

# The Attitude Control Based on Active Disturbance Rejection Control for the Small-Scale Unmanned Helicopter

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## Abstract

*This paper presents a design of three channel controller based on active disturbance rejection control which has a fast response, high control precision, good robustness, strongly adaptability. This design approach does not need accurate mathematics model of the small unmanned helicopter. Input-output feedback linearization is intended primarily for the system output and system control input to establish a direct relation. The final aim is to develop a robust tracking control scheme which ensures that the outputs of closed-loop system track the given expectations. There results of simulations show the satisfactory tracking performance and the great ability to suppress disturbances.*

**Keywords:** *Small-scale unmanned helicopter, active disturbance rejection control, attitude control*

## 1. Introduction

In recent years, the small-scale unmanned helicopter with its small size, low noise, good flexibility, high concealment, *etc.*, began to be widely paid attention and its application scope extends to military, civil and scientific research [1]. Many experts and scholars have also begun to research and develop the small-scale unmanned helicopter [2-4], but its system is very complex, especially its control system, with multivariable, strong coupling, time-varying and non-linear characteristics, make it unusually difficult to control [5].

To solve this problem, this paper puts forward an idea that applies Active Disturbance Rejection Controller (ADRC) [6] to the small unmanned helicopter control system. ADRC has a fast response, high control precision, good robustness, strongly adaptability and does not need accurate mathematics model. The key issues of ADRC, whether it is a system of linear or nonlinear, is not important. And the most essential structural features of ADRC is its basic conditions: observable controllable. This caused a good foundation for the development of ADRC[7-8].

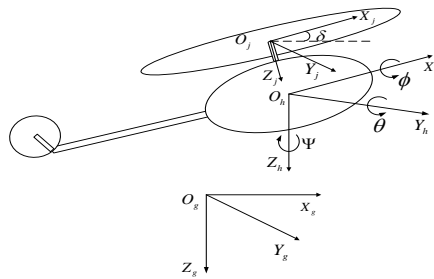
First, this paper gives a brief introduction about small-scale unmanned helicopter's structures and characteristics. Then we analyze in detail the unmanned helicopter platform, each components' forces and moments, their mathematical expression. Secondly based on the research of ADRC algorithm and its characteristics of small unmanned helicopter, we design the three-channel decoupling ADRC, and set the parameters of the controller. Finally, we use simulation tool to build a small unmanned helicopter close-loop control system simulation model. Simulation tests include unmanned helicopter attitude control test. The test results show that the active disturbance rejection controller applied to the control system of small unmanned helicopter is entirely feasible. It can be

accurate and efficient control the flight status of small unmanned helicopter, and gives an accurate estimation and compensation about the internal and external disturbances of the system, so as to achieve the ideal control effect.

## 2. Helicopter Nonlinear Model

### 2.1. Reference Frame

This section describes the nonlinear dynamic model of the small-scale unmanned helicopter. There are two reference frames: body-fixed reference frame, ground coordinates, as follows:



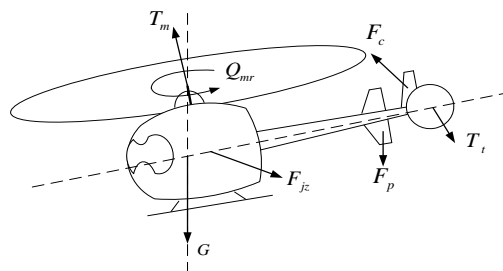
**Figure 1. System of Coordinates Defined**

$O_g X_g Y_g Z_g$  is ground coordinates. helicopter speeds, attitudes and the positions information of the system of coordinates can be determined. Where axis  $X_g$  direction can be determined according to the flight direction in the horizontal plane. Axis  $Y_g$  is determine in accordance with the right hand rule to determine the direction, Axis direction  $Z_g$  is in the same direction of the acceleration of gravity.

