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Improved MUSIC Algorithm for DOA Estimation of Coherent Signals via Toeplitz and Fourth-order-cumulants

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Abstract

In this paper we propose an improved MUSIC (Multiple Signal Classification) algorithm applicable for direction of arrival (DOA) estimation of coherent signals in the presence of one-dimensional uniform linear array (ULA), which is based on Toeplitz matrix theory and Fourth-order-cumulants (Foc). In the signal model, a new Toeplitz construction method combining SVD (singular value decomposition) with mean calculation is explored to reconstruct the covariance matrix of array output. The DOA estimation problem can be addressed when the covariance matrix is full-rank. Foc theory is used to eliminate the Gaussian noise in the signals, after that the space of the array matrix is changed, which determines the final signal subspace and noise subspace. According to the subspace, we can adopt the conventional MUSIC to estimate the DOAs of coherent signals. Simulation results show that this algorithm provides a significant performance in comparison with other de-correlation algorithms. It has a better resolution under the condition of small angle interval. In addition, a lower root-mean-square error (RMSE) is obtained at low signal-to-noise (SNR) situation.

Keywords: *MUSIC, DOA, Coherent Signal, Toeplitz, Fourth-Order-Cumulants*

1. Introduction

Direction of arrival (DOA) estimation is an important research direction in modern signal processing field, many applications in wireless communication, navigation, geological, and radar systems adopt this theory. MUSIC (Multiple Signal Classification) algorithm, as one of the most famous DOA estimation algorithms, is paid close attention to extensively when solving high resolution problems in radar systems. Recently, with the of development high resolution technology, numerous studies have demonstrated that the conventional MUSIC algorithm is not satisfied for estimating coherent signals, a great deviation occurs between estimated results and real datas. Therefore many researchers are constantly studying this algorithm for DOAs estimation of coherent signals [1-5].

In the course of researching eigen-decomposition, we find that DOAs of coherent signals can be estimated by this method combining with MUSIC, consequently correlation coefficients of sources are decreased at the same time [6]. Zuckerman *et al*. [7] have proposed an algorithm using spatial difference smoothing theory on the basis of MUSIC, which effectively estimated the DOAs of coherent signals. A modified spatial smoothing technique is introduced in [8], the authors make cross-

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correlation of all the auto-correlation matrixes of sub-array output, and take estimation to equivalent spatial smoothing matrix after forward and backward correlation matrixes averaged. Simulations demonstrate that good fruits are obtained by this method. Toeplitz matrix is applied to estimation [9-10], DOAs of coherent signals are successfully estimated with reconstructing auto-correlation matrix of array by Toeplitz theory. In [11], an improved MUSIC based on Fourth-order-cumulants (Foc) is derived, Foc theory is used by the authors to transform the auto-correlation matrix of array into an alternative form, then MUSIC is adopted to estimate DOA of signals, experiments show that the algorithm has an excellent anti-colored noise performance.

The algorithms above have scored some remarkable achievements, but it is a pity that there are certain limitations for them. Although DOAs can be estimated effectively with the algorithms in [6, 7, 9], signal intervals need to be set to greater than or equal to 10° in these works. Besides, rank-defect happens when adopting spatial smoothing method, which leads to loss of array aperture. Algorithms in [8, 10] can not only obtain low root mean square errors (RMSEs), but also high success rates. [11] has a good capability of anti-noise, however, both of them need higher signal to noise ratio (SNR), consequently estimated angles go awry under the condition of the SNR $\leq$ 0dB.

The first time Toeplitz and Foc have been used for 2-D DOA estimation is in 2014. It is shown that this method has low computational complexity and yields better performance in terms of maximum probability of success (MPS), maximum root mean square error (MRMSE) of incoming signals in both white noise and color Gaussian noise situations. Meanwhile, the DOAs can be estimated accurately at a low signal-to-noise ratio and small number of snapshots situation [12].

