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Sufficient Conditions for Global Asymptotic Stability of Delayed Cohen-Grossberg Neural Networks

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Abstract

This paper investigates the global asymptotic stability for the Cohen-Grossberg neural networks with time-varying delays. By constructing suitable Lyapunov functional and employing the nonsmooth analysis, some sufficient conditions are obtained without demanding the boundedness and differentiability of the activation function. Moreover, one example is demonstrated to illustrate the effectiveness of the proposed criterion.

Keywords: *Cohen-Grossberg neural system, Global asymptotic stability, Nonsmooth analysis, time-varying delay*

1. Introduction

In [1], Cohen and Grossberg first proposed a kind of neural networks, which are called Cohen-Grossberg neural networks and described by the following ordinary differential equations:

$$\frac{dx_i(t)}{dt} = -d_i(x_i(t))\left[c_i(x_i(t)) - \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + J_i\right], \ i = 1, 2, \ldots, n, \tag{1}$$

where n is the number of the neurons in the networks, $x_i(t)$ is the state variable of the ith neuron, $d_i(\cdot)$ represents an amplification function, $c_i(\cdot)$ is the behaved function, $f_j(\cdot)$ is called an activation function indicating how the jth neuron responses to its input. The Cohen-Grossberg neural networks have important applications in various areas such as classification, associative memory, parallel computing, especially in solving some optimization problems. In these applications, it is very important to study the stability of neural networks.

In addition, time-delays are unavoidably encountered in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. As a consequence, many researchers have focused their attention on the study of stability of the neural networks with delays [2-11]. In recent years, some sufficient conditions were presented to ensure the global asymptotic stability (GAS) of the delayed Cohen-Grossberg neural networks [3-11]. But in [4-8], the activation functions are assumed to be bounded and differentiable. However, in practice the activation functions are not always to be differentiable and bounded. In addition, the authors in [9] have studied the following Cohen-Grossberg neural networks with the constant delays

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$$\frac{dx_i(t)}{dt} = -d_i\left(x_i(t)\right)\left[c_i\left(x_i(t)\right) - \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) - \sum_{j=1}^{n} b_{ij} f_j\left(x_j\left(t-\tau_j\right)\right) + J_i\right], i = 1, 2, \ldots, n, \quad (2)$$

and the GAS conditions are obtained by employing nonsmooth analysis. But in practice, time delay is usual time-varying, which can even largely change the dynamics of system in some cases. Therefore, their methods have a conservatism which can be improved upon.

In this paper, by using a new method based on the nonsmooth analysis, we obtain an improved sufficient condition for the GAS of the equilibrium point without demanding the boundedness and differentiability of the activation functions. One example is provided to show the effectiveness and the benefits of the proposed method.

Notation: Throughout this paper, we use $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to denote the minimum and maximum eigenvalue of a real symmetric matrix, respectively. The superscript T represents the transpose. The notation $\|v\|$ denotes a vector norm defined by $\|v\| = (v^T v)^{1/2}$, where $v = (v_1, v_2, \ldots, v_n)^T \in R^n$ and $|v| = (|v_1|, |v_2|, \ldots, |v_n|)^T$, while $\|A\|$ denotes a matrix norm defined by $\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}$, where A is a matrix and $|A| = (|a_{ij}|)$. Let $\mu_2(A) = \frac{1}{2}\lambda_{\max}\left(A + A^T\right)$, *i.e.*, $\mu_2(A)$ is the largest eigenvalue of the symmetric part of A. It is well known that $\mu_2(A) \le \|A\|_2$.

2. Problem Statement

Consider the following Cohen-Grossberg neural networks with time-varying delays:

$$\frac{dx_i(t)}{dt} = -d_i\left(x_i(t)\right)\left[c_i\left(x_i(t)\right) - \sum_{j=1}^{n} a_{ij} f_j\left(x_j(t)\right) - \sum_{j=1}^{n} b_{ij} f_j\left(x_j\left(t-\tau_j(t)\right)\right) + J_i\right],$$

$$i = 1.2, \ldots, n, \quad (3)$$

where $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively; $\tau_j(t)$ is a time-varying delay, and it is assumed to satisfy

$$0 \le \tau_j(t) \le \tau, \quad 0 \le \dot{\tau}_j(t) \le \sigma < 1, \quad (4)$$

where τ, σ are constants.

