

# A Novel Image Reconstruction Algorithm based on Population Entropy and Adaptive Differential Evolution for Electrical Capacitance Tomography

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## Abstract

*To solve the "soft field" effect and the ill-posed problem in electrical capacitance tomography technology, a novel image reconstruction algorithm based on population entropy and adaptive differential evolution for Electrical Capacitance Tomography is proposed in this study. The algorithm uses all the gray pixels as the initial population's individual. After finite iterations, the algorithm mutates and makes crossover of the population in order to obtain the optimal species populations. That is the optimal value for the ECT imaging pixels. The population entropy and the variation factor make the range of each searching generation decreasing. In the simulation, the improved adaptive differential evolution algorithm will be compared with the LBP algorithm. The result shows that the new algorithm has better image quality and more stable boundary than the LBP Algorithm, which provides a new way to reconstruct images for ECT.*

**Keywords:** *electrical capacitance tomography; image reconstruction algorithm; adaptive differential evolution algorithm; entropy of the population*

## 1. Introduction

Process capacitance tomography as a new technology in the field of scientific research today is paid more and more attention. Especially, it has a lot of active and wide applications in the electrical capacitance tomography. It also has many advantages, such as simple structure, low cost, non-contact measurement *etc.* It is widely used in the industrial process of multiphase nondestructive testing [1-2].

Whether ECT measurement technology can be successfully applied or not depends on the speed and accuracy of image reconstruction to some degree. At the present stage, ECT image reconstruction algorithms mainly include linear back projection algorithm (LBP), Tikhonov regularization method [3] and singular value decomposition (SVD) [4], these methods are non-iterative algorithm. In addition, there are conventional iterative image reconstruction algorithms, such as Landweber iterative method [5], Conjugate Gradient method [6] and the novel intelligent optimization image reconstruction algorithms (artificial neural network and genetic algorithm) [7]. The LBP algorithm is widely applied in ECT image reconstruction because of its high speed of reconstruction and rehabilitation thought, but it has a large error in the image reconstruction. The standard Tikhonov regularization method is one of effective means to solve the ill-posed problem, but its effect is not satisfactory in ECT. The reason is the excessive smooth that means it tends to generate a

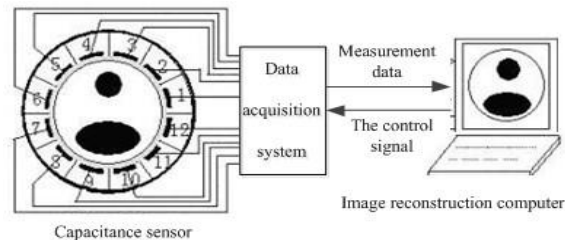
smooth approximation solution. So it will lose some unsmooth information, and the lost detailed information can lead the image spatial resolution to be low.

Landweber algorithm has good image reconstruction quality, which is one of the steepest descent methods. Although it has better image quality, the disadvantage of slow convergence is clearly not suitable for imaging environment which requires higher real-time. Genetic algorithm is an intelligent algorithm which simulates biological evolution and genetic. This algorithm gets an individual with the best fitness through the selection and evolution of the initial population. Then it uses it as the optimal solution of genetic algorithm. However, this algorithm has the problem of premature convergence. The problem is embodied in premature loss of the population diversity, and it will fall into locally optimal solution. The fitness stopped changing and it also cannot find the global optimal solution. The genetic algorithm may have the problem of slow convergence, and it even cannot converge to the global optimal solution.

This study presents an adaptive differential evolution algorithm which is applying to the image reconstruction of electrical capacitance tomography after analyze the ECT principle. The algorithm is not only based on real coding with the characteristics of global optimization but also overcomes the problem of precocity of genetic algorithm. The population entropy and various mutagenic factors reduce the searching range of each generation. Compared with the above iterative and non-iterative algorithms, this algorithm do not need calculate derivative. It has less control parameters and it is easier to understand and realize.

## 2. The System Principles of Electrical Capacitance Tomography

Typical capacitance tomography system mainly includes three parts: capacitive sensors, data acquisition systems and imaging computer. 12-electrode capacitance system is used as an example, and the structure of the system is shown in Figure 1 [8-10]. The principle of ECT image reconstruction is that when the distribution of the medium in the pipe changes, the capacitance between the capacitor plates also will change, thus it can calculate the media distribution in the pipe by using the inversion algorithm according to the measured capacitance value.



**Figure 1. Structure of the System**

The electrode capacitance sensors in the clockwise are marked as 1, 2 ....12. For the N electrode sensor system, the total number M of independent electrode can be gotten according to the formula (1):

$$M = C_N^2 = N \cdot (N - 1) / 2 \quad (1)$$

In the 12-electrode capacitance system, 66 independent capacitance measurement data can be obtained.