Through researching on these results mentioned above, we propose an improved MUSIC (T-Foc MUSIC) based on relevant theories in [12] to achieve a high resolution estimation method for coherent signals under the condition of one-dimensional uniform linear array (ULA). A new construction method of Toeplitz matrix is proposed in this work, consequently the covariance matrix of array output can be transformed into an alternative matrix with a characteristic of full-rank. After that we utilize Foc theory to eliminate the Gaussian noise of the matrix like [12]. Our proposal makes an efficient implementation when combining with the conventional MUSIC, and good experimental results are obtained by simulations. It is shown that the algorithm has a high resolution and a low and steady RMSE at low SNR, moreover DOAs of coherent signals can be estimated accurately even the angle interval is less than or equal to 5°.

2. Related theory

2.1. Toeplitz Matrix

Toeplitz matrix is a special matrix proposed by German mathematician Toeplitz, its specific characteristic is that elements on the every diagonal of the matrix are equal to each other [13]. Consider $\boldsymbol{T}$ represents matrix, which can be modeled as

$$\boldsymbol{T} = \begin{bmatrix} t_0 & t_1 & t_2 & \cdots & t_n \\ t_{-1} & t_0 & t_1 & \cdots & t_{n-1} \\ t_{-2} & t_{-1} & t_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_1 \\ t_{-n} & t_{-n+1} & \cdots & t_{-1} & t_0 \end{bmatrix} \tag{1}$$

As long as conjugate symmetric is satisfied in plural Toeplitz matrix, that is called Hermite Toeplitz matrix, which is shown as

$$\boldsymbol{T_0}^* = \begin{bmatrix} t_0 & t_1^* & t_2^* & \cdots & t_n^* \\ t_1 & t_0 & t_1^* & \cdots & t_{n-1}^* \\ t_2 & t_1 & t_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & t_1^* \\ t_n & t_{n-1} & \cdots & t_1 & t_0 \end{bmatrix} \tag{2}$$

The ranks of $\boldsymbol{T}$ and $\boldsymbol{T_0}$ are equal to $N+1$, that is called full-rank. Depending on this important feature, it is often used in DOA estimation.

2.2. Fourth Order Cumulants

Foc is applied to eliminating some special noise in the signal. Assume n-dimensional stationary random process $\boldsymbol{x}$ with a zero-mean, whose Foc is defined as

$$\begin{aligned} C_{4,x}(k_1,k_2,k_3,k_4) &= cum\left\{x_{k_1},x_{k_2},x_{k_3},x_{k_4}\right\} \\ &= E\left\{x_{k_1},x_{k_2},x_{k_3},x_{k_4}\right\} - E\left\{x_{k_1},x_{k_3}\right\}E\left\{x_{k_2},x_{k_4}\right\} \\ &\quad - E\left\{x_{k_1},x_{k_4}\right\}E\left\{x_{k_2},x_{k_3}\right\} - E\left\{x_{k_1},x_{k_2}\right\}E\left\{x_{k_3},x_{k_4}\right\} \end{aligned} \tag{3}$$

Here $E\left\{x_{k_1},x_{k_2},x_{k_3},x_{k_4}\right\}$ is the fourth moment of $\boldsymbol{x}$, and $E\left\{x_{k_i},x_{k_j}\right\}$ represents the second moment. According to [1, 5], consider an array vector $\boldsymbol{X}(t)$, expressed as (4)

$$\boldsymbol{X}(t) = \boldsymbol{A}\cdot\boldsymbol{S}(t) + \boldsymbol{N}(t) = \sum \boldsymbol{a}_i s_i + \boldsymbol{N}(t) \tag{4}$$

Let $\boldsymbol{A}$ be the steering matrix consisting of i vectors, $\boldsymbol{S}(t)$ denotes the source signal vector, and $\boldsymbol{n}(t)$ is the Gaussian noise vector generated at each array element with a zero-mean. The Foc of $\boldsymbol{X}(t)$ is defined as

$$\begin{aligned} C_{4,x}(k_1,k_2,k_3,k_4) &= cum(x_{k_1},x_{k_2},x_{k_3},x_{k_4}) \\ &= cum\left(\sum_{i=1}^{P}\boldsymbol{a}_i(k_1)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_2)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_3)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_4)s_i(t)\right) \\ &\quad + cum\left(n_{k_1}(t),n_{k_2}(t),n_{k_3}(t),n_{k_4}(t)\right) \\ &= cum\left(\sum_{i=1}^{P}\boldsymbol{a}_i(k_1)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_2)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_3)s_i(t),\sum_{i=1}^{P}\boldsymbol{a}_i(k_4)s_i(t)\right) \\ &= \sum_{i=1}^{P}\sum_{j=1}^{P}\sum_{m=1}^{P}\sum_{n=1}^{P}\boldsymbol{a}_i(k_1)\boldsymbol{a}_j(k_2)\boldsymbol{a}_m(k_3)\boldsymbol{a}_n(k_4)cum\left(s_i(t),s_j(t),s_m(t),s_n(t)\right) \end{aligned} \tag{5}$$

