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Disturbance Attenuation for a Class of Exponentially Uncertain Switched Linear Systems with State Delay

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Abstract

In this paper, we address the disturbance attenuation problem for a class of state delay switched linear systems with exponential uncertainties via switched state feedback and switched dynamic output feedback, respectively. By using Taylor series approximation and convex polytope technique, exponentially uncertain switched linear systems with state delay is transformed into an equivalent switched polytopic model with additive norm bounded uncertainty. For such equivalent switched model, we design its switching strategy and associated state feedback sub-controllers or dynamic output feedback sub-controllers so that whole switched model is asymptotical stabilization with H_∞ *disturbance attenuation based on multiple Lyapunov function technology and LMI approach.*

Keywords: *Switched Linear Systems with State Delay; Exponential Uncertainty; Disturbance Attenuation; State Feedback; Dynamic Output Feedback; LMIs*

1. Introduction

As we know, state delay is always in practical control system due to the action speed limitation of mechanism and electronics. Such as paper-making process, chemical process, long transmission lines in pneumatic systems and communication channels, etc. On the one hand, the phenomenon of state delay is commonly the source of instability and bad-performance in practical; on the other hand, due to the state delay, it is getting more difficult and complex to control such systems effectively. Hence, analysis and synthesis system with state delay is always the hot spot issues of research field in the control theory and control engineering. In recent decade, the linear systems with state delay have been extensively investigated [1-3]. However, so far there have been few results for switched linear systems with state delay. [4, 5] studied the controllability of switched linear systems with state delay and discrete-time switched linear systems with state delay, respectively. [6] considered the stabilization of switched linear systems with state delay based on convex combination technique. [7] addressed the method of delay-dependent robust H-infinity control for a class of uncertain switched systems with state delay, however, in this case, the condition of matrix inequality is not linear matrix inequality, its solving have to make use of iterative method or enforced constraint. Besides, [8-10] studied the stability and stabilization for a class of switched linear systems with state delay.

In order to design a computer based control for switched linear system (whose coefficient matrix is M_σ and N_σ), L. Hetel, *et al.,* [11, 12] show that sampled model of system is derived and discrete time control methods are applied. Under the case that sampling and actuation are periodic and synchronous with the periodicity, the coefficient matrix of the sampled model is given by

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$$A_\sigma = e^{M_\sigma \rho},\ B_\sigma = \left(\int_0^\rho e^{M_\sigma \tau} d\tau\right) N_\sigma$$

where ρ is sampling periodicity. It is well know that, in many control problems, the sampling periodicity of system is often affected by some delays (delays between the sensor and the digital control, computing delays in the controller, communication delays between the controller and the actuator, and so on). Furthermore, these delays are often unknown, time-varying and bounded [13]. Therefore, exponential uncertainty is inevitable in the process of modeling system. Hence, the control synthesis of switched linear systems with state delay subject to exponential uncertainties is a very important and challenging problem because of the practical background of exponential uncertainty. Generally speaking, exponential uncertainty represents the terms like $e^{M\rho}$ or $\int_0^\rho e^{M\tau} d\tau$ that depend on an unknown, possibly time-varying and bounded parameter ρ . In the literature [14], exponential uncertainty is treated by assuming estimable delay uncertainties. Andrea Balluchi, *et al.*, [15] treats the uncertain exponential terms as bounded uncertainties. Hetel, *et al.*, [11, 12] introduces exponential uncertainties to hybrid linear system and treats the uncertain exponential terms as polytopic uncertainties. Under the arbitrary switching rule, Hetel, *et al.*, [11] design state feedback sub-controller for the stabilization problem of a class of hybrid linear systems with exponential uncertainties in the case where the switching and the sampling are synchronous. And then the obtained results in the literature [11] are extended to cope with network controlled systems [12]. In this manuscript, Motivated by the reference [11, 12], we intend to investigate the disturbance attenuation problem for a class of exponentially uncertain switched linear systems with state delay. Throughout this note, it is assumed that the switching strategy is picked in such a way that there are finite switches in finite time and the state of system does not jump in the instantaneous switching.

Our goal is, for a class of state delay switched linear systems subject to exponential uncertainty, to design a switching strategy $\sigma(t)$ and associated state feedback sub-controllers or dynamic output feedback sub-controllers such that the resulting closed-loop system is asymptotical stabilization with a prescribed H_∞ disturbance attenuation level for all admissible uncertainties. Firstly, we show that exponentially uncertain switched system with state delay is transformed into an equivalent polytopic model with an additive norm bounded uncertainty based on Taylor series approximation and convex polytope technique. And then, by taking advantage of multiple Lyapunov function technology and LMI approach, the robust H_∞ disturbance attenuation property of such equivalent switched model is investigated via switched state feedback and switched dynamic output feedback, respectively.

The remainder of this paper is organized as follows: The problem statement and some preliminaries are described in Section 2, while in Section 3 the asymptotical stabilization with H_∞ disturbance attenuation for exponentially uncertain switched linear system with state delay is investigated via switched state feedback. In Section 4, we discuss the asymptotical stabilization with H_∞ disturbance attenuation for a class of state delay switched linear systems subject to exponential uncertainties via switched dynamic output feedback. Two numerical examples are presented in Section 5 to illustrate our results. Finally, some conclusions are drawn in Section 6.

Notations: We use standard notations throughout this paper. Matrix P^T stands for the transpose of the matrix P . $L_2[0,T]$ （$0 \le T < \infty$） denotes the space of square integrable functions on $[0,T]$ and

$$\|\omega\|_{L_2[0,T]} = \left(\int_0^T \omega^T(t)\omega(t)dt\right)^{1/2},\ \forall\, \omega \in L_2[0,T]$$

The symmetric terms in a symmetric matrix are denoted by $*$, $\Re^{\perp}$ denote any matrix whose columns form bases of the null space of $\Re$ and $\lambda_{\min}(P)$ denote the minimum eigenvalues of a symmetric matrix P.

2. Problem Statements and Some Lemmas

In this paper, based on multiple Lyapunov function technology and LMI approach, we investigate the asymptotical stabilization with H_{∞} disturbance attenuation for the exponentially uncertain state delay switched linear systems (1) and (2) via switched state feedback and switched dynamic output feedback, respectively.

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(\rho(t))x(t) + A_{d\sigma(t)}(\rho(t))x(t-d) + B_{1\sigma(t)}(t)\omega(t) + B_{\sigma(t)}(\rho(t))u(t) \\ z(t) = C_{\sigma(t)}(t)x(t) + D_{\sigma(t)}(t)u(t) \\ x(t) = \varphi(t), \forall t \in [-d, 0] \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(\rho(t))x(t) + A_{d\sigma(t)}(\rho(t))x(t-d) + B_{1\sigma(t)}(t)\omega(t) + B_{\sigma(t)}(\rho(t))u(t) \\ z(t) = C_{1\sigma(t)}(t)x(t) + D_{\sigma(t)}(t)u(t) \\ y(t) = C_{2\sigma(t)}(t)x(t) \\ x(t) = \varphi(t), \forall t \in [-d, 0] \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$ is the system state; $u \in \mathbb{R}^m$ is the continuous control input; $\omega \in \mathbb{R}^r$ is the exogenous disturbance input that belongs to $L_2[0, +\infty]$; $y \in \mathbb{R}^p$ is the measured output; $z \in \mathbb{R}^q$ is the regulated output; $d > 0$ is a constant positive delay, $\varphi(t) \in \mathbb{C}([-d, 0], \mathbb{R}^n)$ is a given continuous vector valued initial function. The switching signal $\sigma(\cdot):[0,\infty) \to \bar{\mathbb{N}} = \{1, 2, \cdots, N\}, N < \infty$ stands for the piecewise constant switching strategy to be designed. $A_{\sigma}(\rho(t)) = e^{M_{\sigma}\rho(t)}$, $A_{d\sigma}(\rho(t)) = \left(\int_0^{\rho(t)} e^{M_{\sigma}s} ds\right) R_{\sigma}$, $B_{\sigma}(\rho(t)) = \left(\int_0^{\rho(t)} e^{M_{\sigma}s} ds\right) N_{\sigma}$. Gives a particular index *i* indicating the active system regime. The uncertain parameter $\rho(t)$ is positive, time varying, bounded and $0 < \underline{\rho} < \rho(t) < \bar{\rho}$. $M_i \in \mathbb{R}^{n\times n}$, $R_i \in \mathbb{R}^{n\times n}$ and $N_i \in \mathbb{R}^{n\times m}$ are three families of matrices, $B_{1i}, C_i, C_{1i}, C_{2i}$ and D_i are constant matrices with appropriate dimensions ($i \in \bar{\mathbb{N}}$).

The objective of this paper is, for any given $\gamma > 0$, to find N switched state feedback controllers $u(t) = K_i x(t), i = 1, 2, \cdots N$ (or N switched output feedback controllers $u(t) = K_i(s)y, i = 1, 2, \cdots N$) so that the state delay switched linear system (1) (or the state delay switched linear system (2)) satisfies:

a) With zero disturbance input condition $\omega \equiv 0$, it is asymptotically stable for all admissible uncertainties.

b) With initial condition $x(t) = 0, \forall t \in [-d, 0]$, $\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt$ for all nonzero $\omega \in L_2[0, T]\ (0 \le T < \infty)$ and all admissible uncertainties.