The system (3) can be written as the following vector-matrix form:

$$\dot{x}(t) = -D\left(x(t)\right)\left[C\left(x(t)\right) - Af\left(x(t)\right) - Bf\left(t-\tau(t)\right) + J\right], \quad (5)$$

where

$$x(t) = \left(x_1(t), x_2(t), x_3(t), \ldots, x_n(t)\right)^T,$$

$$D\left(x(t)\right) = diag\left(d_1\left(x_1(t)\right), d_2\left(x_2(t)\right), \ldots, d_n\left(x_n(t)\right)\right),$$

$$C\left(x(t)\right) = \left(c_1\left(x_1(t)\right), c_2\left(x_2(t)\right), \ldots, c_n\left(x_n(t)\right)\right)^T,$$

$$f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$$
$$f(x(t-\tau(t))) = (f_1(x_1(t-\tau_1(t))), f_2(x_2(t-\tau_2(t))), \ldots, f_n(x_n(t-\tau_n(t))))^T,$$
$$J = (J_1, J_2, \ldots, J_n)^T.$$

In order to establish the stability conditions for system (3), we first give some usual assumptions and lemma.

(M_1) Each function $d_i(\cdot)$ is positive, continuous and bounded.

(M_2) Each function $c_i : R \to R$ is locally Lipschitz, and there exists l_i such that $c_i'(x) \ge l_i$ for all $x \in R$ at which $c_i(\cdot)$ is differentiable, where $l = \min_{1\le i\le n}\{l_i\}$.

(M_3) Each function $f_i : R \to R$ is nondecreasing and globally Lipschitz with a constant $k_i > 0$, *i.e.*,

$$\left| f_i(u) - f_i(v) \right| \le k_i \left| u - v \right|, \qquad \forall u, v \in R, \quad i = 1, 2, \ldots, n,$$

where $k = \max_{1\le i\le n}\{k_i\}$.

Lemma 1[10] For each input vector $u \in R^n$, system (3) has a unique equilibrium point if there exists a positive diagonal $D = diag(d_1, d_2, \ldots, d_n)$ such that

$$\mu_2(D(A+B)) < \left(\frac{l}{k}\right)\lambda_{\min}(D) \tag{6}$$

under the conditions (M_1), (M_2) and (M_3).

3. Results and Discussion

In this section, we present the new result for the GAS of the equilibrium point of (3).

Theorem 1. Suppose that the conditions (M_1), (M_2) and (M_3) are satisfied and there exist two positive diagonal matrices $D = diag(d_1, d_2, \ldots, d_n)$ and $\Sigma = diag(\gamma_1, \gamma_2, \ldots, \gamma_n)$ and a positive η such that

$$\mu_2(DA) < \frac{l}{k}\lambda_{\min}(D) - \frac{\eta}{2}(1-\sigma)\lambda_{\max}(\Sigma^2) - \frac{\|DB\|_2^2}{2\eta(1-\sigma)\lambda_{\min}(\Sigma^2)} \tag{7}$$

Is satisfied. Then, each input vector $u \in R^n$, (3) has a unique equilibrium point which is GAS.

Proof. Firstly, under the condition of Lemma 1, we will prove system (3) has a unique equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T \in R^n$. It is noted that $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ on the space of symmetric matrices are convex. Then, according to strict inequality (7), we get

$$\mu_2(D(A+B)) \le \mu_2(DA) + \mu_2(DB)$$
$$< \frac{l}{k}\lambda_{\min}(D) - \frac{\eta}{2}(1-\sigma)\lambda_{\max}(\Sigma^2) - \frac{\|DB\|_2^2}{2\eta(1-\sigma)\lambda_{\min}(\Sigma^2)} + \|DB\|_2^2$$

$$< \frac{l}{k}\lambda_{\min}(D) - \frac{\eta}{2}(1-\sigma)\left(\lambda_{\max}(\Sigma^2) - \lambda_{\min}(\Sigma^2)\right)$$
$$- \frac{1}{2\eta(1-\sigma)\lambda_{\min}(\Sigma^2)}\left(\|DB\|_2^2 - \eta(1-\sigma)\lambda_{\min}(\Sigma^2)\right)^2 < \frac{l}{k}\lambda_{\min}(A) \cdot \quad (8)$$