$O_h X_h Y_h Z_h$  is body-fixed reference coordinates, which determine the attitude of a small unmanned aerial coordinates.  $O_h$  is the center of gravity of the helicopter.  $X_h$  is Point forward along the longitudinal axis of fuselage aircraft.  $Y_h$  is perpendicular to the longitudinal plane of symmetry UAV. The direction of  $Y_h$  is determined in accordance with the right-hand rule.  $Z_h$  is downward and perpendicular to the  $O_h X_h Y_h$ .

### 2.2. The Force Analysis of Small Unmanned Helicopter

We can look into a small unmanned helicopter consists of several components. Body force and torque of Small unmanned helicopter are complicated. Thus, the forces analysis on individual parts will be done separately. Then, the analysis results are synthesized. Furthermore, the overall analysis is completed. Figure 2 shows the helicopter several major forces and torque.



**Figure 2. Helicopter Force and Torque**

$T_m$  is the pull generated by the main rotor.  $Q_{mr}$  is Anti-torque generated by the main rotor blades.  $F_{jz}$  is Small unmanned helicopter airframe's resistance.  $G$  is the gravity.  $F_c$  is the force generated by the tail.  $F_p$  is the force generated by empennage.  $T_r$  is the pull generated by the tail rotor.

### 2.3. 6-DOF Rigid Body Dynamics Equations of the Small-scale Unmanned Helicopter

We just take it for granted that is a 6-DOF rigid model. So its rigid body dynamics equations can be get from following Newton-Euler equations

$$\begin{aligned} I_{3 \times 3} \dot{v} + \omega \times m v &= F \\ I_m \dot{\omega} + \omega \times I_m \omega &= \tau \end{aligned} \quad (1)$$

where

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}, \quad \tau = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}, \quad v = \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \quad I_m = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

is the inertia matrix. The reference frame is longitudinally symmetrical in the body-fixed reference frame, so that  $I_{xy} = I_{yz} = 0$ .  $I_{xz}$  can generally be negligible because  $I_{xz}$  is smaller than the other. The above equation can be expanded into helicopter rigid body dynamics equations as follows:

$$u = vr - wq - g \sin \theta + F_x / m \quad (2)$$

$$v = wp - ur + g \sin \phi \cos \theta + F_y / m \quad (3)$$

$$w = uq - vp + g \cos \phi \cos \theta + F_z / m \quad (4)$$

$$p = qr(I_{yy} - I_{zz}) / I_{xx} + M_x / I_{xx} \quad (5)$$

$$q = pr(I_{zz} - I_{xx}) / I_{yy} + M_y / I_{yy} \quad (6)$$

$$r = pq(I_{xx} - I_{yy}) / I_{zz} + M_z / I_{zz} \quad (7)$$

where,  $u, v, w$  are three-component velocities in body-fixed reference frame.  $p, q, r$  are the three-components of the angular velocities.  $\phi, \theta, \psi$  are attitude angles. The relation between attitude angles and angular velocities is described by:

$$\dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \quad (8)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (9)$$

$$\dot{\psi} = q \sin \phi / \cos \theta + r \cos \phi / \cos \theta \quad (10)$$

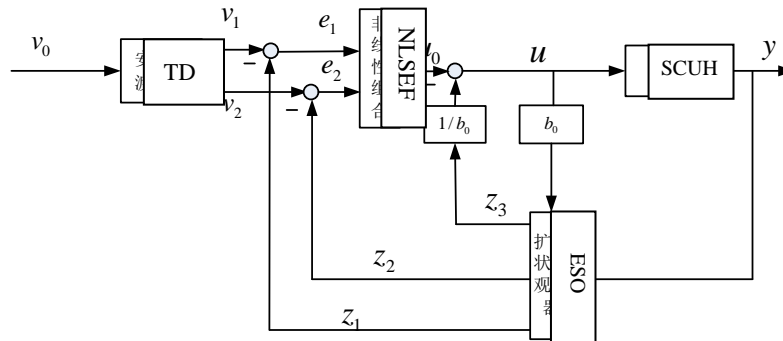
We can see from the above equations a coupling between these attitude angles.