Let $a_i(k)$ denote the k-th element of the i-th steering vector, and s_i ($i=1,2,\cdots P$) are independent of each other, hence (5) can be simplified with another expression.

$$\begin{aligned} C_{4,x}(k_1,k_2,k_3,k_4) &= \sum_{i=1}^{P}\boldsymbol{a}_i(k_1)\boldsymbol{a}_i(k_2)\boldsymbol{a}_i(k_3)\boldsymbol{a}_i(k_4)cum\left(s_i(t),s_j(t),s_m(t),s_n(t)\right) \\ &= \sum_{i=1}^{P}\boldsymbol{a}_i(k_1)\boldsymbol{a}_i(k_2)\boldsymbol{a}_i(k_3)\boldsymbol{a}_i(k_4)\gamma_{4s_i} \end{aligned} \tag{6}$$

Here γ_{4s_i} denotes Foc of the i-th signal ($i=1,2,\ldots,K$), and k_i ($i=1,2,3,4$) satisfies $1\le k_i \le M$. Let M be the sensor number so that it has M^4 combinations. Assume a matrix $\boldsymbol{R_4}$ of size $M^4 \times M^4$ where k_i is placed, and the new definition of $C_{4,x}$ is as follows

$$\begin{aligned} C_{4,x}(k_1,k_2,k_3,k_4) = \boldsymbol{R_4}\left((k_1-1)M+k_3,(k_2-1)M+k_4\right) \\ = E[(x\otimes x^*)(x\otimes x^*)^H] - E[x\otimes x^*]E[(x\otimes x^*)^H] \\ - E[xx^H]\otimes E[(xx^H)^*] \end{aligned} \tag{7}$$

Here "$\otimes$" represents Kronecker product, white and Gaussian noise are applied to MUSIC as noise model, hence some researchers use Foc to improve the anti-noise performance. Meanwhile the sensor number can be added up to M^2-M+1, and we can estimate M^2-M+1 signals at most, that is to say the number of array aperture is equivalently extended.

3. Signal Model and T-FOC MUSIC

3.1. Signal Model

According to [1, 5, 7], the signal model is established. Assume a uniform linear array (ULA) composed of M isotropic sensors, there are P ($P \le M$) correlated narrowband source signals, from directions (θ_1, θ_2, … , θ_p), impinging on array in the far filed. The $M \times 1$ signal vector is as follows

$$\boldsymbol{X}(t) = \boldsymbol{A}(\theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{8}$$

Where $\boldsymbol{S}(t)$ represents the source signal vector consisting of P different incidence signals, $\boldsymbol{A}(\theta)$ denotes the steering matrix consisting of steering vectors, and $\boldsymbol{N}(t)$ is a Gaussian noise vector which is generated by each array element with a zero mean and variance of σ^2. They are respectively defined as

$$\boldsymbol{S}(t) = [s_1(t), s_2(t), \cdots s_P(t)]^T \tag{9}$$

$$\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots \boldsymbol{a}(\theta_P)] \tag{10}$$

$$\boldsymbol{N}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T \tag{11}$$

Let λ be the carrier wavelength, and the i-th steering vector $\boldsymbol{a}(\theta_i)$ can be denoted as

$$\boldsymbol{a}(\theta_i) = \left[1, e^{j2\pi \sin\theta_i d/\lambda}, \cdots, e^{j2\pi(M-1)\sin\theta_i d/\lambda}\right]^T \tag{12}$$

According to (8), the covariance of $\boldsymbol{X}(t)$ is

$$\boldsymbol{R} = E[\boldsymbol{X}(t)\boldsymbol{X}(t)^H] = \boldsymbol{A}\boldsymbol{R_s}\boldsymbol{A}^H + \sigma^2 \boldsymbol{I}_N \tag{13}$$