So, the existence and uniqueness of equilibrium points are ensured under the Lemma 1. Next, Let $x^* = \left(x_1^*, x_2^*, \ldots, x_n^*\right)^T$ be an equilibrium point of (3) and let $z(t) = x(t) - x^*$. From (3) and replaying $x_i(t)$ in (3) by $z_i(t) + x_i^*$, it is easy to obtain:

$$\dot{z}(t) = -\alpha(z(t))\left[\beta(z(t)) - A\phi(z(t)) - B\phi(z(t-\tau(t)))\right], \quad i = 1.2, \ldots, n, \quad (9)$$

where

$$z(t) = (z_1(t), \ldots, z_n(t))^T \in R^n,$$
$$\alpha(z(t)) = diag\left(\alpha_1(z_1(t)), \ldots, \alpha_n(z_n(t))\right)^T \in R^{n\times n},$$
$$\beta(z(t)) = \left(\beta_1(z_1(t)), \ldots, \beta_n(z_n(t))\right)^T \in R^n,$$
$$\phi(z(t)) = \left(\phi_1(z_1(t)), \ldots, \phi_n(z_n(t))\right)^T \in R^n,$$
$$\alpha_i(z_i(t)) = d_i(z_i(t) + x^*),$$
$$\beta_i(z_i(t)) = c_i(z_i(t) + x^*) - c_i(x^*) \cdot$$

From (M_3), it is not difficult to see that

$$\|\phi(z(\cdot))\|^2 \le k_i z^T(\cdot)\phi(z(\cdot)) \text{ and } \phi_i(0) = 0 \quad (10)$$

for all $i = 1.2, \ldots, n$. To obtain the stability result, we choose a suit Lyapunov functional candidate for the system (3) to be

$$V(t) = \sum_{i=1}^{n} \varsigma(1-\sigma)\int_0^{z_i}\frac{2s}{\alpha_i(s)}ds + \frac{2}{\eta}\sum_{i=1}^{n}\int_0^{z_i}\frac{d_i\phi_i(s)}{\alpha_i(s)}ds + \sum_{i=1}^{n}\int_{t-\tau(t)}^{t}\gamma_i^2\phi_i^2(z_i(s))ds$$
$$+ \frac{\varsigma}{l}\int_{t-\tau(t)}^{t}\phi^T(z(s))(BB^T)\phi(z(s))ds, \quad (11)$$

where $\varsigma > 0$ is chosen appropriately later on and η is the constant in Theorem 1. Obviously, $V(z(t))$ is positive except at the origin, and it is radially unbounded in the sense that $V(z(t)) \to \infty$ as $\|z(t)\| \to \infty$. By calculating the time derivative of $V(z(t))$ along the trajectories of the system (9), we obtain

$$\frac{dV(t)}{dt} = \varsigma(1-\sigma)\sum_{i=1}^{n}\frac{2z_i(t)}{\alpha_i(z_i(t))}\dot{z}_i(t) + \frac{2}{\eta}\sum_{i=1}^{n}\frac{d_i\phi_i(z_i(t))}{\alpha_i(z_i(t))}\dot{z}_i(t)$$
$$+\sum_{i=1}^{n}\gamma_i^2\left(\phi_i^2(z_i(t)) - (1-\dot{\tau}(t))\phi_i^2(z_i(t-\tau(t)))\right) + \frac{\varsigma}{l}\phi^T(z(t))(BB^T)\phi(z(t))$$
$$-\frac{\varsigma}{l}(1-\dot{\tau}(t))\phi^T(z(t-\tau(t)))(BB^T)\phi(z(t-\tau(t)))$$
$$= -2\varsigma(1-\sigma)z^T(t)\left(C(z(t)+x^*) - C(x^*)\right) + 2\varsigma(1-\sigma)z^T(t)A\phi(z(t))$$

$$+2\varsigma(1-\sigma)z^T(t)B\phi(z(t-\tau(t)))-\frac{2}{\eta}\phi^T(z(t))D\left(C\left(z(t)+x^*\right)-C\left(x^*\right)\right)$$
$$+\frac{2}{\eta}\phi^T(z(t))DA\phi(z(t))+\frac{2}{\eta}\phi^T(z(t))DB\phi(z(t-\tau(t)))$$
$$+\phi^T(z(t))\Sigma^2\phi(z(t))-(1-\dot{\tau}(t))\phi^T(z(t-\tau(t)))\Sigma^2\phi(z(t-\tau(t)))$$
$$+\frac{\varsigma}{l}\phi^T(z(t))BB^T\phi(z(t))-\frac{\varsigma}{l}(1-\dot{\tau}(t))\phi^T(z(t-\tau(t)))BB^T\varphi(z(t-\tau(t))). \quad (12)$$

