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Dynamic Multi-Attribute Decision Making Based on Advantage Retention Degree

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Abstract

To solve the traditional dynamic decision making time relationship problem, this paper introduced a new dynamic multi-attribute decision making method based on advantage retention degree (ARD). According to the logic continuity of decision result at adjoining periods, the concepts of dynamic decision making and retain ability were described, then the dynamic decision making was realized by the calculation of ordering vector based on ARD. The analysis of numerical examples shows that the distinction among components of ordering vector with retain ability is increased, and dynamic multi-attribute decision making based on ARD can solve the problem that the decision couldn't be made if the components are same during the traditional static decision making process. Finally, the rationality of new decision method is proved by the decision results.

Keywords: *Dynamic Multi-Attribute Decision Making; Advantage Retention Degree; Retain Ability; Dynamic Ordering Vector*

1. Introduction

Dynamic multiple attribute decision making (DMADM) whose input arguments collected from different periods plays an important role in modern decision science [1-5]. DMADM is frequently encountered in real-life situations, such as multi-stage investment decision making, medical diagnosis, personnel dynamic examination, and military system efficiency dynamic evaluation, etc [6-10].

To solve the traditional dynamic decision making time relationship problem, we first present a brief introduction of dynamic multi-attribute decision making. The developed approach to solve the DMADM problems where all the attribute values provided at different periods are expressed in triangular fuzzy numbers is extended. An illustrative example is shown for proving the rationality of new decision method.

2. DMADM based on Advantage Retention Degree

2.1. Retain Ability

Definition 1. DMADM result $\boldsymbol{V}^{t_i}$ of the alternative $x_i \in \boldsymbol{X}$ with respect to attribute $u_j \in \boldsymbol{U}$ by decision maker d at the period $t_i \in \boldsymbol{T}$ has retain ability, where

$$\boldsymbol{V}^{t_i} = f(\boldsymbol{V}_0^{t_i}, \boldsymbol{V}_{retain}^{t_1}, \boldsymbol{V}_{retain}^{t_2}, \ldots, \boldsymbol{V}_{retain}^{t_{i-1}}) \tag{1}$$

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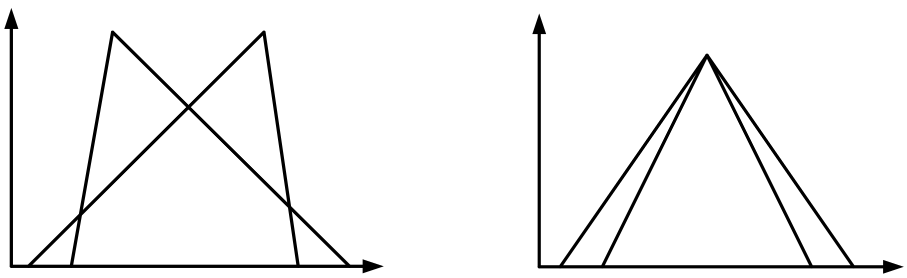
Where $v_0^{t_i}$ is the static decision item at t_i period which only has relation with evalution state at t_i period, $v_{retain}^{t_j}$ (j=1,2,⋯,i-1) are the dynamic retention decision item at t_i period, which are the item influenced by dynamic decision result v^{t_i} at t_j period.

Definition 2. Let $a_1 = [a_1^L, a_1^M, a_1^U]$ and $a_2 = [a_2^L, a_2^M, a_2^U]$ be two triangular fuzzy linguistic variables which could be without the state of cross contain or inside contain, as illustrated in Figure 1 [11].



(a) Cross contain **(b) Inside contain**

Figure 1. Cross and Inside Contain of Triangular Fuzzy Number

We compare a_1 with a_2 by using the following possibility-degree formula based on area proportion for triangular fuzzy number:

$$p(a_1 \ge a_2) = \begin{cases} 1, & a_1^L \ge a_2^U, \\ \dfrac{(\min(a_1^U, a_2^U) - \max(a_1^L, a_2^L))^2}{(l_{21} + l_{12})(l_{11} + l_{22} + l_{21} + l_{12})}, & a_1^L < a_2^L < a_1^U < a_2^U, \\ 1 - \dfrac{(\min(a_1^U, a_2^U) - \max(a_1^L, a_2^L))^2}{(l_{21} + l_{12})(l_{11} + l_{22} + l_{21} + l_{12})}, & a_2^L < a_1^L < a_2^U < a_1^U, \\ 0, & a_1^U \le a_2^L. \end{cases} \quad (2)$$

where $l_{i1} = a_i^M - a_i^L$, $l_{i2} = a_i^U - a_i^M$ (i=1,2).