## 3. The Controller Design Outline

### 3.1. The Principle of ADRC

In order to get the states of uncertain, also to estimate the system within and outside interference, and compensation, ADRC is used in the control system design. It is a nonlinear controller, the principle diagram of ADRC is shown in Figure 3. ADRC consists of three components: the extended state observer and the tracking differentiator and the error nonlinear feedback control law. It can detect the effect of the model and the external disturbances or uncertain and then automatically compensate them in real time.

The controller parameters of extended state observer need to be adjusted. Second-order tracking-differentiator (TD) is similar to those of second linear low-pass filter. TD has smaller phase-shift in passing band, and resonant phenomenon does not occur. TD can provide a smooth input signal for controller. The output signal of TD is continuous, and not produce overshoot, such that the stability of the entire system is enhanced. The relationships of frequency characteristics with tracking parameter and amplitude of sinusoidal input are just the properties of parallel-shifting.



**Figure 3. The ADRC Controller Schematics**

(1) The discretization equations of TD are described by

$$\begin{cases} v_1(k+1) = v_1(k) + hv_2(k) \\ v_2(k+1) = v_2(k) + hfhan(v_1(k) - v_0(k), v_2(k), r, h) \end{cases} \quad (11)$$

where the input signal  $v_0(k)$  is set to be the expected value, the transient  $v_1(k)$ . And it's differential  $v_2(k)$ .  $h$  is integral step.  $r$  is the parameter of the track pace deciding to the track velocity. Optimal function  $fhan$  is given by:

$$\begin{cases} d = rh \\ d_0 = hd \\ y = (v_1 - v_0) + hv_2 \\ a_0 = \sqrt{d^2 + 8r|y|} \\ a = \begin{cases} v_2 + \frac{a_0 - d}{2} \text{sign}(y) & |y| > d_0 \\ v_2 + \frac{y}{h} & |y| \leq d_0 \end{cases} \\ fhan = - \begin{cases} r \text{sign}(a) & |a| > d \\ r \frac{a}{d} & |a| \leq d \end{cases} \end{cases} \quad (12)$$

(2) Extended state observer

Extended state observer (ESO) is described by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2) + w(t) + bu \\ y = x_1 \end{cases} \quad (13)$$

Let  $x_3(t) = f(x_1, x_2) + w(t)$ , the disturbances  $w(t)$  and acceleration  $f(x_1, x_2)$  are expanded into a new state variable  $x_3$ , meanwhile let  $x_3(t) = a(t)$ , (13) is change into the forms as follows:

$$\begin{cases} \square \\ x_1 = x_2 \\ \square \\ x_2 = x_3 + bu \\ \square \\ x_3 = a(t) \\ y = x_1 \end{cases} \quad (14)$$

If  $f(x_1, x_2) + w(t)$  is bounded, We can construct a extended state observer as follows:

$$\begin{cases} \square \\ e = z_1 - y \\ \square \\ z_1 = z_2 - \beta_{01}e \\ \square \\ z_2 = z_3 - \beta_{02} |e|^{0.5} \text{sign}(e) + bu \\ \square \\ z_3 = -\beta_{03} |e|^{0.25} \text{sign}(e) \end{cases} \quad (15)$$

where  $\beta_{01}$ ,  $\beta_{02}$ ,  $\beta_{03}$  are the parameters of extended state observer.  $b$  is adjustable parameter. If the right parameter  $\beta_{01}$ ,  $\beta_{02}$ ,  $\beta_{03}$  are selected. The states  $x_1(t)$ ,  $x_2(t)$  can be properly estimated by extended state observer. Then the state  $x_3(t)$  can be estimated as well. Namely,

$$z_1(t) \rightarrow x_1(t), z_2(t) \rightarrow x_2(t), z_3(t) \rightarrow x_3(t)$$

Then the controller is given by

$$u = \frac{u_0 - z_3(t)}{b} \quad (16)$$

Substituting it into the original second-order system, The original nonlinear control system becomes an integral tandem linear system:

$$\begin{cases} \square \\ x_1 = x_2 \\ \square \\ x_2 = f(x_1, x_2) + w(t) + bu \\ y = x_1 \end{cases} \Rightarrow \begin{cases} \square \\ x_1 = x_2 \\ \square \\ x_2 = u_0 \\ y = x_1 \end{cases} \quad (17)$$

We can use a unified approach to deal with nonlinear, time-varying and uncertain control problems.