According to the Lebourg Theorem [12], we have

$$C\left(z(t)+x^*\right)-C\left(x^*\right)=\bar{C}(z(t)), \quad (13)$$

where $\bar{C}\in\bigcup_{y\in[x^*,x^*+z(t)]}\partial C(y)$. In accordance with the definition of C, matrix $\bar{C}$ is diagonal, and denoted as $\bar{C}=diag(c_1,c_2,\ldots,c_3)$. Clearly, $c_i\ge l$ for $i=1,2,\ldots,n$. So we have

$$z^T(t)\left(C\left(z(t)+x^*\right)-C\left(x^*\right)\right)=\sum_{i=1}^{n}c_iz_i^2(t)\ge l\|z(t)\|^2, \quad (14)$$

and

$$\frac{dV(t)}{dt}\le-2\varsigma l(1-\sigma)\|z(t)\|^2+2\varsigma(1-\sigma)z^T(t)A\phi(z(t))$$
$$+2\varsigma(1-\sigma)z^T(t)B\phi(z(t-\tau(t)))-\frac{2}{\eta}\frac{l}{k}\phi^T(z(t))D\phi(z(t))$$
$$+\frac{2}{\eta}\phi^T(z(t))DA\phi(z(t))+\frac{2}{\eta}\phi^T(z(t))DB\phi(z(t-\tau(t)))$$
$$+\phi^T(z(t))\Sigma^2\phi(z(t))-(1-\sigma)\phi^T(z(t-\tau(t)))\Sigma^2\phi(z(t-\tau(t)))$$
$$+\frac{\varsigma}{l}\phi^T(z(t))BB^T\phi(z(t))-\frac{\varsigma}{l}(1-\sigma)\phi^T(z(t-\tau(t)))BB^T\phi(z(t-\tau(t))). \quad (15)$$

For convenience, we write the following equalities:

$$-l\varsigma(1-\sigma)\|z(t)\|^2+2\varsigma(1-\sigma)z^T(t)A\phi(z(t))$$
$$=-\varsigma(1-\sigma)\left\|\sqrt{l}z(t)-\frac{1}{\sqrt{l}}A\phi(z(t))\right\|^2+\frac{\varsigma(1-\sigma)}{l}\phi^T(z(t))AA^T\phi(z(t)), \quad (16)$$

$$-l\varsigma(1-\sigma)\|z(t)\|^2+2\varsigma(1-\sigma)z^T(t)B\phi(z(t-\tau(t)))$$
$$=-\varsigma(1-\sigma)\left\|\sqrt{l}z(t)-\frac{1}{\sqrt{l}}B\phi(z(t-\tau(t)))\right\|^2+\frac{\varsigma(1-\sigma)}{l}\phi^T(z(t-\tau(t)))BB^T\phi(z(t-\tau(t))), \quad (17)$$

$$\phi^T(z(t))DB\phi(z(t-\tau(t)))$$
$$=\frac{1}{2}\left(\eta(1-\sigma)\phi^T(z(t-\tau(t)))\Sigma^2\phi(z(t-\tau(t)))\right)+\frac{1}{2\eta(1-\sigma)}\phi^T(z(t))DB\Sigma^{-2}B^TD\phi(z(t))$$
$$-\frac{1}{2}\left\|\sqrt{\eta(1-\sigma)}\Sigma\phi(z(t-\tau(t)))-\frac{1}{\sqrt{\eta(1-\sigma)}}\Sigma^{-1}B^TD\phi(z(t))\right\|^2. \quad (18)$$

Rearranging the terms in (15) and using the above equalities from (16) to (18), we obtain

$$\frac{dV(t)}{dt} \le \left(\frac{1}{\eta^2(1-\sigma)} \lambda_{\max}(\Sigma^{-2}) \|DB\|_2^2 + \lambda_{\max}(\Sigma^2) - \frac{2}{\eta}\frac{l}{k}\lambda_{\min}(D) + \frac{2}{\eta}\mu_2(DA) \right) \|\phi(z(t))\|^2$$

$$+ \frac{\varsigma}{l}\left(\|A\|_2^2 + \|B\|_2^2 \right) \|\phi(z(t))\|^2 \qquad (19)$$