Here $\boldsymbol{R_s} = E[\boldsymbol{S}(t)\boldsymbol{S}(t)^H]$ is the covariance matrix of source signals. Let L represent the number snapshots, the array covariance matrix is

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{i=1}^{L} \boldsymbol{X}(i)\boldsymbol{X}(i)^H \tag{14}$$

3.2. T-Foc MUSIC Algorithm

We learn that $\hat{\boldsymbol{R}}$ is a matrix of size $M \times M$ by the mathematical deduction above, let

$$\hat{\boldsymbol{R}}=\begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \cdots & r_{1,M} \\ r_{2,1} & r_{2,2} & r_{2,3} & \cdots & r_{2,M} \\ r_{3,1} & r_{3,2} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & r_{M-1,M-1} & r_{M-1,M} \\ r_{M,1} & r_{M,2} & \cdots & r_{M,M-1} & r_{M,M} \end{bmatrix} \tag{15}$$

So (15) can be expressed in a form of the diagonal.

$$\hat{\boldsymbol{R}}=\begin{bmatrix} r_{0,1} & r_{1,1} & r_{2,1} & \cdots & r_{M-1,1} \\ r_{-1,1} & r_{0,2} & r_{1,2} & \cdots & r_{M-2,2} \\ r_{-2,1} & r_{-1,2} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & r_{0,M-1} & r_{1,M-1} \\ r_{-M+1,1} & r_{-M+2,2} & \cdots & r_{-1,M-1} & r_{0,M} \end{bmatrix} \tag{16}$$

Here $r_{i,j}$ $(i=-M+1,\ldots,0,\ldots,M-1,\ j=1,2,\cdots,M-|i|)$ denote the *j*-th element in the *i*-th diagonal. $\hat{\boldsymbol{R}}$ can be transformed into a Toeplitz matrix using a new construction method proposed by us, which is based on SVD and mean calculation. The steps of the method are as follows

a) A new diagonal matrix $\boldsymbol{S}$ ($\boldsymbol{S}=\begin{bmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_M \end{bmatrix}$) is obtained by SVD on the array matrix output $\hat{\boldsymbol{R}}$.

$$[\boldsymbol{U},\boldsymbol{S},\boldsymbol{V}]=svd(\hat{\boldsymbol{R}}) \tag{17}$$

We can work out the mean of s_i ($i=1,2\cdots M$), which is denoted with $\bar{s}$.

b) According to (18), we work out the means of positive diagonals and negative diagonals (r_i' and r_i'') of the matrix $\hat{\boldsymbol{R}}$.

$$r=\frac{1}{M-|i|}\sum_{i=-M+1}^{M-1} r_{i,j} \tag{18}$$

Here r_i' ($i=-M+1,\ldots 0,\ldots,M-1$) represent the means of the *i*-th positive diagonal and r_i'' denote the means of the *i*-th positive diagonal.

c) Some specified values of r_i' $(i=1,2,\cdots M-1)$ and r_i'' ($i=-M+1,-M+2,\cdots,-1$) are extracted, so a new Toeplitz matrix $\boldsymbol{R_T}$ is constructed, defined as

$$\boldsymbol{R_T}=\begin{bmatrix} \bar{s} & r_1' & r_2' & \cdots & r_{M-1}' \\ r_{-1}'' & \bar{s} & r_1' & \cdots & r_{M-2}' \\ r_{-2}'' & r_{-1}'' & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \bar{s} & r_1' \\ r_{-M+1}'' & r_{-M+2}'' & \cdots & r_{-1}'' & \bar{s} \end{bmatrix} \tag{19}$$