### (3) Nonlinear state error feedback control law (NLESF)

The traditional theory of PID control is weighted together and form of organization. Howere, ADRC uses a nonlinear state error feedback. The tracking differentiator outputs  $v_1$ ,  $v_2$  respectively compare with outputs of ECO  $z_1$ ,  $z_2$ , then the errors  $e_1$ ,  $e_2$  are obtained. Next, the form of non-linear combinations are varied as follows

$$\begin{aligned} u_0 &= \beta_1 e_1 + \beta_2 e_2 \\ u_0 &= \beta_1 \text{fal}(e_1, \alpha_1, \delta) + \beta_2 \text{fal}(e_2, \alpha_2, \delta) \quad 0 < \alpha_1 < 1 < \alpha_2 \\ u_0 &= -\text{fhan}(e_1, e_2, r, h_1) \end{aligned} \quad (18)$$

Every nonlinear combinations have their own characteristics. Where  $r, h_1$  are adjustable parameters.  $\text{fhan}$  is the same form as (12). These non-linear control combinations have a good combination of robustness and adaptability, which can be a good disturbance rejection and enhance dynamic characteristics of closed-loop response.

### 3.2. Input-output Feedback Linearization

Input-output feedback linearization is intended primarily for the system output and system control input to establish a direct relation, and remove the coupling portion of the system. Under the different experimental conditions, the linearized results are different, but the process is the same, here for small unmanned helicopter attitude control as an example. Choose three attitude angle of unmanned helicopter as system outputs: roll angle  $\phi$ , yaw  $\theta$ , pitch angle  $\psi$ .

$$y = [y_1 \quad y_2 \quad y_3]^T = [\phi \quad \theta \quad \psi]^T$$

*Problem statement:* to develop a robust tracking control approach which ensures that the outputs of closed-loop system track a given expectations  $(\phi_d, \theta_d, \psi_d)$ .

According to (8),(9),(10), here we perform input-output feedback linearization[9]. The small unmanned helicopter flight dynamics nonlinear mathematical model is described by

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) \\ g_{21}(x) & g_{22}(x) & g_{23}(x) \\ g_{31}(x) & g_{32}(x) & g_{33}(x) \end{bmatrix} u \quad (19)$$

$$= f(x) + G(x)u$$

where the controller and the states of the dynamics system are shown as follows:

$$u = [\theta_{t,o} \quad \theta_z \quad \theta_h]^T, \quad x = [x_g \quad y_g \quad z_g \quad u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \theta \quad \psi]^T$$

### 3.3. The Design of Three Channel ADRC

For the ADRC, its control objects are generally SISO systems. Here the control object is a MIMO system, so we want to use ADRC algorithm to design a controller, the equation (19) is treated as follows

$$U = [U_1 \quad U_2 \quad U_3]^T = G(x)u$$

Then (19) is rewritten as

$$\phi = f_1(x) + U_1 \quad (20)$$

$$\theta = f_2(x) + U_2 \quad (21)$$

$$\psi = f_3(x) + U_3 \quad (22)$$

Where  $U_i$  ( $i = 1, 2, 3$ ) are the inputs, and  $y_i$  ( $\phi, \theta, \psi$ ) are new output. We can respectively design the ADRC controller for the second-order system (20).  $f_i(x)$  ( $i = 1, 2, 3$ ) are the unknown parts including three-channel coupling, the unmodeled dynamics, the external disturbances or uncertain. In addition, we make the following assumption that the small unmanned helicopter during the flight of the various state variables are bounded. Meanwhile  $f_i(x)$  is bounded as well. ADRC is able to estimate the uncertainties of system