Proof:

Let $A_1(a_1^L, 0)$, $B_1(a_1^M, 1)$, $C_1(a_1^U, 0)$, $A_2(a_2^L, 0)$, $B_2(a_2^M, 1)$, $C_2(a_2^U, 0)$.

1) $a_2^L < a_2^U \le a_1^L < a_1^U$, obviously, $p(a_1 \ge a_2) = 1$;

2) $a_1^L < a_1^U \le a_2^L < a_2^U$, obviously, $p(a_1 \ge a_2) = 0$;

3) $a_1^L < a_2^L < a_1^U < a_2^U$, as illustrated in Fig. 2, $P(a,b)$ is the point of intersection of B_1C_1 and A_2B_2, so:

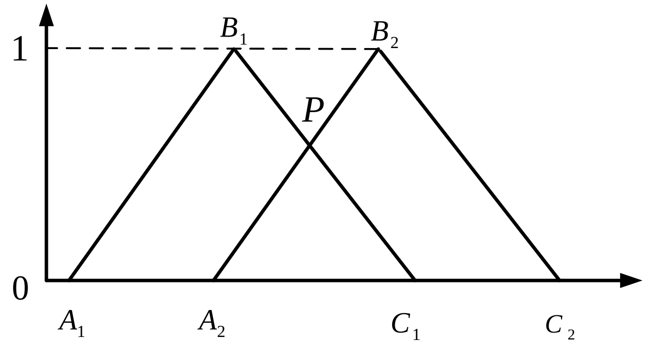


Figure 2. Legend of Possibility Degree Calculation

$$p(a_1 \geq a_2) = \frac{S_{\Delta A_2 P C_1}}{S_{\Delta A_1 B_1 C_1} + S_{\Delta A_2 B_2 C_2}} = \frac{\frac{1}{2}(a_1^U - a_2^L)b}{\frac{1}{2}(a_1^U - a_1^L) + \frac{1}{2}(a_2^U - a_2^L)}$$

$$= \frac{\frac{1}{2}(a_1^U - a_2^L)b}{\frac{1}{2}(a_1^U - a_1^L) + \frac{1}{2}(a_2^U - a_2^L)}$$

$$= \frac{(a_1^U - a_2^L)^2}{[(a_1^U - a_1^M) + (a_2^M - a_2^L)][(a_1^U - a_1^L) + (a_2^U - a_2^L)]}$$

$$= \frac{(a_1^U - a_2^L)^2}{(l_{21} + l_{12})(l_{11} + l_{22} + l_{21} + l_{12})} \tag{3}$$

4) $a_2^L < a_1^L < a_2^U < a_1^U$, $Q(c,d)$ is the point of intersection of B_2C_2 and A_1B_1, so:

$$p(a_1 \geq a_2) = 1 - \frac{S_{\Delta A_1 Q C_2}}{S_{\Delta A_1 B_1 C_1} + S_{\Delta A_2 B_2 C_2}} = 1 - \frac{(a_2^U - a_1^L)^2}{(l_{21} + l_{12})(l_{11} + l_{22} + l_{21} + l_{12})} \tag{4}$$

2.2. Advantage Retention Degree

Definition 3. Let $\boldsymbol{\omega}_{\boldsymbol{P}}^{t_i} = (\omega_1^{t_i}, \omega_2^{t_i}, \ldots, \omega_n^{t_i})^{\mathrm{T}}$ be the ordering vector of matrix $\boldsymbol{P}$ at the period t_i,

$$ard_j^{t_i} = \frac{\omega_j^{t_i}}{\sum_{j=1}^{n} \omega_j^{t_i}} \times 100 . \tag{5}$$

is defined as Advantage Retention Degree (ARD) of $\omega_j^{t_i} \in \boldsymbol{\omega}_{\boldsymbol{P}}^{t_i}$ at the period t_i. The dynamic ordering vector based on ARD $\boldsymbol{\omega}_{\boldsymbol{ard}}^{t_i} = (\omega_{ard1}^{t_i}, \omega_{ard2}^{t_i}, \ldots, \omega_{ardn}^{t_i})^{\mathrm{T}}$, where $\omega_{ardj}^{t_i}$ (j=1,2,…,n) is expressed as:

$$\omega_{ardj}^{t_i} = \omega_j^{t_i} \times ard_j^{t_{i-1}} = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} p_{ij} + \frac{n}{2} - 1\right) \times \frac{\omega_{ardj}^{t_{i-1}}}{\sum_{j=1}^{n} \omega_{ardj}^{t_{i-1}}} \times 100 \quad (6)$$