Now, we introduce some support lemmas and the concept of switching sequence, which can be use in the later.

Lemma 2.1 ([12]): Consider the uncertain polynomial parameter dependent n-order matrix

$$L(\rho) = L_0 + L_1\rho + L_2\rho^2 + \cdots + L_h\rho^h$$

with the uncertain parameter ρ is positive, bounded and $0 < \underline{\rho} < \rho < \bar{\rho}$ (where $\underline{\rho}$ and $\bar{\rho}$ are known constants). Then we can find a convex polytope with $h+1$ vertices that envelopes $L(\rho)$, i.e. there exist parameters $\mu_j(\rho),\ j=1,2,\cdots,h+1$ satisfying $\sum_{j=1}^{h+1}\mu_j(\rho)=1$, $\mu_j(\rho)>0$ such that

$$L(\rho)=\sum_{j=1}^{h+1}\mu_j(\rho)U_j$$

where $U_j,\ j=1,2,\cdots,h+1$ represent the polytope vertices given as follows.

$$\begin{cases} U_1 = L_0 + L_1\underline{\rho} + L_2\underline{\rho}^2 + \cdots + L_h\underline{\rho}^h,\ U_2 = L_0 + L_1\overline{\rho} + L_2\underline{\rho}^2 + \cdots + L_h\underline{\rho}^h \\ U_3 = L_0 + L_1\overline{\rho} + L_2\overline{\rho}^2 + \cdots + L_h\underline{\rho}^h,\cdots,U_{h+1} = L_0 + L_1\overline{\rho} + L_2\overline{\rho}^2 + \cdots + L_h\overline{\rho}^h \end{cases}$$

The relation between the uncertain parameter ρ and $\mu_j(\rho)$ is given by

$$\mu_j(\rho)=\begin{cases} 1-(\rho-\underline{\rho})\big/(\overline{\rho}-\underline{\rho}), & j=1 \\ (\rho^{j-1}-\underline{\rho}^{j-1})\big/(\overline{\rho}^{j-1}-\underline{\rho}^{j-1})-(\rho^{j}-\underline{\rho}^{j})\big/(\overline{\rho}^{j}-\underline{\rho}^{j}), & j=2,3,\cdots,h \\ (\rho^{h}-\underline{\rho}^{h})\big/(\overline{\rho}^{h}-\underline{\rho}^{h}), & j=h+1 \end{cases}$$

Lemma 2.2 (Schur complement [16]): For constant matrices M, L and Q with appropriate dimensions where M and Q are symmetric and $Q>0$, then $M+L^TQL<0$ if and only if

$$\begin{pmatrix} M & L^T \\ L & -Q^{-1} \end{pmatrix}<0 \text{ or } \begin{pmatrix} -Q^{-1} & L \\ L^T & M \end{pmatrix}<0.$$

Lemma 2.3 ([17]): Given a symmetric matrix $\Psi\in\mathbb{R}^{n\times n}$ and two matrices Γ,Ξ of column dimension m, consider the problem of finding some matrix Θ of compatible dimensions such that

$$\Psi+\Gamma^T\Theta^T\Xi+\Xi^T\Theta\Gamma<0$$

Then the above matrix inequality is solvable for Θ if and only if

$$\begin{cases} \Gamma^{\perp T}\Psi\Gamma^{\perp}<0 \\ \Xi^{\perp T}\Psi\Xi^{\perp}<0 \end{cases}$$

Lemma 2.4([17]): Given symmetrically positive definite matrices $X\in\mathbb{R}^{n\times n}$ and $Y\in\mathbb{R}^{n\times n}$, n_k is a positive integer, then there exist matrices $X_2,Y_2\in\mathbb{R}^{n\times n_k}$ and symmetrically matrices $X_3,Y_3\in\mathbb{R}^{n_k\times n_k}$ are satisfied

$$\begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix}>0,\begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix}^{-1}=\begin{bmatrix} Y & Y_2 \\ Y_2^T & Y_3 \end{bmatrix}$$

if and only if

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix}\geq 0, rank\begin{bmatrix} X & I \\ I & Y \end{bmatrix}\leq n+n_k$$

Definition2.1 (switching sequence): The sequence $\{(t_m,r_m)\mid r_m\in\overline{\mathbb{N}};\ m=1,2,\cdots\}$ is said to be switching sequence, if

(i): $\sigma(t_m^-)\neq\sigma(t_m^+)$,

(ii): $\sigma(t)=\sigma(t_m^+)=r_m, t\in[t_m,t_{m+1})$

Moreover, the constant t_m is said to be dwell time length of the r_m-th subsystem.

3. Robust H_∞ Control via Switched State Feedback

In this section, the robust H_∞ problem of state delay switched linear system (1) is investigated via switched state feedback. Before the design of switched stabilization controller, we firstly show how the system (1) can be expressed as a switched polytopic system with an additive norm bounded uncertainty. Such a formulation makes the robust stabilization problem tractable.

According to the Lemma 2.1 and the properties of exponential matrix,

$$e^{Mx}=\sum_{i=0}^{\infty}\frac{M^{i}}{i!}x^{i},\quad \int_{0}^{x}e^{Ms}ds=\sum_{i=1}^{\infty}\frac{M^{i-1}}{i!}x^{i}$$

The following Lemma is obvious.

Lemma 3.1: The state delay switched linear system (1) subject to exponential uncertainties can be expressed as:

$$\begin{cases}\dot{x}(t)=\left(A_{\sigma}^{h}(\rho(t))+\Delta A_{\sigma}^{h}(\rho(t))\right)x(t)+\left(A_{d\sigma}^{h}(\rho(t))+\Delta A_{d\sigma}^{h}(\rho(t))\right)x(t-d)\\ \qquad +\left(B_{\sigma}^{h}(\rho(t))+\Delta B_{\sigma}^{h}(\rho(t))\right)u(t)+B_{1\sigma}\omega(t)\\ z(t)=C_{\sigma}x(t)+D_{\sigma}u(t)\\ x(t)=\varphi(t),\forall t\in[-d,0]\end{cases}\tag{3}$$

where
$$A_{\sigma}^{h}(\rho(t))=\sum_{j=0}^{h}\frac{M_{\sigma}^{j}}{j!}\rho^{j}(t)=\sum_{j=1}^{h+1}\mu_{j}(\rho(t))U_{\sigma j}^{Ah}\tag{4}$$

$$A_{d\sigma}^{h}(\rho(t))=\left(\sum_{j=1}^{h}\frac{M_{\sigma}^{j-1}}{j!}\rho^{j}(t)\right)R_{\sigma}=\left(\sum_{j=1}^{h+1}\left(\mu_{j}(\rho(t))U_{\sigma j}^{Bh}\right)\right)R_{\sigma}\tag{5}$$

$$B_{\sigma}^{h}(\rho(t))=\left(\sum_{j=1}^{h}\frac{M_{\sigma}^{j-1}}{j!}\rho^{j}(t)\right)N_{\sigma}=\left(\sum_{j=1}^{h+1}\left(\mu_{j}(\rho(t))U_{\sigma j}^{Bh}\right)\right)N_{\sigma}\tag{6}$$

$$\sum_{j=1}^{h+1}\mu_{j}(\rho)=1,\ \mu_{j}(\rho)>0,\ j=1,2,\ldots,h+1.\tag{7}$$

$$\begin{cases}U_{\sigma 1}^{Ah}=I+M_{\sigma}\underline{\rho}+\cdots+\frac{M_{\sigma}^{h}}{h!}\underline{\rho}^{h},\quad U_{\sigma 2}^{Ah}=I+M_{\sigma}\overline{\rho}+\cdots+\frac{M_{\sigma}^{h}}{h!}\underline{\rho}^{h},\\ U_{\sigma 3}^{Ah}=I+M_{\sigma}\overline{\rho}+\cdots+\frac{M_{\sigma}^{h}}{h!}\underline{\rho}^{h},\ldots,U_{\sigma,h+1}^{Ah}=I+M_{\sigma}\overline{\rho}+\cdots+\frac{M_{\sigma}^{h}}{h!}\overline{\rho}^{h}\end{cases}\tag{8}$$

$$\begin{cases}U_{\sigma 1}^{Bh}=\overline{\rho}I+\frac{M_{\sigma}}{2!}\underline{\rho}^{2}+\cdots+\frac{M_{\sigma}^{h}}{(h+1)!}\underline{\rho}^{h+1},\quad U_{\sigma 2}^{Bh}=\overline{\rho}I+\frac{M_{\sigma}}{2!}\overline{\rho}^{2}+\cdots+\frac{M_{\sigma}^{h}}{(h+1)!}\underline{\rho}^{h+1},\\ U_{\sigma 3}^{Bh}=\overline{\rho}I+\frac{M_{\sigma}}{2!}\overline{\rho}^{2}+\cdots+\frac{M_{\sigma}^{h}}{(h+1)!}\underline{\rho}^{h+1},\ldots,U_{\sigma,h+1}^{Bh}=\overline{\rho}I+\frac{M_{\sigma}}{2!}\overline{\rho}^{2}+\cdots+\frac{M_{\sigma}^{h}}{(h+1)!}\overline{\rho}^{h+1}\end{cases}\tag{9}$$

