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Characteristics of Exponential Distribution with Respect to Preventive Maintenance

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Abstract

Statistical distributions are useful in predicting patterns of failure and hence, are employed in failure and reliability analysis. Reliability analysis is relatively easier when time variance is linear. However, for a non-linear progression of time such as the case in preventive maintenance, the reliability analysis becomes complex. This paper primarily investigates characteristics of exponential distribution taking into account periodic preventive maintenance. As a secondary consideration, the paper also investigates characteristics of Weibull distribution to provide a platform for comparison.

Keywords: *Reliability, exponential distribution, Weibull distribution, preventive maintenance*

1. Introduction

Components of engineering systems have useful life (also referred to as risk time), within the useful life, the component's performance is acceptable and usually depreciates as the age of the component approaches end of its useful life. In reliability analysis, without the effect of maintenance, the analysis is performed on a linear progression of time; that is, the component age at time t. For instance, if a given component was new at time t = 0 and its useful life is 100 time units, the reliability analysis from t = 0..t = 100 will be an evaluation at time 0, 1, 2, 3, 4, .. , 96, 97, 98, 99, 100. In other words, the evaluation is solely performed using the calendar age of the component.

The exponential distribution is widely used in the application area of component lifetime (MIL-HDBK-338B, 1998). It is used to model reliability distribution where the failure rate is considered or assumed to be constant throughout the time scale under consideration. It is also easy to implement and thus, widely used.According to Pham [3], the exponential distribution has been used to model the lifetime ofelectronic and electrical components, and systems, and it is appropriatewhen a used component that has not failed is as good as a new component - which is arather restrictive assumption. This restrictive assumption is as a result of the assumption of a constant failure rate. The exponential distribution has a memoryless property. Supposing a waiter in a restaurant waits x time for the first customer to arrive and to be served, the memoryless property implies that the waiter will also wait x time before the arrival of the next customer. Therefore, the exponential distribution must be used diplomatically sincenumerous applications exist where the restriction of the memoryless property maynot apply [3].

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Maintenance is known to improve the reliability and performance of a component, and also slows down component aging [1]. This means that there is a form of rejuvenation that occurs as a result of maintenance actions. The implication here is that when a component is maintained at time$t$, there is a rejuvenation oft_r time units, meaning that the age of the component assumes $t - t_r$ time units. In failure or reliability analysis, the next or subsequent time unit that will be used in the analysis will not be $t + 1$ but $(t - t_r) + 1$. This concept of age modeling is further described in section 2. The sufficiency of the exponential distribution in modeling a preventive maintenance problem is exactly the focus of this paper. The proportional age reduction (PAR) model which is widely used in modeling maintenance problems is considered. To achieve this, the Weibull model which is alsowidely used for lifetime modeling [4] is considered for comparison. Thus, the remainder of this paper is organised as follows. Section 2 presents discussion on preventive maintenance modeling and establishes the exponential and Weibull model for preventive maintenance. Section 3 discusses the evaluation, and results are presented in Section 4. Conclusions are drawn in Section 5.

2. Preventive Maintenance

Preventive maintenance is the act of performing certain activities on the component of a system with the aim to improve its reliability, availability, safety and performance. Maintenance also improves safety in the sense that when a system such as power plant, car brake system, etcis maintained, it reduces the likelihood of failure which could result into loss of life, damage to environment or economic loss. In terms of performance, a good example is factory equipment; when the equipment is maintained, productivity could be restored as a result of restored performance of the equipment. Several activities of maintenance exist, some of which are oiling, tightening, cleaning, topping, minimal repair, *etc*. When preventive maintenance actions improve a component to as good as new, this is termed as perfect preventive maintenance (PPM). However, when the component's condition is improved to a certain degree, it is referred to as imperfect preventive maintenance. The characteristics of the exponential distribution are investigated with respect to these two types of preventive maintenance, and are each discussed next.

2.1. Perfect Preventive Maintenance

Firstly, to establish the model of PPM, the general concept of maintenance using the PAR model is presented in Figure 1.