3. Application of DMADM

Let $\boldsymbol{X} = \{x_1, x_2, \ldots x_m\}$ be a discrete set of m feasible alternatives, and $\boldsymbol{U} = \{u_1, u_2, \ldots u_n\}$ is a finite set of attributes. Suppose that there are l periods t_k (k=1,2,..,l), and $\boldsymbol{\omega} = \{\omega_1^{(k)}, \omega_2^{(k)}, \ldots \omega_n^{(k)}\}^{\mathrm{T}}$ is the weight vector of attribute u_j(j=1,2,…,n) at the period t_k, where $\omega_j \geq 0$, $\sum_{j=1}^{n} \omega_j^{(k)} = 1$. Construct $\boldsymbol{R_k} = (r_{ij}^{(k)})_{m \times n}$ a decision matrix (see Table 1).

Table 1. Decision Matrix R_k

	u_1	u_2	…	u_n
x_1	$r_{11}^{(k)}$	$r_{12}^{(k)}$	…	$r_{1n}^{(k)}$
x_2	$r_{21}^{(k)}$	$r_{22}^{(k)}$	…	$r_{2n}^{(k)}$
…	…	…	…	…
x_m	$r_{m1}^{(k)}$	$r_{m2}^{(k)}$	…	$r_{mn}^{(k)}$

As what follows, we develop an approach to DMADM based on ARD.

Step 1. Utilize the TFLWA operator to aggregate the attribute value $r_{ij}^{(1)}$ in the ith column of the decision matrix $\boldsymbol{R}_1$ into a overall attribute value of the alternative x_i (i=1,2,…,m) at the period t_1:

$$r_i^{(1)} = \mathrm{TFLWA}_{\boldsymbol{\omega}}(r_{i1}^{(1)}, \ldots, r_{in}^{(1)}) = \omega_1 r_{i1}^{(1)} + \ldots + \omega_n r_{in}^{(1)}. \quad (7)$$

Step 2. Compare each $r_i^{(1)} (i \in M)$ with all $r_j^{(1)} (j \in M)$ by using the possibility-degree formula(4), then construct the possibility-degree matrix $\boldsymbol{P}^{(1)} = (p_{ij}^{(1)})_{m \times m}$.

Step 3. Calculate the static ordering vector $\boldsymbol{\omega}^{(1)} = (\omega_1^{(1)}, \omega_2^{(1)}, \ldots, \omega_m^{(1)})^{\mathrm{T}}$ of fuzzy complementary judgment matrix $\boldsymbol{P}^{(1)}$ at the period t_1 by using the Eq (9), then get the ARD $ard_i^{(1)}$ of $\omega_i^{(1)}$ at the period t_1.

Step 4. Repeat Step 3 and Step 4, calculate the attribute value $r_{ij}^{(k)}$ (k=1,2,…,l) into a overall attribute value of the alternative x_i at the period t_k. utilizing the TFLWA operator. The possibility-degree matrix $\boldsymbol{P}^{(k)} = (p_{ij}^{(k)})_{m \times m}$ $(k \in T, k > 1)$ can be established.

Step 5. Calculate the ordering vector based on ARD:

$$\boldsymbol{\omega}_{\boldsymbol{ard}}^{(\boldsymbol{k})} = (\omega_{ard1}^{(k)}, \omega_{ard2}^{(k)}, \ldots, \omega_{ardn}^{(k)})^{\mathrm{T}} = (\omega_1^{(k)} \times ard_1^{(k-1)}, \omega_2^{(k)} \times ard_2^{(k-1)}, \ldots, \omega_n^{(k)} \times ard_n^{(k-1)}). \quad (8)$$

until the $\boldsymbol{\omega}_{\boldsymbol{ard}}^{(\boldsymbol{l})}$ at last period t_l.

Step 6. Calculate the advantage retention degree $ard_i^{(k)}$ of $\omega_{ardi}^{(k)}$ which is the element of dynamic ordering vector $\boldsymbol{\omega}_{ard}^{(k)}$ at the period t_k, until the $\boldsymbol{ard}^{(l-1)}$ at period t_{l-1}.