The remainders of the Taylor series approximation $\Delta A_{\sigma}^{h}(\rho(t))$, $\Delta A_{d\sigma}^{h}(\rho(t))$ and $\Delta B_{\sigma}^{h}(\rho(t))$, are given as follows:

$$\begin{cases}\Delta A_{\sigma}^{h}(\rho(t))=e^{M_{\sigma}\rho(t)}-\sum_{j=0}^{h}\frac{M_{\sigma}^{j}}{j!}\rho^{j}(t)\\ \Delta A_{d\sigma}^{h}(\rho(t))=\left(\int_{0}^{\rho(t)}e^{M_{\sigma}s}ds-\sum_{j=1}^{h+1}\frac{M_{\sigma}^{j-1}}{j!}\rho^{j}(t)\right)R_{\sigma}\\ \Delta B_{\sigma}^{h}(\rho(t))=\left(\int_{0}^{\rho(t)}e^{M_{\sigma}s}ds-\sum_{j=1}^{h+1}\frac{M_{\sigma}^{j-1}}{j!}\rho^{j}(t)\right)N_{\sigma}\end{cases}\tag{10}$$

The relation between the uncertain parameter $\rho(t)$ and the coordinates $\mu_{j}(\rho)$ is given by lemma 2.1.

Notice that the uncertain items $\Delta A_{\sigma}^{h}(\rho(t))$, $\Delta A_{d\sigma}^{h}(\rho(t))$ and $\Delta B_{\sigma}^{h}(\rho(t))$ are bounded while $0<\underline{\rho}<\rho(t)<\overline{\rho}$. Therefore one can write

$$\left\|\Delta A_{\sigma}^{h}(\rho(t))\right\|\le\gamma_{A},\left\|\Delta A_{d\sigma}^{h}(\rho(t))\right\|\le\gamma_{A_d},\left\|\Delta B_{\sigma}^{h}(\rho(t))\right\|\le\gamma_{B}\tag{11}$$

where

$$\begin{cases} \gamma_A = \sup\limits_{\underline{\rho}<\rho(t)<\bar{\rho}} \max\limits_{1\le i\le N} \left\| e^{M_i\rho(t)} - \sum_{j=0}^{h} \frac{M_i^j}{j!}\rho^j(t) \right\| \\ \gamma_{A_d} = \sup\limits_{\underline{\rho}<\rho(t)<\bar{\rho}} \max\limits_{1\le i\le N} \left\| \left(\int_0^{\rho(t)} e^{M_i s} ds - \sum_{j=1}^{h+1} \frac{M_i^{j-1}}{j!}\rho^j(t) \right) R_i \right\| \\ \gamma_B = \sup\limits_{\underline{\rho}<\rho(t)<\bar{\rho}} \max\limits_{1\le i\le N} \left\| \left(\int_0^{\rho(t)} e^{M_i s} ds - \sum_{j=1}^{h+1} \frac{M_i^{j-1}}{j!}\rho^j(t) \right) N_i \right\| \end{cases} \tag{12}$$

Remark 3.1: the describing and proof of lemma 3.1 is come from the same idea of literature [12].

By the above analysis, switched dynamic model (3) is actually a switched polytopic system subject to an additive norm bounded uncertainty. As lemma 3.1 has showed, such a switched model can be used to represent the state delay switched system (1). Therefore, stabilizing switched dynamic model (3) is equivalent to doing the system (1).

Our goal in this section, for the uncertain state delay switched linear system (3), is to investigate stabilization with H_∞ disturbance attenuation level γ via switched state feedback. It is stated as follows: for any given $\gamma > 0$, design a switching strategy $\sigma(t)$ and N associated state feedback sub-controller $u(t) = K_i(t)x(t), (i = 1, 2, \cdots N)$ such that the resulting closed-loop system (3) satisfies the following:

a) With zero disturbance input condition $\omega \equiv 0$, it is asymptotically stable for all admissible uncertainties.

b) With initial condition $x(t) = 0, \forall t \in [-d, 0]$, $\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt$ for all nonzero $\omega \in L_2[0,T]\,(0 \le T < \infty)$ and all admissible uncertainties.

For disturbance attenuation performance of system (3), we have the following result.

Theorem 3.1: Given any constant $\gamma > 0$, the uncertain state delay switched linear system (3) is asymptotically stabilization with H_∞ disturbance attenuation γ via switched state feedback if there exist symmetrically positive definite matrices X_i and matrices Y_i with $i \in \overline{¥}$ such that the following linear matrix inequality is satisfied for any $i = 1, 2, L\ , N$.

$$\begin{bmatrix} (A_i^h X_i + B_i^h Y_i) + (A_i^h X_i + B_i^h Y_i)^T & * & * & * & * & * & * \\ (A_{di}^h)^T & (\gamma_{A_d}^2 - 1)I & 0 & 0 & 0 & 0 & 0 \\ \gamma^{-1} B_{1i}^T & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{\gamma_A^2 + 1} X_i & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma_B Y_i & 0 & 0 & 0 & -I & 0 & 0 \\ \sqrt{3} I & 0 & 0 & 0 & 0 & -I & 0 \\ C_i X_i + D_i Y_i & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{13}$$

In this case, the state feedback sub-controller gain and switching strategy are taken as

$$\begin{cases} K_i = Y_i X_i^{-1} \\ \sigma(k) = \arg\min\limits_{i\in ¥} x^T(t) \begin{bmatrix} X_i^{-1} A_{Ci} + A_{Ci}^T X_i^{-1} + (1-\gamma_{A_d}^2)^{-1} X_i^{-1} A_{di}^h (A_{di}^h)^T X_i^{-1} \\ +\gamma^{-2} X_i^{-1} B_{1i} B_{1i}^T X_i^{-1} + (\gamma_A^2 + 1)I + \gamma_B^2 K_i^T K_i + 3X_i^{-2} + C_{Ci}^T C_{Ci} \end{bmatrix} x(t) \end{cases} \tag{14}$$

where $A_{Ci}(\rho(t)) = A_i^h(\rho(t)) + B_i^h(\rho(t))K_i, C_{Ci} = C_i + D_i K_i$.

Proof: Setting $X_i = P_i^{-1}, Y_i = K_i P_i^{-1}$, then the matrix inequality (13) is equivalent with the following matrix inequality

$$\begin{bmatrix} (A_i^h + B_i^h K_i)P_i^{-1} + P_i^{-1}(A_i^h + B_i^h K_i)^T & * & * & * & * & * & * \\ (A_{di}^h)^T & (\gamma_{A_d}^2 - 1)I & 0 & 0 & 0 & 0 & 0 \\ \gamma^{-1}B_{1i}^T & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{\gamma_A^2 + 1}P_i^{-1} & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma_B K_i P_i^{-1} & 0 & 0 & 0 & -I & 0 & 0 \\ \sqrt{3}I & 0 & 0 & 0 & 0 & -I & 0 \\ (C_i + D_i K_i)P_i^{-1} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{15}$$

Pro-multiplying and post-multiplying the matrix $\mathrm{diag}(P_i, I, I, I, I, I, I)$ in left-side of matrix inequality (15), the matrix inequality implies the following matrix inequality.

$$\begin{bmatrix} P_i A_{Ci} + A_{Ci}^T P_i & * & * & * & * & * & * \\ (A_{di}^h)^T P_i & (\gamma_{A_d}^2 - 1)I & 0 & 0 & 0 & 0 & 0 \\ \gamma^{-1}B_{1i}^T P_i & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{\gamma_A^2 + 1}I & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma_B K_i & 0 & 0 & 0 & -I & 0 & 0 \\ \sqrt{3}P_i & 0 & 0 & 0 & 0 & -I & 0 \\ C_{Ci} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{16}$$

In view of Lemma 2.2, the matrix inequality (16) is equivalent to the following inequality.

$$\begin{bmatrix} P_i A_{Ci} + A_{Ci}^T P + \gamma^{-2} P_i B_{1i} B_{1i}^T P_i + (\gamma_A^2 + 1)I + \gamma_B^2 K_i^T K_i + 3P_i^2 + C_{Ci}^T C_{Ci} & P_i A_{di}^h \\ (A_{di}^h)^T P_i & (\gamma_{A_d}^2 - 1)I \end{bmatrix} < 0 \tag{17}$$

Consider the notations:

$$\alpha_i(t) = \begin{cases} 1 & t \in \Omega_i \\ 0 & t \notin \Omega_i \end{cases}$$

$$\Omega_i = \{ t \mid \text{the i-th subsystem is active at time instant } t \},\ i = 1, 2, \cdots, N.$$

The closed-loop dynamic of system (3) with state feedback controller $u(t) = K_{\sigma(t)}x(t)$ is given by:

$$\begin{cases} \dot{x}(t) = \left(A_{C\sigma} + \Delta A_{C\sigma}\right)x(t) + \left(A_{d\sigma}^h + \Delta A_{d\sigma}^h\right)x(t-d) + B_{1\sigma}\omega(t) \\ z(t) = C_{C\sigma}x(t) \\ x(t) = \varphi(t), \forall t \in [-d, 0] \end{cases} \tag{18}$$

where $\Delta A_{C\sigma} = \Delta A_\sigma^h + \Delta B_\sigma^h K_\sigma$

Consider switched parameter dependent Lyapunov-like function

$$V(x_t) = x^T(t)\left(\sum_{i=1}^N \alpha_i(t)P_i\right)x(t) + \int_{t-d}^t x^T(\tau)x(\tau)d\tau \tag{19}$$

where $x_t = x(t+\theta), \theta \in [-d, 0]$ and $P_i\ (i \in \square)$ are symmetrical positive definite matrices.