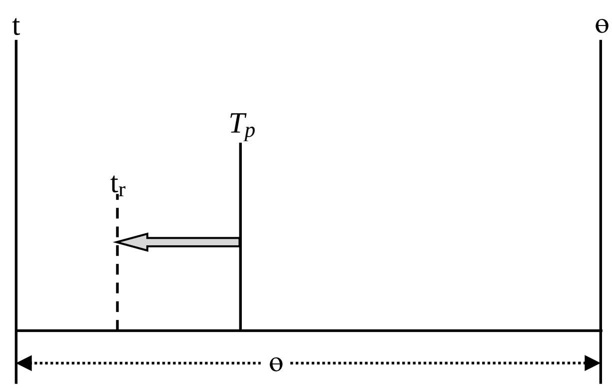

Figure 1. Concept of Maintenance Using PAR Model

Under periodic maintenance, a component is maintained at an interval call preventive maintenance time or simply referred to as PM time T_p. In Figure 1, the scale of time considered is denoted by ɵ which could equally be the useful life of the component. The PAR model assumes that each maintenance activity reduces proportionally the age gained from previous maintenance [6]. Therefore, it implies that each PM activity is assumed to only reduce a portion of the component age. The scale of age reduction is dependent on the effectiveness of the maintenance, this is usually referred to as improvement factor *f*, where $0 \leq f \leq 1$. In Figure 1, the component was new at $t = t_0$, when the component is maintained atT_p, the age is reduced to t_r. This implies that the reliability after the maintenance action at T_p will be evaluated at t_r. The next PM time for the component will be at $t_r + T_p$ and the reliability evaluation will be performed at $2t_r$. In general, the mathematical representation of Figure 1isexpressed as in equation 1 below, also similar to an equation found in [6].

$$W^{+} = (1 - f)T_p \tag{1}$$

Where: $f$is the improvement factor
$T_p$is the PM time
$W^{+}$is the new age after maintenance or effective age

Under PPM, the component's condition is as good as new implying that improvement factor $f = 1$. Hence, the exponential modeling of PPM is discussed next.

2.1.1. Exponential Modeling of PPM: According to Tsai *et al.*, [7], the reliability model of a system at the *j-th* PM stage is modeled as shown in equation 2.

$$R_j(t) = R_{0j}R_{vj} \tag{2}$$

Where R_{oj} is the probability of surviving until the *j-th* PM stage, and R_{vj} is the probability of surviving the remaining time. A component which was new at $t_0 = 0$ and with PM intervals t_1, t_2, .., t_{n-1}, t_n are statistically independent [8]. Hence, successive reliabilities at PM stages are independent. Under PPM, each maintenance stage returns the component to as good as new; implying that the effective age of the component after maintenance at each stage is 0. Hence, the reliability at the next PM stage is evaluated between 0 andT_p. This means that if there are *n* PM stages, there will be *n* number of cumulative reliabilities, each evaluated between 0 and T_p. In mathematical form, it implies that the probability of surviving the *n-th* PM stage can be represented as in equation 3.

$$R_n(t) = R_1(T_p).R_2(T_p).R_3(T_p)\ldots R_{n-1}(T_p).R_n(T_p) \tag{3}$$

Equation 3 can be reduced to equation 4 as shown below.

$$R_n(t) = R(T_p)^n \tag{4}$$

Assuming the calendar time is$t$, the remaining time after the *n-th* PM stage is $t - nT_p$. Therefore, the probability of surviving the remaining time R_{rem} is modeled as shown in equation 5.

$$R_{rem}(t) = R(t - nT_p) \quad (5)$$

According to equation 2, the reliability under perfect preventive maintenance $R_{ppm}(t)$ ismodeled as shown in equation 6 below.

$$R_{ppm}(t) = R_n(t).R_{rem}(t) = R(T_p)^n.R(t - nT_p) \quad (6)$$

The reliability of a component using exponential distribution without the effect of PM is given by equation 7[8].

$$R(t) = e^{-\lambda t} \quad (7)$$

Substituting equation 6 in 7 yields the exponential reliability under PPM as follows.

$$R_{ppm}(t) = e^{(-\lambda T_p)^n} e^{-\lambda(t - nT_p)} \quad (8)$$

Equation 8 simplifies to:

$$R_{ppm}(t) = e^{-n\lambda T_p} e^{-(\lambda t - n\lambda T_p)}$$

$$R_{ppm}(t) = e^{-n\lambda T_p} e^{(-\lambda t + n\lambda T_p)}$$

$$R_{ppm}(t) = e^{-n\lambda T_p} e^{-\lambda t} e^{n\lambda T_p}$$; applying the laws of indices

$$R_{ppm}(t) = e^{-n\lambda T_p} e^{n\lambda T_p} e^{-\lambda t}$$

Further application of the laws of indices gives:

$$R_{ppm}(t) = e^{-\lambda n T_p + \lambda n T_p} e^{-\lambda t}$$

$$R_{ppm}(t) = e^0 e^{-\lambda t}$$

$$R_{ppm}(t) = 1 . e^{-\lambda t}$$

$$R_{ppm}(t) = e^{-\lambda t} \quad (9)$$

Equation 9 is not different from 7, which is the reliability R(t) of a component using exponential distribution under no PM policy. This case reveals that the exponential distribution is not sufficient for modelling a real life problem under PM using the proportional age reduction model. A similar case is also found in Ebeling(pp204) [9]. Therefore, it will be interesting to investigate how the exponential distribution will model imperfect preventive maintenance; discussed in the next section.