The array output $\hat{\boldsymbol{R}}$ is transformed into $\boldsymbol{R_T}$, whose rank is equal to *M*, according to the Classical-Toeplitz algorithm [14], the DOAs of coherent signals can be estimated. After that, Foc is applied to $\boldsymbol{R_T}$, according to (7), $\boldsymbol{R}_4$ is defined as (20)

$$\begin{aligned}\boldsymbol{R}_4 &= C_{4,x}(k_1,k_2,k^*{}_3,k^*{}_4)\\ &= E\{(\boldsymbol{R_T}\otimes\boldsymbol{R_T}^*)(\boldsymbol{R_T}\otimes\boldsymbol{R_T}^*)^H\} - E\{(\boldsymbol{R_T}\otimes\boldsymbol{R_T}^*)\}E\{(\boldsymbol{R_T}\otimes\boldsymbol{R_T}^*)^H\}\\ &\quad - E\{(\boldsymbol{R_T}\boldsymbol{R_T}^H)\}\otimes E\{(\boldsymbol{R_T}\boldsymbol{R_T}^H)^*\}\\ &= \boldsymbol{B}(\theta)\boldsymbol{C_s}\boldsymbol{B}^H(\theta)\end{aligned} \tag{20}$$

Here

$$\boldsymbol{B}(\theta) = [b(\theta_1), b(\theta_2),\cdots,b(\theta_P)] = [a(\theta_1)\otimes a(\theta_1),\cdots,a(\theta_P)\otimes a(\theta_P)] \tag{21}$$

$$b(\theta_i) = a(\theta_i)\otimes a(\theta_i)\quad (i=1,2,\cdots,P) \tag{22}$$

The MUSIC spectrum is computed by performing an eigen-decomposition on the matrix $\boldsymbol{R_4}$, the space spanned by M^2 eigenvectors produces two disjoint subspaces: signal and noise subspaces [14].

$$\boldsymbol{E_s} = [e_1, e_2,\cdots e_P] \tag{23}$$

$$\boldsymbol{E_N} = [e_{P+1}, e_{P+2},\cdots e_{M^2}] \tag{24}$$

Here $\boldsymbol{E_s}$ denotes the signal subspace consisting of P eigenvectors, and $\boldsymbol{E_N}$ is the noise subspace consisting of M^2-P eigenvectors, so the T-Foc MUSIC spectrum is defined as

$$P(\theta) = \frac{1}{\boldsymbol{b}^H(\theta)\boldsymbol{E_N}\boldsymbol{E_N}^H\boldsymbol{b}(\theta)} \tag{25}$$

We work out the mean of $P(\theta)$ to guarantee the reliability, and the mean $\bar{P}(\theta)$ can be obtained after K calculations.

$$\bar{P}(\theta) = \frac{1}{K}\sum_{i=1}^{K} P_i(\theta) \tag{26}$$

Here $\bar{P}(\theta)$ is the mean value of $P(\theta)$, meanwhile it is the finial spectrum function.

In the convention MUSIC, the reason why DOAs of coherent signals can't be estimated is that correlation of the signals leads to rank defect of the array matrix, and a deviation comes out between signal and noise subspaces. The proposed algorithm is a deterministic method. When the array matrix $\boldsymbol{R_T}$ has a characteristic of full rank, the phenomenon of rank defect mentioned above can be avoided. Meanwhile, the array matrix is only related to the steering matrix, the problem of loss of array aperture generated by the spatial smoothing algorithm can be effectively solved [15].

4. Simulation Analysis

To illustrate the validity of the proposed algorithm, we evaluate the performance of T-Foc MUSIC in compar- ison with Classical-Foc [16], Spatial smoothing [17], and Classical-Toeplitz [14] with several experiments, which are in terms of resolution, root mean square err- or (RMSE) and calculation time, the source signal can be expressed as

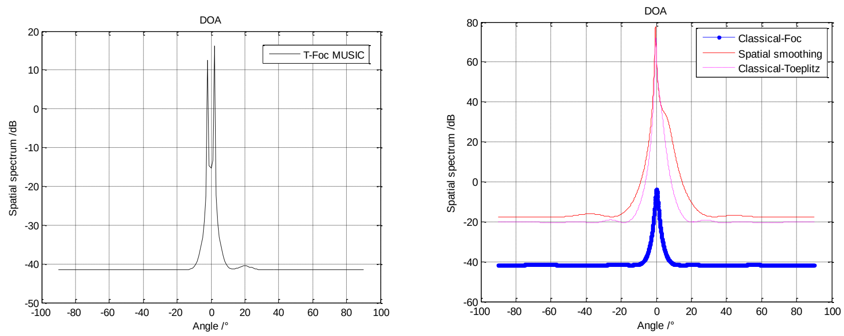
$$s_i = a_i e^{(j\omega_i l+\varphi_i)} \tag{27}$$

a_i, ω_i and φ_i signify the power, the frequency and the initial phase of s_i, respectively. Assume n is the snapshot number, and f_s denotes the sampling rate.