Then for any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the time-derivative of Lyapunov-like function (19) along with the trajectory of the closed-loop dynamic system (18) is given by

$$\dot{V}(x_t) = \dot{x}^T(t)P_{r_m}x(t) + x^T(t)P_{r_m}\dot{x}(t) + x^T(t)x(t) - x^T(t-d)x(t-d)$$
$$= x^T(t)\left[A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + \Delta A_{Cr_m}^T P_{r_m} + P_{r_m}\Delta A_{Cr_m} + I\right]x(t) + x^T(t-d)(A_{dr_m}^h)^T P_{r_m}x(t) + x^T(t)P_{r_m}A_{dr_m}^h x(t-d)$$
$$+ x^T(t-d)(\Delta A_{dr_m}^h)^T P_{r_m}x(t) + x^T(t)P_{r_m}\Delta A_{dr_m}^h x(t-d) + \omega^T B_{1r_m}^T P_{r_m}x(t) + x^T(t)P_{r_m}B_{1r_m}\omega - x^T(t-d)x(t-d)$$
$$\le x^T(t)\left[A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + \frac{1}{2}\Delta A_{Cr_m}^T \Delta A_{Cr_m} + 2P_{r_m}^2 + I\right]x(t) + x^T(t-d)(A_{dr_m}^h)^T P_{r_m}x(t)$$
$$+ x^T(t)P_{r_m}A_{dr_m}^h x(t-d) + x^T(t-d)(\Delta A_{dr_m}^h)^T \Delta A_{dr_m}^h x(t-d) + x^T(t)P_{r_m}^2 x(t)$$
$$+\gamma^2\omega^T\omega + \gamma^{-2}x^T(t)P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m}x(t) - x^T(t-d)x(t-d) - z^T(t)z(t) + x^T(t)C_{Cr_m}^T C_{Cr_m}x(t)$$

By means of (11), we have

$$\Delta A_{Cr_m}^T \Delta A_{Cr_m} = (\Delta A_{r_m}^h(\rho) + \Delta B_{r_m}^h(\rho)K_{r_m})^T(\Delta A_{r_m}^h(\rho) + \Delta B_{r_m}^h(\rho)K_{r_m})$$
$$\le 2(\Delta A_{r_m}^h(\rho))^T \Delta A_{r_m}^h(\rho) + 2(\Delta B_{r_m}^h(\rho)K_{r_m})^T(\Delta B_{r_m}^h(\rho)K_{r_m}) \le 2\gamma_A^2 I + 2\gamma_B^2(K_{r_m})^T K_{r_m}$$

and $(\Delta A_{dr_m}^h)^T \Delta A_{dr_m}^h \le \gamma_{Ad}^2 I$

Consequently,

$$z^T(t)z(t) - \gamma^2\omega^T\omega + \dot{V}(x_t)$$
$$\le x^T(t)\left[A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_m}^T K_{r_m} + 3P_{r_m}^2 + \gamma^{-2}P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m}\right]x(t)$$
$$+ x^T(t-d)(A_{dr_m}^h)^T P_{r_m}x(t) + x^T(t)P_{r_m}A_{dr_m}^h x(t-d) + (\gamma_{A_d}^2 - 1)x^T(t-d)x(t-d)$$
$$= \begin{bmatrix} x^T(t) & x^T(t-d)\end{bmatrix}\begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_m}^T K_{r_m} + 3P_{r_m}^2 + \gamma^{-2}P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & P_{r_m}A_{dr_m}^h \\ (A_{dr_m}^h)^T P_{r_m} & (\gamma_{A_d}^2 - 1)I\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}$$

Assume $x(t) = 0, \forall t \in [-d, 0]$ and introduce the performance

$$J = \int_0^T (z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t))dt$$

Let $\{(t_m, r_m) \mid r_m \in \overline{M}; m = 1, 2, \ldots, s; 0 = t_1 \le t_2 \le \ldots \le t_s \le T\}$ be switching sequence in the interval $[0, T]$ that is generated by the switching strategy (14). Noting that $x(t_1) = 0$ and $x(t_1 - d) = 0$, then for every $\omega \in L_2[0, T]$.

$$J = \sum_{m=1}^{s-1}\int_{t_m}^{t_{m+1}}(z^T z - \gamma^2\omega^T\omega + \dot{V}(x_t))dt + \int_{t_s}^T(z^T z - \gamma^2\omega^T\omega + \dot{V}(x_t))dt - V(x_T)$$
$$\le \sum_{m=1}^{s-1}\int_{t_m}^{t_{m+1}}(z^T z - \gamma^2\omega^T\omega + \dot{V}(x_t))dt + \int_{t_s}^T(z^T z - \gamma^2\omega^T\omega + \dot{V}(x_t))dt$$
$$\le \sum_{m=1}^{s-1}\int_{t_m}^{t_{m+1}}\begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}^T\begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_m}^T K_{r_m} + 3P_{r_m}^2 + \gamma^{-2}P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & P_{r_m}A_{dr_m}^h \\ (A_{dr_m}^h)^T P_{r_m} & (\gamma_{A_d}^2 - 1)I\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}dt$$
$$+\int_{t_s}^T\begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}^T\begin{bmatrix} A_{Cr_s}^T P_{r_s} + P_{r_s}A_{Cr_s} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_s}^T K_{r_s} + 3P_{r_s}^2 + \gamma^{-2}P_{r_s}B_{1r_s}B_{1r_s}^T P_{r_s} + C_{Cr_s}^T C_{Cr_s} & P_{r_s}A_{dr_s}^h \\ (A_{dr_s}^h)^T P_{r_s} & (\gamma_{A_d}^2 - 1)I\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-d)\end{bmatrix}dt$$

By virtue of the matrix inequality (17), it follows that $J < 0$. That is to say

$$\int_0^T z^T(t)z(t)dt < \gamma^2\int_0^T \omega^T(t)\omega(t)dt, \quad \forall \omega \in L_2[0, T].$$

Now, we prove that the switched system (18) with $\omega = 0$ is asymptotically stable.

Let $\{(t_m, r_m) \mid r_m \in \overline{M}; m = 1, 2, \ldots; 0 = t_1 \le t_2 \le \ldots\}$ be switching sequence in the interval $[0, \infty)$ that is generated by the switching strategy (14).

For any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the derivative of Lyapunov-like function (19) along with the trajectory of the switched system (18) with $\omega = 0$ is given by

$$
\begin{aligned}
\dot V(x_t) &= \dot x^T(t)P_{r_m}x(t) + x^T(t)P_{r_m}\dot x(t) + x^T(t)x(t) - x^T(t-d)x(t-d) \\
&= x^T(t)\left[A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + \Delta A_{Cr_m}^T P_{r_m} + P_{r_m}\Delta A_{Cr_m} + I\right]x(t) + x^T(t-d)(A_{dr_m}^h)^T P_{r_m}x(t) \\
&\quad + x^T(t)P_{r_m}A_{dr_m}^h x(t-d) + x^T(t-d)(\Delta A_{dr_m}^h)^T P_{r_m}x(t) + x^T(t)P_{r_m}\Delta A_{dr_m}^h x(t-d) - x^T(t-d)x(t-d) \\
&\le x^T(t)\begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_m}^T K_{r_m} \\ +3P_{r_m}^2 + \gamma^{-2}P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} \end{bmatrix} x(t) \\
&\quad + x^T(t-d)(A_{dr_m}^h)^T P_{r_m}x(t) + x^T(t)P_{r_m}A_{dr_m}^h x(t-d) + (\gamma_{A_d}^2 - 1)x^T(t-d)x(t-d) \\
&= \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m}A_{Cr_m} + (\gamma_A^2+1)I + \gamma_B^2 K_{r_m}^T K_{r_m} & \\ +3P_{r_m}^2 + \gamma^{-2}P_{r_m}B_{1r_m}B_{1r_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & P_{r_m}A_{dr_m}^h \\ (A_{dr_m}^h)^T P_{r_m} & (\gamma_{A_d}^2 - 1)I \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}
\end{aligned}
$$

By means of (17), for any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the time derivative of Lyapunov-like functions (19) alone with the trajectory of the switched system (18) with $\omega = 0$ is less than zero.