From the above, we have separately ADRC tracking differentiator, the extended state observer and the nonlinear state error feedback control law for a detailed description. Here we will directly give the designed ADRC in the following discrete forms:

$$\begin{cases} v_{i1}(k+1) = v_{i1}(k) + hv_{i2}(k) \\ v_{i2}(k+1) = v_{i2}(k) + hfhan((v_{i1}(k) - v_{id}(k), v_{i2}(k), r_{i0}, h) \\ e_i(k) = z_{i1}(k) - y_i(k) \\ z_{i1}(k+1) = z_{i1}(k) + h(z_{i2}(k) - \beta_{i1}e_i(k)) \\ z_{i2}(k+1) = z_{i2}(k) + h(z_{i3}(k) - \beta_{i2}fal(e_i(k), 0.5, \delta_i) + U_i(k)) \\ z_{i3}(k+1) = z_{i3}(k) - h\beta_{i3}fal(e_i(k), 0.25, \delta_i) \\ e_{i1}(k) = z_{i1}(k) - v_{i1}(k), e_{i2}(k) = z_{i2}(k) - v_{i2}(k) \\ U_{i0}(k) = -fhan(e_{i1}(k), e_{i2}(k), r_{i1}, h_{i1}) \\ U_i(k) = U_{i0}(k) - z_{i3}(k) \end{cases} \quad (23)$$

where,  $v_{id}$  ( $i=1, 2, 3$ ) respectively denote  $\phi_d, \theta_d, \psi_d$ . The following controller is ultimately applied to the control object

$$u = G^{-1}(x)U \quad (24)$$

## 4. Simulation Results

In this section, the purpose of numerical simulation is to demonstrate the performances of the proposed control system.

### 4.1. Adjustment of the Parameters of ADRC

There are many parameters in ADRC, including the parameters  $r_{i0}, h$  in tracking differentiator;  $\beta_{i1}, \beta_{i2}$ , the parameters  $\beta_{i3}, \delta_i$  in extended state observer, the parameters  $r_{i1}, h_{i1}$  in nonlinear state error feedback control law. They are very difficult to be adjusted simultaneously. Since we can follow the principle of separation, every parts can be designed individually.

#### (1) Adjustment of the parameters of TD

Based on the experience,  $r_{i0}$  is a certain relationship with the integration step, then  $r_{i0}$  is given by  $r_{i0} = \frac{0.0001}{h^2}$ . Where  $h$  is integration step.

#### (2) Adjustment of the parameters of ESO

Expanded state ADRC is the core unit. Only when it is valid to estimate the uncertainty of the system state variables, variable differential and systems, The controller can give proper feedback and Compensation

Changes  $\beta_{i1}$  in a range not causing excessive impacts to the system, But too much can also cause the system the divergent shock, too small to affect the tracking results.  $\beta_{i2}$  is selected to make the system self-generated high-frequency interference signals, and too small to make the system oscillation.  $\beta_{i3}$  influences the tracking speed of the system. The main parameters  $\beta_{i1}, \beta_{i2}, \beta_{i3}$  are taken as the following forms:

$$\beta_{i1} = \frac{1}{h}, \beta_{i2} = \frac{1}{3h^2}, \beta_{i3} = \frac{1}{32h^3}$$

$fal$  is nonlinear saturation function with a certain linear range,  $\delta_i$  directly impacts on the size of the linear range. The larger  $\delta_i$  will cause that the entire range of functions have become linear, and losing the advantages of nonlinear. Through several attempts, let  $\delta_i = 0.0001$ . Thus, ESO will be an ideal output.

(3) Adjustment of the parameters of NLSEF

Two parameters of nonlinear feedback part are the time factor  $r_{i1}$  and the integration step  $h_{i1}$ . Their selection principle is similar with tracking differentiator. Here they are given directly in the forms:

$$r_{i1} = \frac{0.5}{h^2}, \quad h_{i1} = 5h$$

The simulation includes the small-scale unmanned helicopter attitude control test based on the initial state of a certain height, the rated speed of the rotor to hover. Unless controller achieves effective attitude control, we can make the small-scale unmanned helicopter more difficult action. The reference signals apply to each channel separately. Then the results of the attitudes tracking are given in Figure 4 and Figure 5. we can see the effects of its track are still desirable attitude angles regulating along with the reference signal.