Experiment 1: the Comparison of Resolution

In the first experiment, consider a ULA of 8 sensors ($M = 8$) with half-wavelength spaced sensors, we set $\omega_i = \pi / 2$, $f_s = 1000$ Hz, n=256, and SNR=0dB.

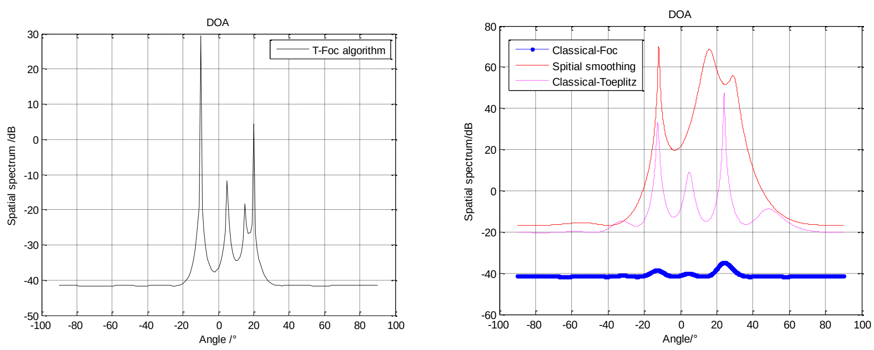
a) There are two correlated narrowband sources impinging on the array from directions of -2° and 2°, The DOA estimation results are as follows as Figure1.



(A) T-Foc MUSIC (B) Other Algorithms

Figure 1. DOA Estimation for SNR=0db, Snapshot Number=256, *K*=8, = [-2°, 2°]

b) There are four correlated narrowband sources impinging on the array from directions of -10°, 5°, 20°and 25°, The DOA estimation results are as follows as Figure2.



(a) T-Foc MUSIC (b) Other Algorithms

Figure 2. DOA Estimation for SNR=0db, Snapshot Number=256, *K*=8, = [-10°, 5, 20, 25°]

Figure1 and Figure2 show that the DOAs are accurately estimated by the proposed algorithm even the intervals of the incidence signals are less than or equal to 5°, so we can get a conclusion that it has a better resolution than other de-correlation algorithms under the condition of low SNR.

Experiment 2: the Comparison of RMSE

The initial parameters are similar to those in experiment 1. The incidence angle is equal to 10°, $\omega_i = \pi/2$, $f_s = 1000\,\text{Hz}$, $M = 8$, and $n = 256$.. The definition of RMSE is given by [18-20], defined as

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left|\theta_n - \hat{\theta}_n\right|^2} \tag{29}$$

Here $\hat{\theta}_n$ is the estimated angle and θ_n is the incidence angle (θ_n=10°). In this part, 200 Monte Carlo experiments are carried out to calculate the RMSE in terms of SNR, sensor number, and snapshot number.

a) We set SNR = [-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10] dB. Calculate RMSE according to (26), and construct the relation between RMSE and SNR, which is shown in Figure3.

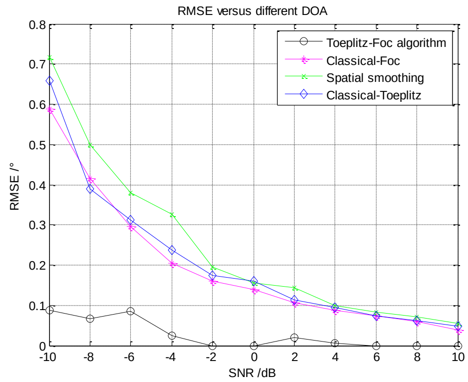


Figure 3. RMSE Performance versus SNR (Incidence Angle=10°, Snapshot Number=256, Sensor Number=8)

It is observed from Figure3 that our work has a lower RMSE in comparison with other algorithms Therefore the proposed algorithm has a better performance in terms of SNR, especially a lower SNR.

b) We set sensor number is equal to [4-6, 8, 10, 12, 14, 16, 18, 20] respectively, the relation between RMSE and sensor number is as shown in Figure 4.