Without of generally, suppose $\sigma(t_{m+1}^+) = r_{m+1}$. Then by Lyapunov-like functions (19), switching strategy (14) and $P_i^{-1} = X_i, i = 1, 2, \cdots, N$.

$$
V(t_{m+1}^+) - V(t_{m+1}^-) = x^T(t_{m+1})\left(P_{r_{m+1}} - P_{r_m}\right)x(t_{m+1}) = x^T(t_{m+1})\left(X_{r_{m+1}}^{-1} - X_{r_m}^{-1}\right)x(t_{m+1}) < 0
$$

Hence under the action of switching strategy $\sigma(t)$ and associated state feedback sub-controllers $u = K_i x(t), i \in \overline{\square}$, the state delay switched linear system (3) is asymptotically stabilization with H_∞ disturbance attenuation level γ via switched state feedback.

4. Robust H_∞ Control via dynamic Output Feedback

In this section, the robust H_∞ problem of state delay switched linear system (2) is investigated via switched output feedback. Before the design of switched stabilization controller, we firstly show how the system (2) can be expressed as a switched polytopic system with an additive norm bounded uncertainty. Such a formulation makes the robust stabilization problem tractable.

By Lemma 3.1, the following lemma is obvious.

Lemma 4.1: The state delay switched linear system (2) subject to exponential uncertainties can be expressed as:

$$
\begin{cases}
\dot x(t) = \left(A_\sigma^h(\rho(t)) + \Delta A_\sigma^h(\rho(t))\right)x(t) + \left(A_{d\sigma}^h(\rho(t)) + \Delta A_{d\sigma}^h(\rho(t))\right)x(t-d) \\
\qquad + \left(B_\sigma^h(\rho(t)) + \Delta B_\sigma^h(\rho(t))\right)u(t) + B_{1\sigma}\omega(t) \\
z(t) = C_{1\sigma}x(t) + D_\sigma u(t) \\
y(t) = C_{2\sigma}x(t) \\
x(t) = \varphi(t), \forall t \in [-d, 0]
\end{cases}
\tag{20}
$$

where $A_\sigma^h(\rho(t))$, $A_{d\sigma}^h(\rho(t))$ and $B_\sigma^h(\rho(t))$ are described as (4)-(6) and (8)-(9); the remainders of the Taylor series approximation $\Delta A_\sigma^h(\rho(t))$, $\Delta A_{d\sigma}^h(\rho(t))$ and $\Delta B_\sigma^h(\rho(t))$ are given by the functions (10)-(12). The description of parameters $\mu_j(\rho(t))$ is given by (7) and Lemma 2.1.

In view of the Lemma 4.1, switched dynamic model (20) is actually a switched polytopic system subject to norm bounded uncertainty. Furthermore, such a switched model can be used to represent the state delay switched linear system (2). Therefore, stabilizing switched dynamic model (20) is equivalent to doing the system (2).

Our goal is, for any given $\gamma > 0$, to find N switched dynamic output feedback sub-controllers $u = k_i(s)y,\ i = 1,2,\cdots N$ such that the resulting closed-loop system (20) satisfies:

a) With zero disturbance input condition $\omega \equiv 0$, it is asymptotically stable for all admissible uncertainties.

b) With initial condition $x(t) = 0, \forall t \in [-d, 0]$, $\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt$ for all nonzero $\omega \in L_2[0,T]\ (0 \le T < \infty)$ and all admissible uncertainties.

For system (20), we are interested in constructing the form of the switched dynamic output-feedback controller as follows:

$$\begin{cases} \dot{\xi}(t) = \hat{A}_{\sigma(t)}\xi(t) + \hat{B}_{\sigma(t)}y(t) \\ u(t) = \hat{C}_{\sigma(t)}\xi(t) + \hat{D}_{\sigma(t)}y(t) \end{cases} \tag{21}$$

where $\xi \in \mathbb{R}^n$.

In this case, the resulting closed-loop system (20) with switched dynamic output-feedback controller (21) is given by

$$\begin{cases} \dot{\tilde{x}}(t) = (A_{C\sigma(t)} + \Delta A_{C\sigma(t)})\tilde{x}(t) + (A_{Cd\sigma(t)} + \Delta A_{Cd\sigma(t)})\tilde{x}(t-d) + B_{C\sigma(t)}\omega(t) \\ z(t) = C_{C\sigma(t)}\tilde{x}(t) \end{cases} \tag{22}$$

where

$$\tilde{x}^T = (x^T, \xi^T),\ A_{C\sigma(t)} = A^0_{\sigma(t)} + B^0_{\sigma(t)}K_{\sigma(t)}C^0_{2\sigma(t)},\ \Delta A_{C\sigma(t)} = \Delta A^0_{\sigma(t)} + \Delta B^0_{\sigma(t)}K_{\sigma(t)}C^0_{2\sigma(t)},$$

$$C_{C\sigma(t)} = C^0_{1\sigma(t)} + D^0_{\sigma(t)}K_{\sigma(t)}C^0_{2\sigma(t)},\ C^0_{1\sigma(t)} = \begin{bmatrix} C_{1\sigma(t)} & 0 \end{bmatrix},\ B_{C\sigma(t)} = B^0_{1\sigma(t)} = \begin{bmatrix} B_{1\sigma(t)} \\ 0 \end{bmatrix},$$

$$A_{Cd\sigma(t)} = \begin{bmatrix} A^h_{d\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, \Delta A_{Cd\sigma(t)} = \begin{bmatrix} \Delta A^h_{d\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, A^0_{\sigma(t)} = \begin{bmatrix} A^h_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Delta A^0_{\sigma(t)} = \begin{bmatrix} \Delta A^h_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, D^0_{\sigma(t)} = \begin{bmatrix} D_{\sigma(t)} & 0 \end{bmatrix}, B^0_{\sigma(t)} = \begin{bmatrix} B^h_{\sigma(t)} & 0 \\ 0 & I \end{bmatrix},$$

$$\Delta B^0_{\sigma(t)} = \begin{bmatrix} \Delta B^h_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, C^0_{2\sigma(t)} = \begin{bmatrix} C_{2\sigma(t)} & 0 \\ 0 & I \end{bmatrix}, K_{\sigma(t)} = \begin{bmatrix} \hat{D}_{\sigma(t)} & \hat{C}_{\sigma(t)} \\ \hat{B}_{\sigma(t)} & \hat{A}_{\sigma(t)} \end{bmatrix}.$$

Next, we give the sufficient conditions for existence of the switching strategy $\sigma(t)$ and associated dynamic output feedback sub-controllers (21) such that the resulting closed-loop system (22) is stabilizable with H_∞ disturbance attenuation level γ.

Theorem 4.1: Given any constant $\gamma > 0$, the uncertain state delay switched linear system (20) is asymptotically stabilization with H_∞ disturbance attenuation γ via switched dynamic output feedback if there exist symmetrically positive definite matrices X_{Ci} with $i = 1,2,\ldots,N$ such that the following matrix inequality is satisfied for any $i = 1,2,\ldots,N$.

$$\begin{bmatrix} A^T_{Ci}X_{Ci} + X_{Ci}A_{X_{Ci}} + X_{Ci} & * & * & * & * & * & * \\ (A^0_{di})^T X_{Ci} & (\gamma^2_{A_d} - \eta_i)I & 0 & 0 & 0 & 0 & 0 \\ \gamma_A I_0 & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{3}I_0X_{Ci} & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma^{-1}B^T_{Ci}X_{Ci} & 0 & 0 & 0 & -I & 0 & 0 \\ \gamma_B E_{Ci} & 0 & 0 & 0 & 0 & -I & 0 \\ C_{Ci} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{23}$$

where $A^0_{di} = \begin{bmatrix} A^h_{di} \\ 0 \end{bmatrix}$, $0 < \eta_i \le \lambda_{\min}(X_{Ci})$, $I_0 = \begin{bmatrix} I & 0 \end{bmatrix}, I_C = I_0^T I_0 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $E_{Ci} = I_0 K_i C^0_{2i}$.

In this case, the dynamic output feedback controller gain matrix

$$K_i = \begin{bmatrix} \hat{D}_i & \hat{C}_i \\ \hat{B}_i & \hat{A}_i \end{bmatrix}, \quad i = 1, 2, \cdots, N \tag{24}$$

The switching strategy $\sigma(t)$ is given by

$$\sigma(t) = \arg\min_{i \in \overline{N}} \left\{ \tilde{x}^T(t) \begin{pmatrix} A_{Ci}^T X_{Ci} + X_{Ci} A_{Ci} + X_{Ci} + (\eta_i - \gamma_{Ad}^2)^{-1} X_{Ci} A_{di}^0 (A_{di}^0)^T X_{Ci} + \\ \gamma_A^2 I_C + 3 X_{Ci} I_C X_{Ci} + \gamma^{-2} X_{Ci} B_{Ci} B_{Ci}^T X_{Ci} + \gamma_B^2 E_{Ci}^T E_{Ci} + C_{Ci}^T C_{Ci} \end{pmatrix} \tilde{x}(t) \right\} \tag{25}$$

Proof: The matrix inequality (23) is transformed into the following matrix inequality.