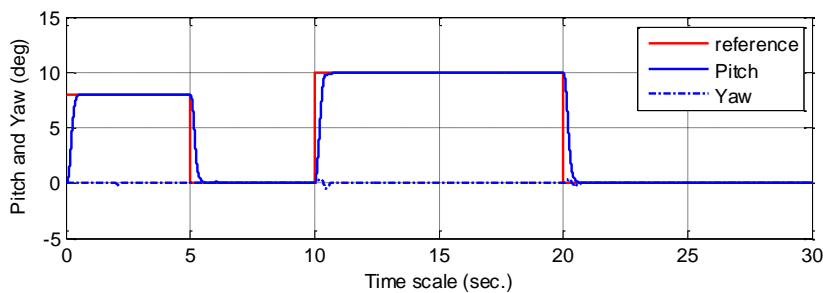


Figure 4. Results of the Pitch Angle and Yaw Angle Tracking

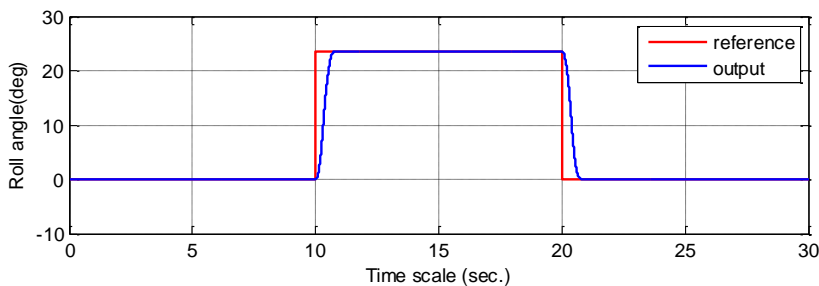


Figure 5. Results of Roll Angles Tracking

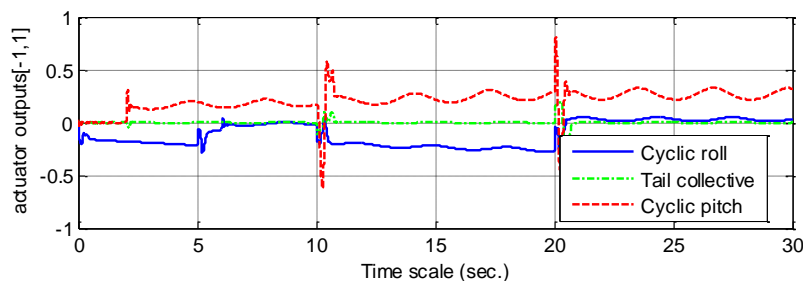
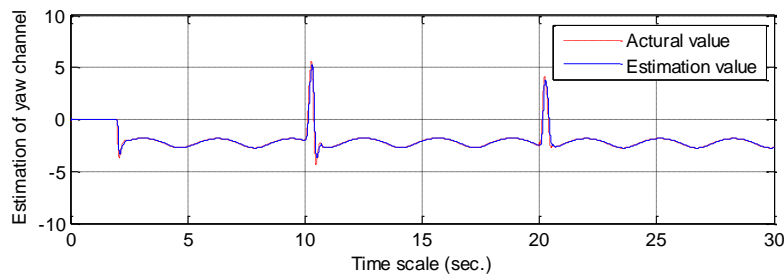


Figure 6. Controller Output Signals to the Actuators

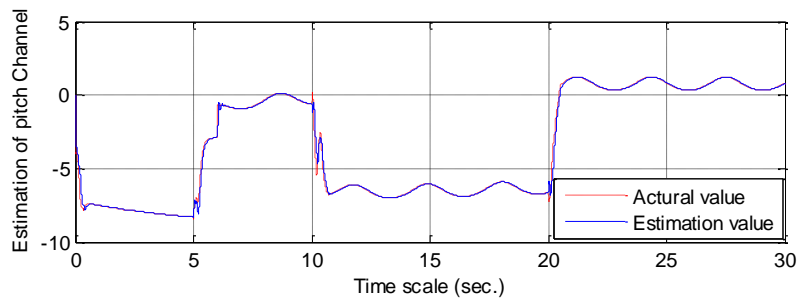
Figure.6 shows that servo-actuators signal regulate along with the attitude angles. Figure 6, Figure 7 and Figure 8 are show the errors between the actual value of three channel and the estimation value by the extended state observer. Although the peak of the differential outputs and tracking slight deviation, but the sizes of the deviation are within