2.2. Imperfect Preventive Maintenance

Unlike PPM, IPM does not return component condition to as good as new, and this makes IPM modeling more complex.

2.2.1. Exponential Modeling of IPM: For an IPM with constant improvement factor f and constant PM time T_p in-between stages, the effect of rejuvenation in equation 1 will be same at all PM stages. Therefore let t_r represent equation 1 as follows.

$$t_r = (1 - f)T_p \tag{10}$$

The reliability of a component from *j = 1* to *j = n*, where *j* is the *j-th* PM stage and *n* is the maximum number of PM stage can be computed as follows.

$$R_{pm} = R(T_p)R(t_r, t_r + T_p)R(2t_r, 2t_r + T_p) \dots R\big((n-2)t_r, (n-2)t_r + T_p\big)R\big((n-1)t_r, (n-1)t_r + T_p\big)R\left(nt_r, nt_r + (t - nT_p)\right) \tag{11}$$

Where $R\big(nt_r, nt_r + (t - nT_p)\big)$ is the probability of surviving the remaining time.

The universal reliability model under preventive maintenance using equation 11 was established by Nggada [11] with which models of reliability under any given PM policy could be modeled using the proportional age reduction model. The universal model is as shown in equation 12.

$$R_u(t) = \left(\prod_{j=1}^{n}\left(1 - R\big((j-1)t_r\big) + R\left((j-1)t_r + T_p\right)\right)\right)\big(1 - R(nt_r) + R(nt_r + (t - nT_p))\big) \tag{12}$$

Using the exponential model in equation 7 to model exponential reliability under IPM R_{ipm} yields equation 13 below.

$$R_{ipm}(t) = \left(\prod_{j=1}^{n}\left(1 - e^{-\lambda((j-1)t_r)} + e^{-\lambda\left((j-1)t_r + T_p\right)}\right)\right)\left(1 - e^{-\lambda(nt_r)} + e^{-\lambda\left(nt_r + (t - nT_p)\right)}\right) \tag{13}$$

3. Evaluation

To further investigate the characteristics of the exponential distribution, equations 9 and 13 are evaluated on a component assuming the following parameters.

Failure rate λ = 5.05E-04

Time scale t = 3000

PM time T_p = 180

Improvement factor f_1 = 0.801

Improvement factor f_2 = 0.501

The rationale of evaluating the component reliability using two different improvement factors is to be able to examine and to suggest the behaviour of the exponential distribution with respect to preventive maintenance.

4. Results

The result of the evaluation of the component reliability under IPM using exponential distribution is shown in Table 1. It illustrates two major result categories; component reliability (i) under PPM and (ii) under IPM policy. The latter contains two sub-categories; (a) component reliability with improvement factor $f = f_1 = 0.801$ and (b) $f = f_2 = 0.501$.Due to space limitation, it is impossible to show the component reliability for time step of 1 unit. Therefore, a time step of 60 units is considered; implying that the time sequence is 0, 60, 120, 180…, 2820, 2880, 2940, 3000.

Table 1. Component Exponential Reliability Under PPM and IPM

Time (t)	*Reliability R(t)*		
	Under No PM = PPM	Under IPM	
		f = 0.801	*f = 0.501*
0	1	1	1
60	0.970154	0.970154	0.970154
120	0.9412	0.9412	0.9412
180	0.913109	0.913109	0.913109
240	0.885857	0.886345	0.887065
300	0.859418	0.86038	0.861799
360	0.833768	0.83519	0.837287
420	0.808884	0.811149	0.814464
480	0.784742	0.787826	0.792323
540	0.761321	0.765198	0.770843
600	0.738599	0.743567	0.750764
660	0.716555	0.722581	0.731284
720	0.695169	0.702222	0.712385
780	0.674421	0.682726	0.694652
840	0.654293	0.663813	0.677447
900	0.634765	0.645464	0.660757
960	0.61582	0.627866	0.645038