$$\begin{bmatrix} A_{Ci}^T X_{Ci} + X_{Ci} A_{X_{Ci}} + X_{Ci} & * & * & * & * & * & * \\ A_{Cdi}^T X_{Ci} & \gamma_{A_d}^2 I_C - \eta_i I & 0 & 0 & 0 & 0 & 0 \\ \gamma_A I_0 & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{3} I_0 X_{Ci} & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma^{-1} B_{Ci}^T X_{Ci} & 0 & 0 & 0 & -I & 0 & 0 \\ \gamma_B E_{Ci} & 0 & 0 & 0 & 0 & -I & 0 \\ C_{Ci} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{26}$$

In view of Lemma 2.2, the matrix inequality (26) is equivalent to the following inequality.

$$\begin{bmatrix} A_{Ci}^T X_{Ci} + X_{Ci} A_{Ci} + X_{Ci} + \gamma_A^2 I_C + 3 X_{Ci} I_C X_{Ci} & \\ + \gamma^{-2} X_{Ci} B_{Ci} B_{Ci}^T X_{Ci} + \gamma_B^2 E_{Ci}^T E_{Ci} + C_{Ci}^T C_{Ci} & X_{Ci} A_{Cdi} \\ A_{Cdi}^T X_{Ci} & \gamma_{A_d}^2 I_C - \eta_i I \end{bmatrix} < 0 \tag{27}$$

Consider the notations:

$$\alpha_i(t) = \begin{cases} 1 & t \in \Omega_i \\ 0 & t \notin \Omega_i \end{cases}$$

$$\Omega_i = \{ t \mid \text{the i-th subsystem is active at time instant } t \}, \ i = 1, 2, \cdots, N.$$

Consider switched parameter dependent Lyapunov-like function

$$V(\tilde{x}_t) = \tilde{x}^T(t) \left(\sum_{i=1}^N \alpha_i(t) P_i \right) \tilde{x}(t) + \int_{t-d}^t \tilde{x}^T(\tau) \left(\sum_{i=1}^N \alpha_i(t) P_i \right) \tilde{x}(\tau) d\tau \tag{28}$$

where $\tilde{x}_t = \tilde{x}(t+\theta), \theta \in [-d, 0]$ and $P_i \ (i \in \overline{N})$ are symmetrical positive definite matrices.

Then for any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the time-derivative of Lyapunov-like function (28) along with the trajectory of the closed-loop dynamic system (22) is given by

$$\begin{aligned} \dot{V}(\tilde{x}_t) &= \dot{\tilde{x}}^T(t) P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} \dot{\tilde{x}}(t) + \tilde{x}^T(t) P_{r_m} \tilde{x}(t) - \tilde{x}^T(t-d) P_{r_m} \tilde{x}(t-d) \\ &= \tilde{x}^T(t) \left[A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \Delta A_{Cr_m}^T P_{r_m} + P_{r_m} \Delta A_{Cr_m} + P_{r_m} \right] \tilde{x}(t) + \tilde{x}^T(t-d) A_{Cdr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} A_{Cdr_m} \tilde{x}(t-d) \\ &\quad + \tilde{x}^T(t-d) \Delta A_{Cdr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} \Delta A_{Cdr_m} \tilde{x}(t-d) + \omega^T B_{Cr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} B_{Cr_m} \omega - \tilde{x}^T(t-d) P_{r_m} \tilde{x}(t-d) \end{aligned}$$

Again in view of (11),

$$\Delta A_{Cr_m}^T P_{r_m} + P_{r_m} \Delta A_{Cr_m} \le \frac{1}{2} \Delta A_{Cr_m}^T \Delta A_{Cr_m} + 2 P_{r_m} I_C P_{r_m} \le \gamma_A^2 I_C + \gamma_B^2 E_{Cr_m}^T E_{Cr_m} + 2 P_{r_m} I_C P_{r_m}$$

$$\Delta A_{Cdr_m}^T P_{r_m} + P_{r_m} \Delta A_{Cdr_m} \le \Delta A_{Cdr_m}^T \Delta A_{Cdr_m} + P_{r_m} I_C P_{r_m} \le \gamma_{Ad}^2 I_C + + P_{r_m} I_C P_{r_m}$$

Consequently,

$$\begin{aligned} \dot{V}(\tilde{x}_t) &\le \tilde{x}^T(t) \left[A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \gamma_A^2 I_C + \gamma_B^2 E_{Cr_m}^T E_{Cr_m} + 2 P_{r_m} I_C P_{r_m} + P_{r_m} \right] \tilde{x}(t) + \tilde{x}^T(t-d) A_{Cdr_m}^T P_{r_m} \tilde{x}(t) \\ &\quad + \tilde{x}^T(t) P_{r_m} A_{Cdr_m} \tilde{x}(t-d) + \gamma_{Ad}^2 \tilde{x}^T(t-d) I_C \tilde{x}(t-d) + \tilde{x}^T(t) P_{r_m} I_C P_{r_m} \tilde{x}(t) + \gamma^2 \omega^T \omega \\ &\quad + \gamma^{-2} \tilde{x}^T(t) P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} x(t) - \tilde{x}^T(t-d) P_{r_m} \tilde{x}(t-d) - z^T(t) z(t) + \tilde{x}^T(t) C_{Cr_m}^T C_{Cr_m} \tilde{x}(t) \end{aligned}$$

By $P_{r_m} \ge \eta_{r_m} I > 0$, we have

$$
\begin{aligned}
& z^T(t)z(t) - \gamma^2 \omega^T \omega + \dot{V}(\tilde{x}_t) \\
& \le \tilde{x}^T(t) \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \gamma_A^2 I_C + P_{r_m} + \gamma_B^2 E_{Cr_m}^T E_{Cr_m} \\ +3P_{r_m} I_C P_{r_m} + \gamma^{-2} P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} \end{bmatrix} \tilde{x}(t) \\
& + \tilde{x}^T(t-d) A_{Cdr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} A_{Cdr_m} \tilde{x}(t-d) + \tilde{x}^T(t-d)(\gamma_{A_d}^2 I_C - \eta_{r_m} I)\tilde{x}(t-d) \\
& = \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}^T \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \gamma_A^2 I_C + P_{r_m} + \gamma_B^2 E_{Cr_m}^T E_{Cr_m} & \\ +3P_{r_m} I_C P_{r_m} + \gamma^{-2} P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & P_{r_m} A_{Cdr_m} \\ A_{Cdr_m}^T P_{r_m} & \gamma_{A_d}^2 I_C - \eta_{r_m} I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}
\end{aligned}
$$

Assume $\tilde{x}(t) = 0, \forall t \in [-d, 0]$ and introduce the performance

$$J = \int_0^T (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t))dt$$

Let $\{(t_m, r_m) \mid r_m \in \bar{M}; m = 1, 2, \cdots, s; 0 = t_1 \le t_2 \le \cdots \le t_s \le T\}$ be switching sequence in the interval $[0, T]$ that is generated by the switching strategy (25). Noting that $\tilde{x}(t_1) = 0$ and $\tilde{x}(t_1 - d) = 0$, then for every $\omega \in L_2[0, T]$.

$$
\begin{aligned}
J &= \sum_{m=1}^{s-1} \int_{t_m}^{t_{m+1}} (z^T z - \gamma^2 \omega^T \omega + \dot{V}(\tilde{x}_t))dt + \int_{t_s}^{T} (z^T z - \gamma^2 \omega^T \omega + \dot{V}(\tilde{x}_t))dt - V(\tilde{x}_T) \\
&\le \sum_{m=1}^{s-1} \int_{t_m}^{t_{m+1}} (z^T z - \gamma^2 \omega^T \omega + \dot{V}(\tilde{x}_t))dt + \int_{t_s}^{T} (z^T z - \gamma^2 \omega^T \omega + \dot{V}(\tilde{x}_t))dt \\
&\le \sum_{m=1}^{s-1} \int_{t_m}^{t_{m+1}} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}^T \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + P_{r_m} + \gamma_A^2 I_C & \\ +\gamma_B^2 E_{Cr_m}^T E_{Cr_m} + 3P_{r_m} I_C P_{r_m} & P_{r_m} A_{Cdr_m} \\ +\gamma^{-2} P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & \\ A_{Cdr_m}^T P_{r_m} & \gamma_{A_d}^2 I_C - \eta_{r_m} I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix} dt \\
&+ \int_{t_s}^{T} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}^T \begin{bmatrix} A_{Cr_s}^T P_{r_s} + P_{r_s} A_{Cr_s} + P_{r_s} + \gamma_A^2 I_C & \\ +\gamma_B^2 E_{Cr_s}^T E_{Cr_s} + 3P_{r_s} I_C P_{r_s} & P_{r_s} A_{Cdr_s} \\ +\gamma^{-2} P_{r_s} B_{Cr_s} B_{Cr_s}^T P_{r_s} + C_{Cr_s}^T C_{Cr_s} & \\ A_{Cdr_s}^T P_{r_s} & \gamma_{A_d}^2 I_C - \eta_{r_s} I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix} dt
\end{aligned}
$$

Setting $P_{r_m} = X_{Cr_m} > 0$, by virtue of the matrix inequality (27), it follows that $J < 0$. That is

$$\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt, \quad \forall \omega \in L_2[0, T].$$

Now, we prove that the switched system (20) with $\omega = 0$ is asymptotically stable.