Step 7. Reorder r_i (i=1,2,…,m) in descending order in accordance with $\omega_{ardi}^{(l)}$. Then we can rank all the alternatives x_i and select the most desirable one in accordance with the value of r_i.

4. Simulation

Aircraft landing process is simulated for five times with aircraft landing system as shown in Figure. 3 [12-17]. The flight states of landing process are indicated in Figure 4, and final distributions of touchdown points are shown in Figure 5. Finally, the landing risk are evaluated.

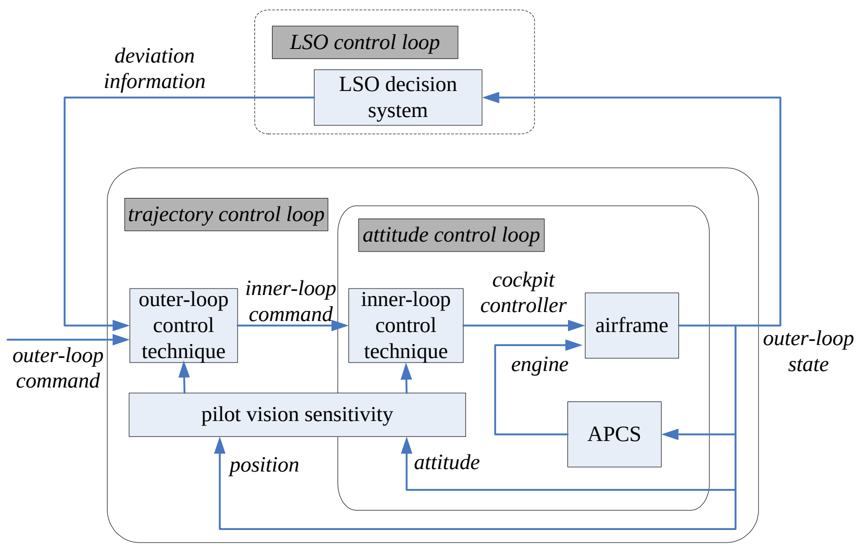
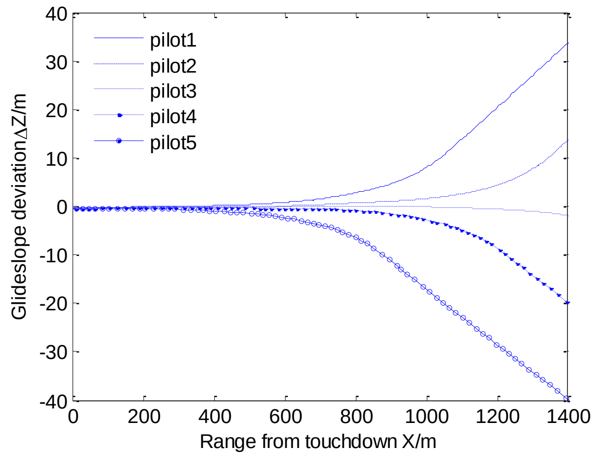


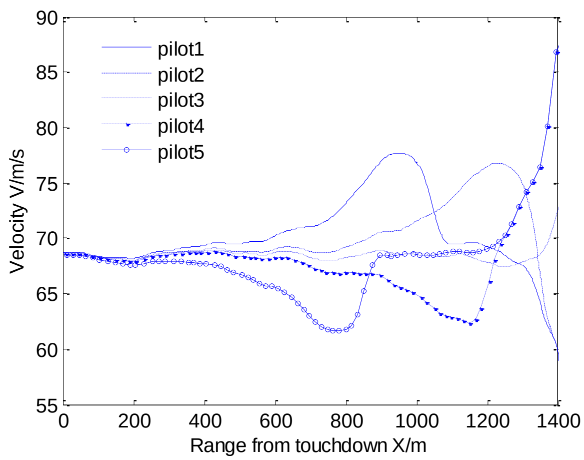
Figure 3. LSO-pilot-aircraft Landing System

Landing signal officer (LSO) uses triangular fuzzy linguistic variables to evaluate the performance of the voyages x_i (i=1,2,3,4,5) at four reference points according to the attributes u_j (j=1,2,3), and constructs, respectively, the decision matrices $\boldsymbol{R}_k$ (k=1,2,3,4, here as listed in Tables 2-5.