Let

$$h(\eta) = \frac{1}{\eta^2(1-\sigma)} \lambda_{\max}(\Sigma^{-2}) \|DB\|_2^2 + \lambda_{\max}(\Sigma^2) - \frac{2}{\eta}\frac{l}{k}\lambda_{\min}(D) + \frac{2}{\eta}\mu_2(DA)$$

Under the condition (7) in Theorem 1, we can get $h(\eta) < 0$. Then, (19) can be rewritten

$$\frac{dV(t)}{dt} \le h(\eta) \|\phi(z(t))\|^2 + \frac{\varsigma}{l}\left((1-\sigma)\|A\|_2^2 + \|B\|_2^2 \right) \|\phi(z(t))\|^2 \qquad (20)$$

Next, we consider the following cases.

(i) $\phi(z(t)) \ne 0$. In this case, $z(t) \ne 0$. It follows from (20) that the choice of ς satisfying

$$0 < \varsigma < \frac{-lh(\eta)}{(1-\sigma)\|A\|_2^2 + \|B\|_2^2}$$

ensures that $\frac{dV(t)}{dt}$ is negative.

(ii) $\phi(z(t)) = 0$, but $z(t) \ne 0$. Then it follows from (15)

$$\frac{dV(t)}{dt} \le -l\varsigma(1-\sigma)\|z(t)\|^2 - \varsigma(1-\sigma)\left\| \sqrt{l}\, z(t) - \frac{1}{\sqrt{l}} B\phi(z(t-\tau(t))) \right\|^2$$

$$-(1-\sigma)\phi^T(z(t-\tau(t)))\Sigma^2\phi(z(t-\tau(t))) < 0.$$

(iii) $z(t) = 0$. Clearly, from (10), we can acquire $\phi(z(t)) = 0$. From (15)

$$\frac{dV(t)}{dt} = -(1-\sigma)\phi^T(z(t-\tau(t)))\Sigma^2\phi(z(t-\tau(t)))$$

$$-\frac{\varsigma}{l}\phi^T(z(t-\tau(t)))BB^T\phi(z(t-\tau(t)))$$

Hence, $\frac{dV(t)}{dt}$ is negative if $\phi(z(t-\tau(t))) \ne 0$, and $\frac{dV(t)}{dt} = 0$ if and only if it happens in the last case where $\phi(z(t-\tau(t))) = \phi(z(t)) = z(t) = 0$. Thus, by the Lyapunov stability theorems [13], we can conclude that the unique equilibrium point x^* of system (3) is GAS. This completes the proof.

Remark 1. Theorem 1 gives an improved sufficient condition for the GAS of the equilibrium point by using the nonsmooth analysis as in [11]. In [4-8], the authors considered the GAS of the equilibrium point of the Cohen-Grossberg neural networks. However, the activation functions are often required to be differentiable and bounded. But, in practice the activation functions are not always to be differentiable and bounded. In addition, the authors in [9] have studied the GAS conditions for the Cohen-Grossberg neural networks with the constant delays by employing nonsmooth analysis. But in practice, time delay is usual time-varying. Then, in this paper we remove the boundedness and differentiability of the activation functions by nonsmooth analysis for the Cohen-Grossberg neural networks with time-varying delays which implies our results to generalize and improve the previous ones.

Let $\tau(t) = \tau$. Then, the system (3) will evolve into the system (2) that Theorem 1 can reduce to the following Corollary 1.

Corollary 1. Suppose that the conditions (M_1) , (M_2) and (M_3) are satisfied and there exist two positive diagonal matrices $D = diag(d_1, d_2, \dots, d_n)$, $\Sigma = diag(\gamma_1, \gamma_2, \dots, \gamma_n)$ and a positive η such that

$$\mu_2(DA) < \frac{l}{k}\lambda_{\min}(D) - \frac{\eta}{2}\lambda_{\max}(\Sigma^2) - \frac{\|DB\|_2^2}{2\eta\lambda_{\min}(\Sigma^2)} \tag{21}$$

is satisfied. Then, for each input vector $u \in R^n$, system (2) has a unique equilibrium point GAS.