Let $\{(t_m, r_m) \mid r_m \in \bar{M}; m = 1, 2, \cdots; 0 = t_1 \le t_2 \le \cdots\}$ be switching sequence in the interval $[0, \infty)$ that is generated by the switching strategy (25).

For any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the derivative of Lyapunov-like function (28) along with the trajectory of the switched system (22) with $\omega = 0$ is given by

$$
\begin{aligned}
\dot{V}(\tilde{x}_t) &= \dot{\tilde{x}}^T(t) P_{r_m} x(t) + \tilde{x}^T(t) P_{r_m} \dot{\tilde{x}}(t) + \tilde{x}^T(t) P_{r_m} \tilde{x}(t) - \tilde{x}^T(t-d) P_{r_m} \tilde{x}(t-d) \\
&= \tilde{x}^T(t) \left[A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \Delta A_{Cr_m}^T P_{r_m} + P_{r_m} \Delta A_{Cr_m} + P_{r_m} \right] \tilde{x}(t) + \tilde{x}^T(t-d) A_{Cdr_m}^T P_{r_m} \tilde{x}(t) \\
&+ \tilde{x}^T(t) P_{r_m} A_{Cdr_m} \tilde{x}(t-d) + \tilde{x}^T(t-d) \Delta A_{Cdr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} \Delta A_{Cdr_m} \tilde{x}(t-d) - \tilde{x}^T(t-d) P_{r_m} \tilde{x}(t-d) \\
&\le \tilde{x}^T(t) \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + \gamma_A^2 I_C + \gamma_B^2 E_{Cr_m}^T E_{Cr_m} + P_{r_m} \\ +3P_{r_m} I_C P_{r_m} + \gamma^{-2} P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} \end{bmatrix} \tilde{x}(t) \\
&+ \tilde{x}^T(t-d) A_{Cdr_m}^T P_{r_m} \tilde{x}(t) + \tilde{x}^T(t) P_{r_m} A_{Cdr_m} \tilde{x}(t-d) + \tilde{x}^T(t-d)(\gamma_{A_d}^2 I_C - \eta_{r_m} I)\tilde{x}(t-d)
\end{aligned}
$$

$$
= \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}^T \begin{bmatrix} A_{Cr_m}^T P_{r_m} + P_{r_m} A_{Cr_m} + P_{r_m} + \gamma_A^2 I_C & \\ +\gamma_B^2 E_{Cr_m}^T E_{Cr_m} + 3 P_{r_m} I_C P_{r_m} & P_{r_m} A_{Cdr_m} \\ +\gamma^{-2} P_{r_m} B_{Cr_m} B_{Cr_m}^T P_{r_m} + C_{Cr_m}^T C_{Cr_m} & \\ A_{Cdr_m}^T P_{r_m} & \gamma_{A_d}^2 I_C - \eta_{r_m} I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{bmatrix}
$$

By means of (27), for any $t \in [t_m, t_{m+1}) \subset \Omega_{r_m}$, the time derivative of Lyapunov-like functions (28) alone with the trajectory of the switched system (22) with $\omega = 0$ is less than zero.

Without of generally, suppose that $\sigma(t_{m+1}^+) = r_{m+1}$. Then by Lyapunov-like functions (28) and switching strategy (25) and $P_i = X_{Ci}, i = 1, 2, \cdots, N$

$$
V(\tilde{x}_{t_{m+1}^+}) - V(\tilde{x}_{t_{m+1}^-}) = \tilde{x}^T(t_{m+1}) \left(X_{Cr_{m+1}} - X_{Cr_m} \right) \tilde{x}(t_{m+1}) + \int_{t_{m+1}-d}^{t_{m+1}} \tilde{x}^T(\tau) \left(X_{Cr_{m+1}} - X_{Cr_m} \right) \tilde{x}(\tau) d(\tau) < 0
$$

Hence under the action of switching strategy $\sigma(t)$ and associated dynamic output feedback sub-controllers (21), the state delay switched linear system (2) is asymptotically stabilization with H_∞ disturbance attenuation level γ via switched dynamic output feedback.

However, the matrix inequality (23) is not linear matrix inequality. Our goal in the following section, for the non-linear matrix inequality (23), is transformed into linear matrix inequalities by the so-called eliminate element method (a kind of classical technique in studying the H_∞ control problem of a non-switched linear system).

The matrix inequality (23) is transformed into the following matrix inequality.

$$
H_{X_{Ci}} + M_{X_{Ci}}^T K_i Q_i + Q_i^T K_i^T M_{X_{Ci}} < 0 \tag{29}
$$

where

$$
H_{X_{Ci}} = \begin{bmatrix} (A_i^0)^T X_{Ci} + X_{Ci} A_i^0 + X_{Ci} & * & * & * & * & * & * \\ (A_{di}^0)^T X_{Ci} & (\gamma_{Ad}^2 - \eta_i) I & 0 & 0 & 0 & 0 & 0 \\ \gamma_A I_0 & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{3} I_0 X_{Ci} & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma^{-1} B_{Ci}^T X_{Ci} & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I & 0 \\ C_{1i}^0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},
$$

$$
M_{X_{Ci}} = \begin{bmatrix} (B_i^0)^T X_{Ci} & 0 & 0 & 0 & 0 & \gamma_B I_0^T & (D_i^0)^T \end{bmatrix}, \quad Q_i = \begin{bmatrix} C_{2i}^0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$

By Lemma 2.3, the matrix inequality (29) is equivalent to the following inequalities.

$$
M_{X_{Ci}}^{\perp T} H_{X_{Ci}} M_{X_{Ci}}^{\perp} < 0 \text{ and } Q_i^{\perp T} H_{X_{Ci}} Q_i^{\perp} < 0 . \tag{30}
$$

For $M_{X_{Ci}}^{\perp T} H_{X_{Ci}} M_{X_{Ci}}^{\perp} < 0$, setting $S_i = diag\{X_{Ci}, I, I, I, I, I, I\}$, then $M_{X_{Ci}} = M_i S_i$, where $M_i = \begin{bmatrix} (B_{2i}^0)^T & 0 & 0 & 0 & 0 & \gamma_B I_0^T & (D_i^0)^T \end{bmatrix}$. So we have $M_{X_{Ci}}^{\perp} = S_i^{-1} M_i^{\perp}$.

The matrix inequality $M_{X_{Ci}}^{\perp T} H_{X_{Ci}} M_{X_{Ci}}^{\perp} < 0$ is transformed into the following inequality:

$$
M_i^{\perp T} T_{X_{Ci}} M_i^{\perp} < 0
$$

where

$$T_{X_{Ci}} = (S_i^{-1})^T H_{X_{Ci}} S_i^{-1} = \begin{bmatrix} X_{Ci}^{-1}(A_i^0)^T + A_i^0 X_{Ci}^{-1} + X_{Ci}^{-1} & * & * & * & * & * & * \\ (A_{di}^0)^T & (\gamma_{Ad}^2 - \eta_i)I & 0 & 0 & 0 & 0 & 0 \\ \gamma_A I_0 X_{Ci}^{-1} & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{3} I_0 & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma^{-1} B_{Ci}^T & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I & 0 \\ C_{1i}^0 X_{Ci}^{-1} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

On the one hand in view of Lemma 2.4 and the definition of A_i^0, A_{di}^0, B_{Ci} and C_{1i}^0, the above matrix is transformed into the following matrix.

$$T_{X_{Ci}} = \begin{bmatrix} A_i^h Y_i + Y_i (A_i^h)^T + Y_i & * & * & * & * & * & * & * \\ Y_{2i}^T (A_i^h)^T + Y_{2i}^T & Y_{3i} & * & * & * & * & * & * \\ (A_{di}^h)^T & 0 & (\gamma_{Ad}^2 - \eta_i)I & 0 & 0 & 0 & 0 & 0 \\ \gamma_A Y_i & \gamma_A Y_{2i} & 0 & -I & 0 & 0 & 0 & 0 \\ \sqrt{3} I & 0 & 0 & 0 & -I & 0 & 0 & 0 \\ \gamma^{-1} B_{1i}^T & 0 & 0 & 0 & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ C_{1i} Y_i & C_{1i} Y_{2i} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

On the other hand by the definition of B_{2i}^0, D_i^0 and I_0, the matrix M_i is transformed into the following matrix.

$$M_i = \begin{bmatrix} (B_i^h)^T & 0 & 0 & 0 & 0 & 0 & \gamma_B I & D_i^T \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, we have

$$M_i^{\perp T} = \begin{bmatrix} ((B_i^h)^{T\perp})^T & 0 & 0 & 0 & 0 & 0 & 0 & (D_i^{T\perp})^T \\ 0 & 0 & I & I & I & I & 0 & 0 \end{bmatrix}$$

Hence,the matrix inequality $M_i^{\perp T} T_{X_{Ci}} M_i^{\perp} < 0$ is transformed into the following inequality.