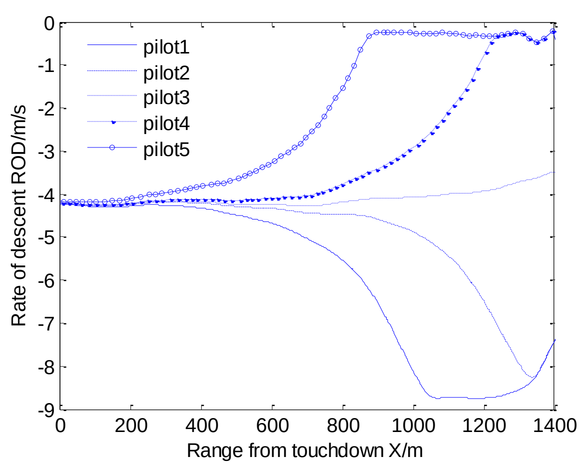
Let $\boldsymbol{\omega} = (0.4, 0.4, 0.2)^{\mathrm{T}}$ be the weight vectors of the attributes u_j (j=1,2,3) at all reference points.



(a) Glideslope deviations family of curves



(b) Speed family of curves



(c) ROD family of curves

Figure 4. Landing States of Aircraft with Different Pilots

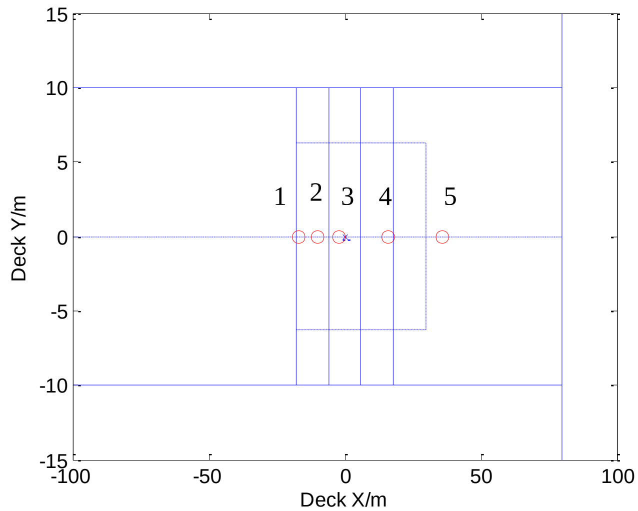


Figure 5. Distributions of Touchdown Points on Deck

Table 2. Evaluation Matrix of Decision Time t_1 (X point)

Pilot\flight state	u_1	u_2	u_3
x_1	[0.1,0.2,0.3]	[0.1,0.2,0.3]	[0.1,0.2,0.3]
x_2	[0.4,0.5,0.6]	[0.2,0.3,0.4]	[0.2,0.3,0.4]
x_3	[0.5,0.6,0.7]	[0.4,0.5,0.6]	[0.5,0.6,0.7]
x_4	[0.3,0.4,0.5]	[0,0.1,0.2]	[0.2,0.3,0.4]
x_5	[0,0.1,0.2]	[0,0.1,0.2]	[0.1,0.2,0.3]

Table 3. Evaluation Matrix of Decision Time t2 (IM point)

Pilot\flight state	u_1	u_2	u_3
x_1	[0.3,0.4,0.5]	[0.1,0.2,0.3]	[0.3,0.4,0.5]
x_2	[0.5,0.6,0.7]	[0.4,0.5,0.6]	[0.5,0.6,0.7]
x_3	[0.6,0.7,0.8]	[0.5,0.6,0.7]	[0.6,0.7,0.8]
x_4	[0.4,0.5,0.6]	[0.2,0.3,0.4]	[0.2,0.3,0.4]
x_5	[0.1,0.2,0.3]	[0.4,0.5,0.6]	[0.1,0.2,0.3]

Table 4. Evaluation Matrix of Decision Time t3 (IC point)

Pilot\flight state	u_1	u_2	u_3
x_1	[0.4,0.5,0.6]	[0.3,0.4,0.5]	[0.5,0.6,0.7]
x_2	[0.6,0.7,0.8]	[0.5,0.6,0.7]	[0.6,0.7,0.8]
x_3	[0.7,0.8,0.9]	[0.6,0.7,0.8]	[0.7,0.8,0.9]
x_4	[0.5,0.6,0.7]	[0.4,0.5,0.6]	[0.4,0.5,0.6]
x_5	[0.4,0.5,0.6]	[0.1,0.2,0.3]	[0.3,0.4,0.5]

Table 5. Evaluation Matrix of Decision Time t4 (AR point)

