force Order after Harmonic Injection**

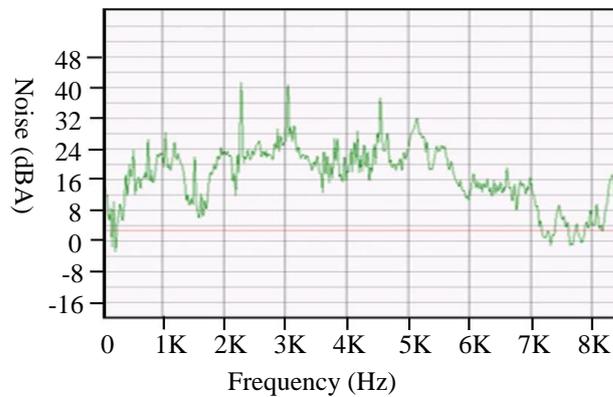
From figure 4-8, the electromagnetic radial force is almost same before and after harmonic injection, and it is also almost same after inject voltage harmonic in different phase angle, form the simulation side, proof the theory conclusion.

## 5. Noise Test Experiment

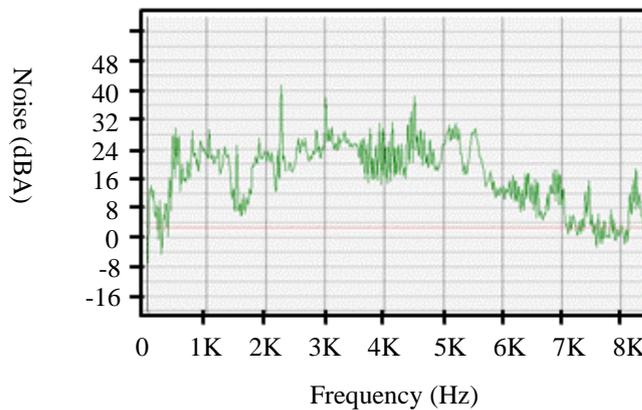
Because electromagnetic radial force is main source of rounded square motor, this paper test the variation of motor noise cause by the 42 order electro-magnetic radial force after harmonic injection, it can validate the motor electromagnetic radial force affected by the harmonic injection. Motor noise test equipment as below



**Figure 9. Motor Noise Test Platform**



**Figure 10. Motor Noise Test on Load before Harmonic Injection**



**Figure 11. Motor Noise Test on Load after Harmonic Injection**

From the test result, the 42 order motor noise almost same after 42 order voltage harmonic injection, and it is also almost same after inject voltage harmonic in different phase angle, so the electromagnetic radial force affected by the harmonic injection can be ignored. Form the experimental side, proof the theory and simulation conclusion.

## 6. Conclusion

The existing Electromagnetic force of Rounded square PMDC motor affected by harmonic injection method is focus on the PMSM and Flux-switching Permanent Magnet Motor, base on the build rounded square motor harmonic injection theoretical model, make the conclusion, the electromagnetic radial force affected by harmonic injection can be ignored. Use the simulation method to analyze the electromagnetic radial force affected by harmonic injection, for the 42 order harmonic force, inject the voltage harmonic in the different initial phase angle, found that the force has not significant change. Because the main source of motor noise is radial force, for the 42 order motor noise, inject voltage harmonic in different initial phase angle, found that motor noise has not significant change, so the electromagnetic radial force affected by the harmonic injection can be ignored. form the experimental side, proof the theory and simulation conclusion. The study provide support for the next step work, motor noise reduction with harmonic injection method.

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