Time (t)	*Reliability R(t)*		
	Under No PM = PPM	Under IPM	
		f = 0.801	*f = 0.501*
1560	0.454844	0.480218	0.515957
1620	0.441269	0.46792	0.505444
1680	0.428099	0.456053	0.495415
1740	0.415322	0.44454	0.485685
1800	0.402927	0.433371	0.476246
1860	0.390901	0.422577	0.467215
1920	0.379235	0.412105	0.458454
1980	0.367916	0.401946	0.449954
2040	0.356936	0.392114	0.4418
2100	0.346283	0.382576	0.43389
2160	0.335948	0.373322	0.426216
2220	0.325921	0.364354	0.418835
2280	0.316194	0.355654	0.411674
2340	0.306757	0.347213	0.404727
2400	0.297601	0.339022	0.398029
2460	0.288719	0.331075	0.391531
2520	0.280102	0.323366	0.385226

1020	0.597441	0.610792	0.629788
1080	0.57961	0.594229	0.614993
1140	0.562311	0.578318	0.601011
1200	0.545529	0.562882	0.587447
1260	0.529247	0.547906	0.574288
1320	0.513451	0.533499	0.561811
1380	0.498127	0.519521	0.549706
1440	0.48326	0.50596	0.537962
1500	0.468837	0.492894	0.526793
2580	0.271743	0.315874	0.379134
2640	0.263632	0.308606	0.373223
2700	0.255764	0.301554	0.367488
2760	0.248131	0.294693	0.361934
2820	0.240725	0.288036	0.356546
2880	0.23354	0.281579	0.351318
2940	0.22657	0.275287	0.346243
3000	0.219808	0.269183	0.34132

To visually depict the effect of IPM, Figure 2 is the graphic representation of Table 1. The graphic representation shown in Figure 2 takes into account a time step of 1 unit. The red plot or R_1 is the reliability of the component with improvement factor $f = 0.501$, the green plot or R_2 is its equivalent reliability with improvement factor $f = 0.801$ while the black plot or R_3 is the reliability under No PM policy.

Table 1 and Figure 2 reveal that (i) IPM is better than PPM, and (ii) component reliability is improved better when the improvement factor value is lower; e.g. component reliability is improved with $f = 0.501$ than with $f = 0.801$. This violates the assertion of the age reduction model and therefore,suggests that the exponential distribution is not sufficient in modeling a PM problem under the age reduction model.