$$\begin{bmatrix} (B_i^h)^{T\perp} & 0 \\ D_i^{T\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_i^h Y_i + Y_i (A_i^h)^T + Y_i & * & * & * & * & * \\ C_{1i} Y_i & -I & 0 & 0 & 0 & 0 \\ (A_{di}^h)^T & 0 & (\gamma_{Ad}^2 - \eta_i)I & 0 & 0 & 0 \\ \gamma_A Y_i & 0 & 0 & -I & 0 & 0 \\ \sqrt{3} I & 0 & 0 & 0 & -I & 0 \\ \gamma^{-1} B_{1i}^T & 0 & 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} (B_i^h)^{T\perp} & 0 \\ D_i^{T\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix} < 0 \quad (31)$$

According to the above same method, the matrix inequality $Q_i^{\perp T} H_{X_{Ci}} Q_i^{\perp} < 0$ is transformed into the following inequality.

$$\begin{bmatrix} C_{2i}^{\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix}^T \begin{bmatrix} X_i A_i^h + (A_i^h)^T X_i + X_i & * & * & * & * & * \\ (A_{di}^h)^T X_i & (\gamma_{Ad}^2 - \eta_i)I & 0 & 0 & 0 & 0 \\ \gamma_A I & 0 & -I & 0 & 0 & 0 \\ \sqrt{3} X_i & 0 & 0 & -I & 0 & 0 \\ \gamma^{-1} B_{1i}^T X_i & 0 & 0 & 0 & -I & 0 \\ C_{1i} & 0 & 0 & 0 & 0 & -I \end{bmatrix} \begin{bmatrix} C_{2i}^{\perp} & 0 \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \\ 0 & I \end{bmatrix} < 0 \quad (32)$$

In summary, for disturbance attenuation performance of system (2), we have the following result.

Theorem 4.2 Given any constant $\gamma > 0$, the uncertain state delay switched linear system (2) is said to be asymptotically stabilizable with H_∞ disturbance attenuation level γ via switched dynamic output feedback, if there exist symmetrically positive definite matrices X_i and Y_i with $i \in$ such that the following linear matrix inequality, linear matrix inequalities (21) and (22) are satisfied for all $i \in$.

$$\begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix} \geq 0 \tag{33}$$

The switching strategy $\sigma(t)$ is given by (25);

In this case, the dynamic output feedback controller gain matrix

$$K_i = \begin{bmatrix} \hat{D}_i & \hat{C}_i \\ \hat{B}_i & \hat{A}_i \end{bmatrix}, \quad i = 1, 2, \ldots, N$$

can be solved by using the following algorithm.

Step 1: To solve the matrices X_i and Y_i by using the conditions (31)-(33).

Step 2: We first solve $X_{2i} \in \mathbb{R}^{n \times n_k}$ via the matrix equality $X_{2i} X_{2i}^T = X_i - Y_i^{-1}$, and then construct the matrix $X_{Ci} = \begin{bmatrix} X_i & X_{2i} \\ X_{2i}^T & I \end{bmatrix} > 0$, where $n_k = rank(X_i - Y_i^{-1})$.

Step 3: To solve K_i via $H_{X_{Ci}} + M_{X_{Ci}}^T K_i Q_i + Q_i^T K_i^T M_{X_{Ci}} < 0$.

Proof: The proof of Theorem 4.2 is obvious.

5. Numerical example

In this section, we use two examples to show the benefits effective of our results.

Example 1: Consider the state delay switched linear system (3) with $N = 2$, $\sigma(t): [0, \infty) \to \{1, 2\}$ and

$$M_1 = \begin{bmatrix} -3 & -2 \\ 1 & -1 \end{bmatrix}, M_2 = \begin{bmatrix} 2 & -7 \\ 0 & 1 \end{bmatrix}, R_1 = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} -1 & 0 \\ 4 & -1 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -18 & -5 \\ -4 & 11 \end{bmatrix}, N_2 = \begin{bmatrix} -5 & 6 \\ -15 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}, B_{12} = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 3 & -5 \\ 1 & -3 \end{bmatrix}, C_2 = \begin{bmatrix} -6 & 2 \\ -1 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} -3 & 1 \\ 1 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix}.$$

Set $\underline{\rho} = 0.1$, $\overline{\rho} = 0.5$, $\rho_1 = 0.3$, $\rho_2 = 0.2$, $h = 3$ and the disturbance attenuation level $\gamma = 0.9$. Then, by theorem 3.1, we have

$$X_1 = \begin{bmatrix} 1.8793 & 0.5732 \\ 0.5732 & 0.2932 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 1.6772 & -0.1442 \\ 1.5326 & -2.3549 \end{bmatrix};$$

$$X_2 = \begin{bmatrix} 0.4778 & -0.3619 \\ -0.3619 & 6.5929 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} -3.2924 & 7.2986 \\ -2.5587 & 1.6661 \end{bmatrix}$$

and by (14), the gain matrices of the state feedback sub-controller are given by

$$K_1 = \begin{bmatrix} 2.5817 & -5.5387 \\ 8.0861 & -23.8392 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -6.3148 & 0.7604 \\ -5.3877 & -0.0430 \end{bmatrix}$$

Example 2: Consider the state delay switched linear system (20) with $N = 2$, $\sigma(t): [0, \infty) \to \{1, 2\}$ and

$$M_1 = \begin{bmatrix} -1 & -15 \\ 0.05 & 1.9 \end{bmatrix}, M_2 = \begin{bmatrix} -1.54 & -99 \\ 0 & 1 \end{bmatrix}, R_1 = \begin{bmatrix} 0.2 & -0.5 \\ 0.1 & -0.3 \end{bmatrix}, R_2 = \begin{bmatrix} 2 & 0.5 \\ 0.5 & -0.1 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 15 & -2 \\ 1 & 7 \end{bmatrix}, N_2 = \begin{bmatrix} -2 & 6 \\ -3 & 1 \end{bmatrix}, B_{11} = \begin{bmatrix} -2 & 2 \\ 4 & -2 \end{bmatrix}, B_{12} = \begin{bmatrix} -4 & 0 \\ 2 & 0 \end{bmatrix}, C_{11} = \begin{bmatrix} -7 & 1 \\ 2 & 0.5 \end{bmatrix},$$

$$C_{12}=\begin{bmatrix}-4 & -1\\ -3 & 1\end{bmatrix},\ C_{21}=\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix},\ C_{22}=\begin{bmatrix}-1 & 0\\ 3 & 0\end{bmatrix},\ D_1=\begin{bmatrix}-2 & 2\\ 0 & 0\end{bmatrix},\ D_2=\begin{bmatrix}-3 & -2\\ 0 & 0\end{bmatrix}.$$

Set $\underline{\rho}=0.1$, $\bar{\rho}=0.5$, $\rho_1=0.3$, $\rho_2=0.2$, $\eta_1=0.01$, $\eta_2=0.05$, $h=3$, and the disturbance attenuation level $\gamma=0.9$.

Then by theorem 4.2, we have

$$X_1=\begin{bmatrix}11.8487 & 1.4274\\ 1.4274 & 0.7331\end{bmatrix},\quad Y_1=\begin{bmatrix}4.2763 & -1.4503\\ -1.4503 & 12.2084\end{bmatrix},$$

$$X_2=\begin{bmatrix}3.5762 & 0.1934\\ 0.1934 & 0.7214\end{bmatrix},\quad Y_2=\begin{bmatrix}3.5608 & -0.0198\\ -0.0198 & 3.8525\end{bmatrix}.$$

Again by Theorem 4.2, system (20) with $N=2$ satisfies robust H_∞ performance with the disturbance attenuation level $\gamma=0.9$ via dynamic output feedback controllers and a switching strategy, where the dynamic output feedback controllers are given by

$$\Gamma_1:\begin{cases}\dot{x}_C=\begin{bmatrix}-56.5085 & 260.8326\\ 146.3184 & -204.0850\end{bmatrix}x_C+\begin{bmatrix}78.5307 & 78.5307\\ 178.9588 & 178.9588\end{bmatrix}y\\ u=\begin{bmatrix}156.9772 & -63.6624\\ 158.7781 & -64.0382\end{bmatrix}x_C+\begin{bmatrix}283.1606 & 283.1606\\ 288.1804 & 288.1804\end{bmatrix}y\end{cases}$$

$$\Gamma_2:\begin{cases}\dot{x}_C=\begin{bmatrix}-961.7952 & 387.2148\\ 302.5135 & -80.7868\end{bmatrix}x_C+\begin{bmatrix}81.9678 & -245.9033\\ -43.3079 & 129.9237\end{bmatrix}y\\ u=\begin{bmatrix}-300.7303 & 31.1283\\ 449.5533 & -45.8000\end{bmatrix}x_C+\begin{bmatrix}59.8047 & -179.4142\\ -89.1940 & 267.5819\end{bmatrix}y\end{cases}$$

It is evident that Example 1 and example 2 are neither of the designed controllers makes the associated subsystem asymptotically stable.

6. Conclusions

The robust H_∞ control problem has been studied via switched state feedback and dynamic output feedback for state delay switched linear systems with exponential uncertainties by using Taylor series approximation, convex polytope technique and LMI method. Sufficient conditions are, by solving linear matrix inequalities, presented to realize the H_∞ control design. How to design a switched controller and associated state feedback sub-controllers to improve the performance of uncertain discrete-time switched linear systems with state delay should be further studied in the future work.

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