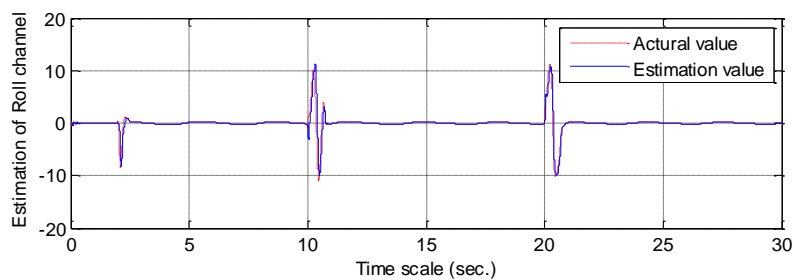
the allowable range, and almost no effect on the controller. These cause that the nonlinear state error feedback laid a good foundation. This point shows in Figure 6.



**Figure 7. The Estimation of Yaw Channel**



**Figure 8. The Estimation of Pitch Channel**



**Figure 9. The Estimation of Roll Channel**

The test results show that the active disturbance rejection controller applied to the control system of small unmanned helicopter is entirely feasible. It can be accurate and efficient control the flight status of small unmanned helicopter, and give an accurate estimation and compensation about the internal and external disturbances of the system, so as to achieve the ideal control effects.

## 5. Conclusion

Combined with the characteristics of small unmanned helicopter, we completed the design of ADRC controller for small unmanned helicopter. Through the ADRC algorithms, the work of anti-rejection controller tuning comes true by the three-channel decoupling ADRC controller. The three-channel controller based on active disturbance rejection control has a fast response, high control precision, good robustness, strongly adaptability and does not need accurate mathematics model to the small unmanned helicopter control system. The test results show that the active disturbance rejection controller applied to the control system of small-scale unmanned helicopter is entirely feasible.

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## References

- [1] K. Dalamagkidis, K. P. Valavans and L. A. Piegal, "Nonlinear Model Predictive Control with Neural Networks Optimization for Autonomous Autorotation of Small Unmanned Helicopters", IEEE Transactions on Control Systems Technology, vol. 19, no. 4, (2011).
- [2] A. Karimoddini, W. C. Guo, B. M. Chen, H. Lin and T. H. Lee, "Multi-Layer Flight Control Synthesis and Analysis of a Small-Scale UAV Helicopter", IEEE Conference on Robotics, Automation and Mechatronics, (2010); Singapore.
- [3] B. Zheng and Y. S. Zhong, "Robust Attitude Regulation of a 3-DOF Helicopter Benchmark: Theory and Experiments", IEEE Transactions on Industrial Electronics, vol. 58, no. 2, (2011).
- [4] I. A. Raptis, K. P. Valavanis and G. J. Vachtsevanos, "Linear Tracking Control for Small-Scale Unmanned Helicopters", IEEE Transactions on Control Systems Technology, vol. 20, no. 4, (2012).
- [5] X. Yang, M. Garratt and H. Pota, "Non-Linear Position Control for Hover and Automatic Landing of Unmanned Aerial Vehicles", IET Control Theory Apply, vol. 6, no. 7, (2012).
- [6] Y. Q. Xia, H. P. Huang and J. Q. Han, "ADRC Control of Uncertain Systems with Time-Delay", Center South University of Technology, vol. 34, no. 4, (2003).
- [7] J. Q. Han, "A new type of controller: NLPID", Control and Decision, vol. 9, no. 6, (1994).
- [8] Y. Huang, K. K. Xu and J. Q. Han, "Flight Control Design Using Extended State Observer and Non-Smooth Feedback", Proceedings of IEEE Conference on Control and Decision, (2001); Orlando, FL.

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