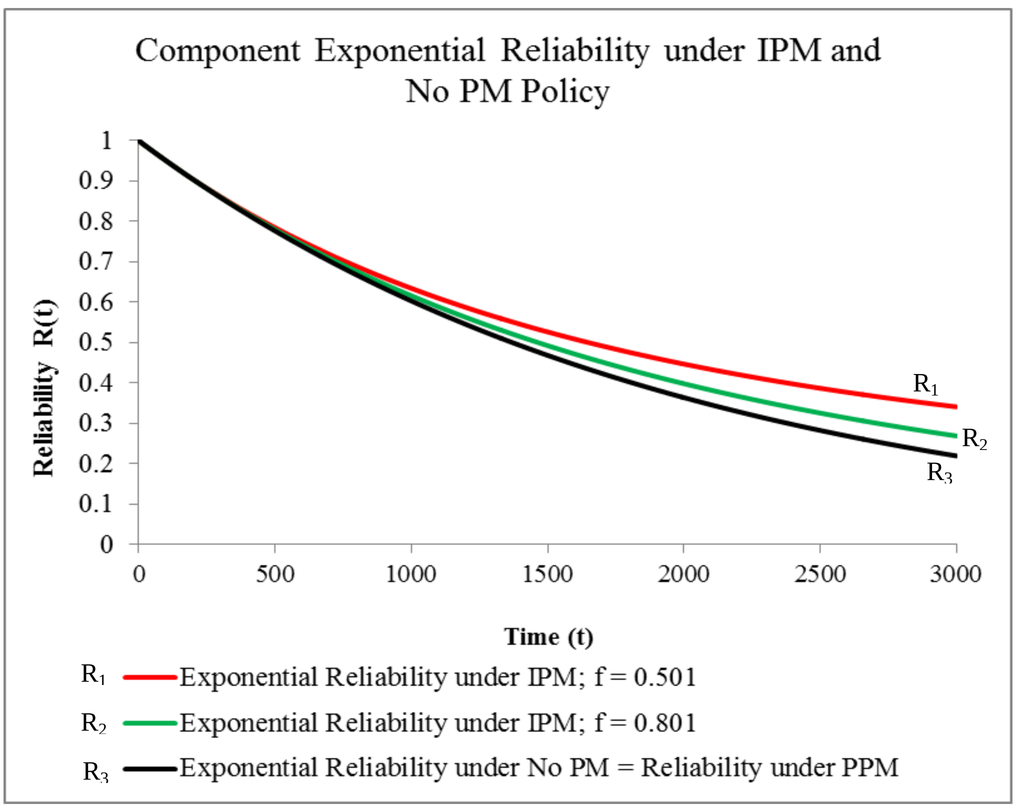

Figure 2. Component Exponential Reliability under IPM and No PM

Weibull distribution is known to model lifetime problems, therefore, to see how the exponential model performs against the Weibull distribution with respect to maintenance modeling, the Weibull model forcomponent reliability under preventive maintenance is considered. Equation 14 is the Weibull model for component reliability under no PM policy [10] while equation 15 is the Weibull model for component reliability under IPM [11].

$$R(t) = exp\left[-\left(\frac{t-\gamma}{\theta}\right)^{\beta}\right] \tag{14}$$

$$R_{ipm}(t) = \prod_{j=1}^{n}\left(1 - exp\left[-\left(\frac{(j-1)t_r-\gamma}{\theta}\right)^{\beta}\right] + exp\left[-\left(\frac{\left((j-1)t_r+T_p\right)-\gamma}{\theta}\right)^{\beta}\right]\right)\left(1 - exp\left[-\left(\frac{nt_r-\gamma}{\theta}\right)^{\beta}\right] + exp\left[-\left(\frac{(nt_r+(t-nT_p))-\gamma}{\theta}\right)^{\beta}\right]\right) \tag{15}$$

The evaluation uses same parameter as used for the exponential distribution. Additional parameter used is Weibull shape parameter β = 2 (failure rate of component is assumed to increase with time). The results obtained are shown in Table 2.

Table 2. Component Weibull Reliability under PPM and IPM

Time (t)	*Reliability R(t)*		
	Under No PM = PPM	Under IPM	
		f = 0.801	*f = 0.501*
0	1	1	1
60	0.9996	0.9996	0.9996
120	0.998401	0.998401	0.998401
180	0.996406	0.996406	0.996406
240	0.99362	0.995533	0.994817
300	0.99005	0.993864	0.992437
360	0.985703	0.991405	0.989272
420	0.980591	0.990063	0.98652
480	0.974725	0.987933	0.982993
540	0.968119	0.985019	0.978697
600	0.960789	0.983218	0.974828
660	0.952753	0.980636	0.970205
720	0.944027	0.97728	0.964838
780	0.934634	0.97503	0.959914
840	0.924595	0.972009	0.954263
900	0.913931	0.968224	0.947898
960	0.902668	0.965538	0.941996
1020	0.890831	0.962092	0.9354
1560	0.763074	0.92511	0.864134
1620	0.747067	0.91979	0.85475
1680	0.730811	0.915528	0.845914
1740	0.714337	0.910566	0.836555
1800	0.697676	0.904915	0.826693
1860	0.680859	0.900309	0.817397
1920	0.663916	0.895021	0.807625
1980	0.646876	0.889064	0.7974
2040	0.62977	0.884137	0.787755
2100	0.612626	0.878547	0.777683
2160	0.595473	0.872308	0.767206
2220	0.578336	0.867084	0.757319
2280	0.561244	0.861217	0.747053
2340	0.544221	0.854721	0.736431
2400	0.527292	0.849225	0.726405
2460	0.510482	0.843107	0.716047
2520	0.493812	0.836381	0.70538
2580	0.477305	0.830638	0.695308

1080	0.878447	0.957895	0.928125
1140	0.865541	0.954788	0.921334
1200	0.852144	0.950934	0.913886
1260	0.838283	0.946342	0.905799
1320	0.823987	0.942831	0.898219
1380	0.809288	0.938587	0.890023
1440	0.794216	0.93362	0.881232
1500	0.778801	0.929724	0.872968

2640	0.46098	0.824294	0.684952
2700	0.444858	0.817364	0.67433
2760	0.428956	0.811399	0.664301
2820	0.413292	0.804856	0.65403
2880	0.397882	0.797749	0.643537
2940	0.38274	0.791589	0.633627
3000	0.367879	0.784872	0.623515

Table 2 shows that reliability is improved better with improvement factor $f = 0.801$ as compared to $f = 0.501$. Overall, it shows that as the improvement factor f approaches unity, reliability is improved better. A graphical and detail representation of Table 2 is shown in Figure 3.