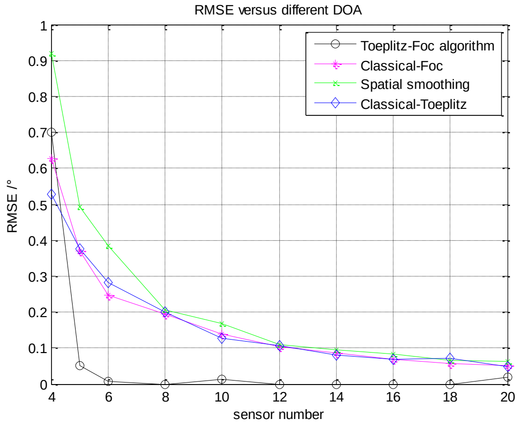


Figure 4. RMSE Performance versus Sensor Number (Incidence Angle=10°, Snapshot Number=256, SNR=0db)

As for the simulation in Figure4, we can also find that our work has a lower RMSE than the other algorithms when the sensor number is greater than or equal to 5, and with the sensor number increasing, the RMSE in this algorithm begins to tend to be steady.

c) We set snapshot number is equal to [50, 100, 150, 200, 250, 300, 250, 300, 350, 400, 450, 500], the relation between RMSE and snapshot number is as follows.

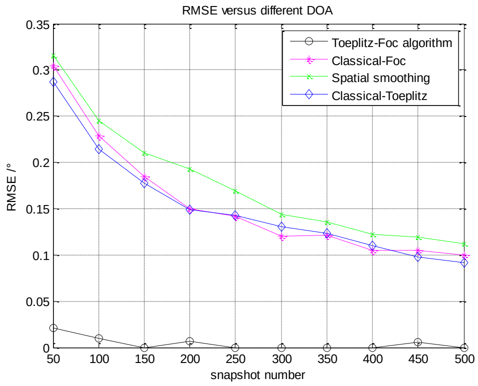


Figure 5. RMSE Performance versus Snapshot Number (Incidence Angle=10°, Sensor Number=8, SNR=0db)

Figure5 illustrates that the RMSE generated by the proposed algorithm has little to do with the snapshot number; it achieves a better experimental performance in terms of RMSE compared to the other algorithms.

Experiment 3: the Comparison of Calculating Time

In this experiment, we compare the calculation time with other algorithms, we set the snapshot number is equal to [50, 100, 150, 200, 250, 300, 250, 300, 350, 400, 450, 500] respectively, θ_n=10°, and SNR= 0dB, the calculation time can be obtained by MATLAB, which is shown in Table 1.

Table 1. The Comparison of Calculating Time

snapshot number	Calculating time (s)			
	T-Foc Algorithm	Classical-Foc	Spatial smoothing	Classical-Toeplitz
50	1.0572	1.0553	0.9168	0.8439
100	1.0677	1.1737	0.9656	0.9010
150	1.0599	1.1493	0.8599	0.9506
200	1.0660	1.4513	0.9308	0.9088
250	1.0548	1.6584	0.9350	0.9154
300	1.0241	1.7412	0.9404	0.8691
150	1.0528	2.0490	0.9593	0.8724
400	1.0371	2.0844	0.9168	0.9301

450	1.0341	2.4927	0.9278	0.8744
500	1.0485	2.5923	0.8889	0.8732

Table 1 lists the calculation time of four algorithms. It is shown that T-Foc MUSIC spends more time than Spatial smoothing and Classical-Toeplitz on the simul- ations, but in comparison with Classical-Foc, it has a shorter time.

5. Conclusion

In the paper we propose a T-Foc MUSIC algorithm for DOAs estimation of coherent signals under the condition of one-dimensional ULA. A new Toeplitz construction method and Foc theory are used to estimate the coherent signals in conjunction with the conventional MUSIC. In contrast to other de-correlation algorithms, the proposed algorithm has some notable advantages. It has a higher resolution under the condition of small angel intervals and low SNR, moreover, a lower and more steady RMSE can be obtained in terms of SNR, sensor number, snapshot number.

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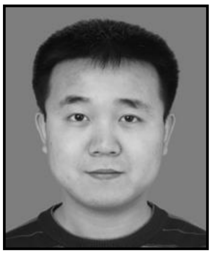

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