Remark 2. Corollary 1 in this paper is equivalent to Theorem 1 in [9], so Theorem 1 in this work also includes Theorem 1 in [9] as a special case.

Next, we give some corollaries when f_i is taken of the form

$$f_i(\delta) = \frac{1}{2}(|\delta + 2| - |\delta - 2|), \tag{22}$$

and

$$f_i(\delta) = \max\{0, \delta\} \tag{23}$$

to illustrate the significance of Theorem 1.

Corollary 2. (With $\eta = \lambda_{\min}(A) - u_2(AP)$) Suppose that each activation function f_i is given by (22) or (23) with $C = I$, where I is identity matrix in $R_{n \times n}$, and there exist two positive diagonal positive diagonal matrixes $D = diag(d_1, d_2, \dots, d_n)$ and $\Sigma = diag(\gamma_1, \gamma_2, \dots, \gamma_n)$ such that

$$\|DB\|_2 < \lambda_{\min}(\Sigma)(\lambda_{\min}(D) - \mu_2(DA))\sqrt{2 - 2\sigma - (1-\sigma)^2\lambda_{\max}(\Sigma^2)} \tag{24}$$

is satisfied. Then, each input vector $u \in R^n$, system (3) has a unique equilibrium point which is GAS.

Proof. From (22), (23) and $C = I$, we can get $l = k = 1$. By (7) that we can acquire

$$\mu_2(DA) < \lambda_{\min}(D) - \frac{\lambda_{\min}(D) - \mu_2(DA)}{2}(1-\sigma)\lambda_{\max}(\Sigma^2)$$
$$- \frac{\|DB\|_2^2}{2(\lambda_{\min}(D) - \mu_2(DA))(1-\sigma)\lambda_{\min}(\Sigma^2)}.$$

Then,

$$\|DB\|_2^2 < 2(\lambda_{\min}(D) - \mu_2(DA))(1-\sigma)\lambda_{\min}(\Sigma^2)\lambda_{\min}(D)$$

$$-2\left(\lambda_{\min}(D)-\mu_2(DA)\right)\mu_2(DA)(1-\sigma)\lambda_{\min}\left(\Sigma^2\right)$$
$$-\left(\lambda_{\min}(D)-\mu_2(DA)\right)^2(1-\sigma)^2\lambda_{\min}\left(\Sigma^2\right)\lambda_{\max}\left(\Sigma^2\right)$$
$$=\left(\lambda_{\min}(D)-\mu_2(DA)\right)^2\lambda_{\min}\left(\Sigma^2\right)\left(2-2\sigma-(1-\sigma)^2\lambda_{\max}\left(\Sigma^2\right)\right).$$

So (24) is satisfied. Next similar to the proof of Theorem 1, the result follows immediately.

Remark 3. By the viewpoint of nonsmooth analysis, we argue that Corollary 2 provides the best possible bounds on $\|B\|_2$. Then, in some cases our results can be seen as an improvement in [11].

Corollary 3. Suppose that the conditions (M_1), (M_2) and (M_3) are satisfied and there exist two positive diagonal matrices $\Sigma = diag(\gamma_1, \gamma_2, ..., \gamma_n)$ and a positive η such that

$$\mu_2(DA) < \frac{l}{k} - \frac{1}{2}\eta(1-\sigma)\lambda_{\max}\left(\Sigma^2\right) - \frac{\|B\|_2^2}{2\eta(1-\sigma)\lambda_{\min}\left(\Sigma^2\right)} \tag{25}$$

is satisfied. Then, each input vector $u \in R^n$, system (2) has a unique equilibrium point which is GAS.

Proof. The proof follows the deduction in Theorem 1 with the matrix $D = I$.

Remark 4. It is noted that Corollary 3 gives an easy way to find feasible matrix Σ with the only consideration of the largest and smallest eigenvalues of Σ. This corollary can be applied to high-dimension neural networks efficiently.

4. Illustrative Example

Now, we give one example to present the merits of our result.