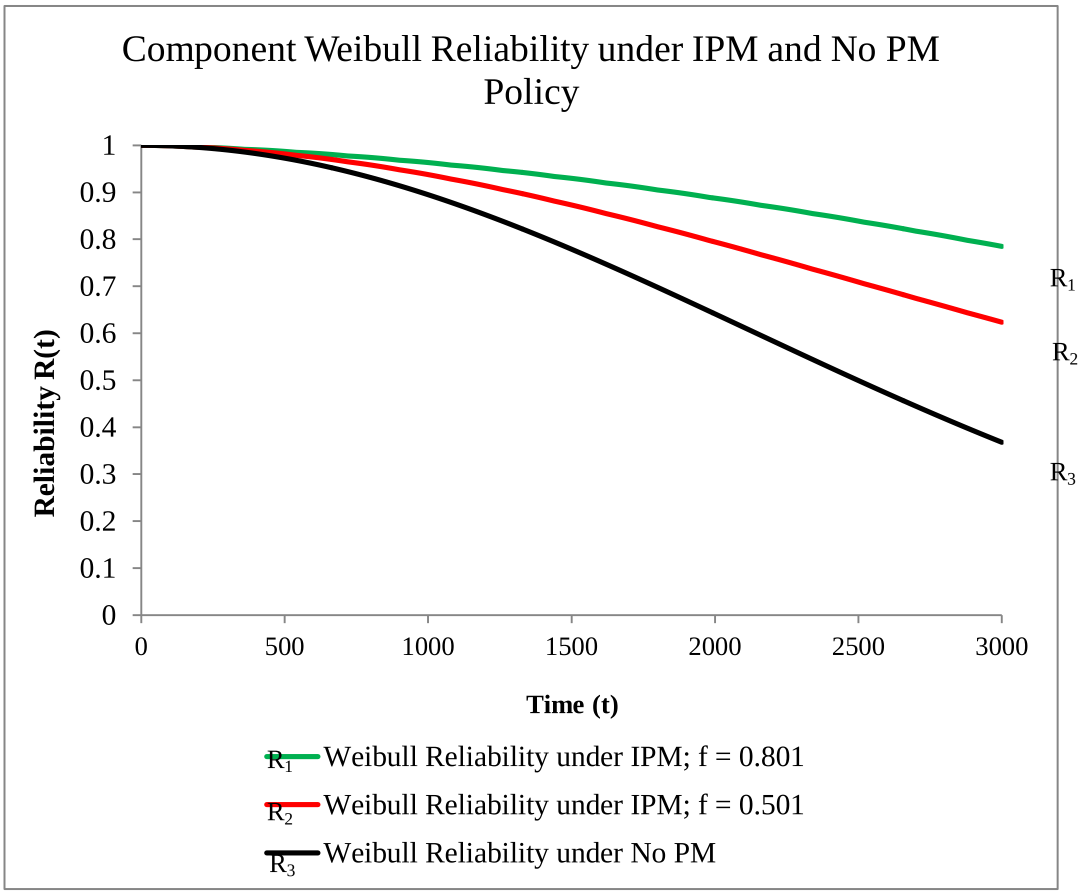

Figure 3. Component Weibull Reliability under IPM and No PM

5. Conclusions

Preventive maintenance modeling is an essential factor in system performance in several industries such as manufacturing, automotive, shipping, aerospace, *etc*. Preventive maintenance modeling could inform the system engineer on system reliability improvement with respect to a given time. The probabilistic distribution model used for the modeling also has impact on the information that the system engineer is exposed to regarding preventive maintenance. This paper has investigated the exponential distribution in modeling preventive maintenance. Typically, as the improvement factor (or effectiveness of maintenance) f approaches unity, the reliability improves more. However, the exponential distribution as usedin this paper showed that perfect preventive maintenance has no effect on component reliability, while component reliability under imperfect preventive maintenance improves better as the improvement factor approaches value 0. This is counter intuitive but mathematically proven. The results obtained under exponential modeling were then compared to those obtained under Weibull distribution model. The Weibull distribution showed that component reliability is improved better as the improvement factor approaches unity. The results obtained in this paper suggest that the exponential distribution model is not sufficient to modeling a preventive maintenance problem using the proportional age reduction model (PAR).

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