Pilot\flight state	u_1	u_2	u_3
x_1	[0.6,0.7,0.8]	[0.5,0.6,0.7]	[0.6,0.7,0.8]
x_2	[0.7,0.8,0.9]	[0.6,0.7,0.8]	[0.7,0.8,0.9]
x_3	[0.8,0.9,1]	[0.7,0.8,0.9]	[0.8,0.9,1]
x_4	[0.6,0.7,0.8]	[0.6,0.7,0.8]	[0.5,0.6,0.7]
x_5	[0.5,0.6,0.7]	[0.5,0.6,0.7]	[0.4,0.5,0.6]

(1) Period t_1:

Attribute value $r_{ij}^{(1)}$:

$$r_1^{(1)}=[0.1,0.2,0.3],\ r_2^{(1)}=[0.28,0.38,0.48],$$

$$r_3^{(1)}=[0.46,0.56,0.66],\ r_4^{(1)}=[0.16,0.26,0.36],\ r_5^{(1)}=[0.02,0.12,0.22].$$

Possibility-degree matrix:

$$\boldsymbol{P}^{(1)}=\begin{bmatrix} 0.5 & 0.005 & 0 & 0.245 & 0.98 \\ 0.995 & 0.5 & 0.005 & 0.92 & 1 \\ 1 & 0.995 & 0.5 & 1 & 1 \\ 0.755 & 0.08 & 0 & 0.5 & 0.955 \\ 0.02 & 0 & 0 & 0.045 & 0.5 \end{bmatrix}.$$

Static ordering vector:

$$\boldsymbol{\omega}^{(1)}=(0.1615,0.246,0.300,0.1895,0.1033)^{\mathrm{T}}.$$

ARD:

$$\boldsymbol{ard}^{(1)}=(16.15,24.59,30.00,18.94,10.33)^{\mathrm{T}}.$$

(2) Period t_k:

Static ordering vector:

$$\boldsymbol{\omega}^{(2)}=(0.1373,0.256,0.2938,0.1768,0.1373)^{\mathrm{T}}.$$

Dynamic ordering vector based on ARD:

$$\boldsymbol{\omega}_{\boldsymbol{ard}}^{(2)}=(\omega_{ard\,1}^{(2)},\omega_{ard\,2}^{(2)},\ldots,\omega_{ard\,5}^{(2)})^{\mathrm{T}}=(\omega_1^{(2)}\times ard_1^{(1)},\omega_2^{(2)}\times ard_2^{(1)},\ldots,\omega_5^{(2)}\times ard_5^{(1)})$$

$$=(2.22,6.30,8.81,3.35,1.42)^{\mathrm{T}}.$$

ARD of $\boldsymbol{\omega}_{\boldsymbol{ard}}^{(2)}$:

$$\boldsymbol{ard}^{(2)}=(10.05,28.51,39.86,15.15,6.43)^{\mathrm{T}}.$$

(3) Final period t_4:

Dynamic ordering vector :

$$\boldsymbol{\omega}_{ard}^{(4)} = (1.11, 7.19, 14.30, 2.27, 0.32)^{\mathrm{T}}.$$

Rank all the alternatives r_i in accordance with the values of $\boldsymbol{\omega}_{ard}^{(4)}$. We can see that the ranking order is

$$r_3 > r_2 > r_4 > r_1 > r_5$$

and the best voyage is r_3.

Judging from static ordering vector $\boldsymbol{\omega}^{(2)}$ at the period t_2, the comprehensive landing effect of group 1 is as same as group 5. But using the new method this paper proposed, the landing effect of group 1 is better than group 5 based on the ARD at period t_1. According to the logic continuity, it can be obtained that the whole flight effect of group 1 is better than group 5 until period t_2.

The decision result shows that the landing risk of group 3 is the least. Comparing with Figure 5, correspondingly the touchdown point locates between the second cable and the third one, and it is closest to the desired touchdown, however, other voyages points distributions are far from the desired one, and the decision risk is increased. The decision result equates with the distributions of touchdown points, and it proves the practicality of this decision method.

5. Conclusion

DMADM based on ARD can satisfy the requirement of logic continuity during the decision process, so the accuracy of decision result is improved effectively. The differences of each ordering element which are obtained through the calculation method of traditional complementary judgment matrix ordering vector are less and can't be distinguished. The differences of each element are increased by improving the formula for ordering vector, and this method overcomes traditional method's defects.

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