Example 1. Consider the following Cohen-Grossberg neural networks:

$$\begin{cases} \frac{dx_1}{dt} = -\left(4+\sin x_1(t)\right)\left[b_1\left(x_1(t)\right)+3f_1\left(x_1(t)\right)-2f_2\left(x_2(t)\right)\right. \\ \qquad \left. +2f_1\left(x_1\left(t-\tau_1(t)\right)\right)+2f_2\left(x_2\left(t-\tau_2(t)\right)\right)-4\right] \\ \frac{dx_2}{dt} = -\left(4+\cos x_2(t)\right)\left[b_2\left(x_2(t)\right)+4f_1\left(x_1(t)\right)+5f_2\left(x_2(t)\right)\right. \\ \qquad \left. +f_1\left(x_1\left(t-\tau_1(t)\right)\right)-f_2\left(x_2\left(t-\tau_2(t)\right)\right)-4\right] \end{cases}, \tag{26}$$

where $b_i\left(x_i(t)\right) = -x_i(t)$, $f_i\left(x_i(t)\right) = \tanh\left(x_i(t)\right)$, $\tau_i(t) = 0.5\sin t$ for $i = 1, 2$. It is easy to see that $l = 1$ and $k = 1$,

$$A = \begin{bmatrix} -3 & 2 \\ -2 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & -2 \\ -1 & 1 \end{bmatrix},$$

we consider the case $D = a\mathrm{I}$ and $\Sigma = b\mathrm{I}$, (I is the identity matrix) which the inequality in Theorem 1 becomes

$$\mu_2(A) < 1 - \frac{\eta(1-\sigma)b^2}{2a} - \frac{a\|B\|_2^2}{2\eta(1-\sigma)b^2}. \tag{27}$$

Let $\eta = 1, \sigma = 0.5$. We can calculate $\mu_2(A) = -3$, $\|B\|_2^2 = 8$ and

$$1 - \frac{\eta(1-\sigma)b^2}{2a} - \frac{a\|B\|_2^2}{2\eta(1-\sigma)b^2} = -2.42$$

by choosing $\frac{b^2}{a} = 3$ which satisfies (27). So by virtue of Theorem 1, System (26) has a unique globally asymptotically stable equilibrium point.

Remark 5. Obviously, the time delays considered in [9, 10] are constant, so their stability criteria cannot be applied to systems with time-varying delays. Then the criteria proposed in this paper are very effective and an improvement over [9, 10]. Furthermore, when $\max \tau_1(t) = \max \tau_2(t) = 0.5$, Figure 1 depicts the time responses of state variables $x_1(t)$ and $x_2(t)$. It confirms that the proposed condition leads to the global asymptotic stability for the model.

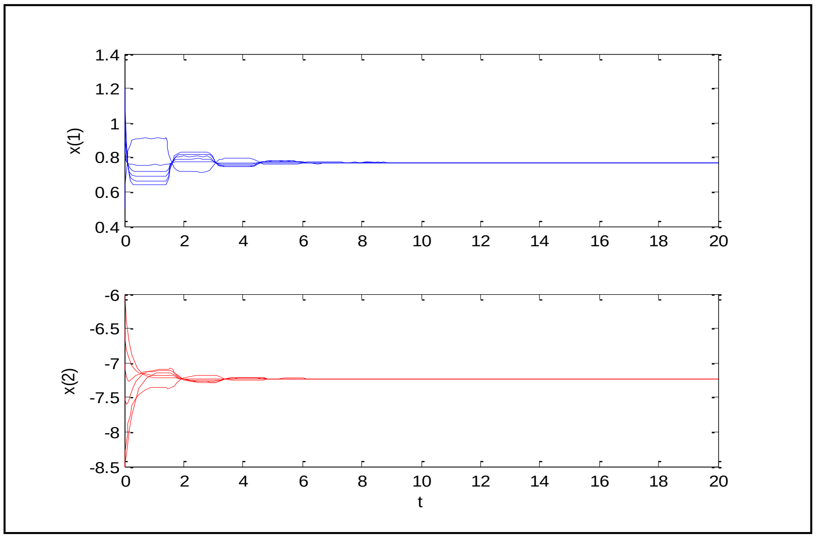

Figure 1. Stability Regions in Example 1 with $\tau_1(t) = \tau_2(t) = 0.5$

5. Conclusion

This paper has discussed the global asymptotic stability for the Cohen-Grossberg neural networks with time-varying delays. By constructing suitable Lyapunov functional and employing nonsmooth analysis, some sufficient conditions are obtained without demanding the boundedness and differentiability of the activation functions. A simulation example is shown the effectiveness of the proposed method.

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