Currently, the image reconstruction method of ECT is based on the linear model which is from permittivity to capacitors, and the model is simplified, discrete and normalized. Then the formula is as follows:

$$SG = C \quad (2)$$

In the formula,  $C$  is normalized capacitance measurement field matrix of  $m \times 1$ ;  $G$  is normalized permittivity distribution field matrix of  $n \times 1$ , and it represents the gray values of the image in the image reconstruction;  $S$  is sensitive field matrix of  $m \times n$ , and it reflect the changes of capacitance  $C$  which is impacted by the distribution of substance  $G$  [11-12].

### 3. Principle of Imaging Algorithm

For ECT inverse problem of pathological features, this study introduces population entropy and variable variation factor's adaptive differential evolution algorithm.  $NG$  is the scale of the population, and it must be set carefully. If  $NG$  is too large, it will slow down the algorithm's convergence rate. But if it is too small, it will speed up the convergence rate quite quickly, and it can also miss the global optimal solution.

This study selects Weighted and Amended Gauss-Newton's (WAGN) iteration as the initial population [13].

WAGN iterative formula is:

$$G_{k+1} = G_k - \omega_k \alpha_k [S^T S + \lambda_k I]^{-1} S^T (SG_k - C) \quad (3)$$

In the formula,  $\lambda > 0$  and it is the regularization parameter.  $I$  is a unit field matrix  $m \times n$ .  $\alpha_k$  is a one-dimensional search factor.  $\omega$  is the weight matrix and its value is:

$$\omega = \beta \cdot \text{diag} \left( \left| \frac{1}{\Delta \rho_1} \right|, \left| \frac{1}{\Delta \rho_2} \right|, \dots, \left| \frac{1}{\Delta \rho_N} \right| \right) \quad (4)$$

$\Delta \rho_i$  is calculated from the  $i$ -th component of  $-[S^T S + \lambda_k I]^{-1} S^T (SG_k - C)$ .  $\beta$  is the scale factor and its value can be obtained by using the following formula:

$$\beta^2 \cdot \sum_{i=1}^N \frac{1}{|\Delta \rho_i|^2} = N^2 \quad (5)$$

The solution of each step of iterative of Gauss-Newton algorithm is used as the simulated initial population  $CG_i (i = 1, 2, \dots, NG)$  of ECT. The  $T$ -th generation and  $i$ -th species is represented as follows:

$$CG_i^t = \{tg_{i1}^t, tg_{i2}^t, tg_{i3}^t, tg_{i4}^t, \dots, tg_{id}^t\} \quad (6)$$

Multiply species: three mutually different integers  $k_1, k_2, k_3$  are selected from populations,  $k$  belongs to  $NG$  and it is different from  $i$ . The selected species are mutated and the variability species can be gotten as follows:

$$VG_i^{t+1} = CG_{k_3}^t + F \cdot (CG_{k_1}^t - CG_{k_2}^t) \quad (7)$$

$$VG_i^t = \{vg_{i1}^t, vg_{i2}^t, vg_{i3}^t, vg_{i4}^t, \dots, vg_{id}^t\} \quad (8)$$

Then it cross the species in order to increase the diversity of the population, and the test vectors can be obtained through the following way:

$$ug_{ij}^{t+1} = \begin{cases} vg_{ij}^{t+1}, & rand(j) \leq CR \text{ or } j = randn(i) \\ cg_{ij}^t, & rand(j) > CR \text{ and } j \neq randn(i) \end{cases} \quad (9)$$

$$UG_i^t = \{ug_{i1}^t, ug_{i2}^t, ug_{i3}^t, ug_{i4}^t, \dots, ug_{id}^t\} \quad (10)$$

$CR$  is the crossover probability constant.  $rand(j) \in [0, 1]$  and it is uniformly distributed random number.  $F$  is variability factor. The speed of convergence is significantly affected by the selection of crossover factor and mutation factor, its value is typically between 0 and 1. If values of crossover factor and mutation factor were great, it is easy to fall into local optimum and premature though it can accelerate the convergence of the algorithm. So in the course of this study it presents a time-varying mutation factor to the adaptive differential evolution algorithm based on population entropy, and it is described as follows:

$NG$  is the scale of the population,  $f_{\min}$  and  $f_{\max}$  are the maximum and minimum fitness values of each generation. The solution space of the objective function is divided into  $N$  different areas as  $U_i, i = (1, 2, 3, \dots, N)$ , and the dividing rules is as follows:

$$U_i \in \left\{ f_{\min} + (i-1) \frac{f_{\max} - f_{\min}}{N}, f_{\min} + i \frac{f_{\max} - f_{\min}}{N} \right\}$$

If there are  $N_i$  groups of species' fitness values belonging to  $U_i$ , so the probability of species appearing in the  $i$ -th area is described as following according to the definition.

$$p_i = \frac{N_i}{N} \quad (11)$$

The  $t$ -th generation of population entropy is defined as:

$$S_t = - \sum_{i=1}^N p_i \ln p_i \quad (12)$$

The control parameter  $S$  is set as follows:

$$S_r = 0.98 \exp \frac{-t}{t_{\max}} \quad (13)$$

Then the value range of single element of each species is as:  $x_j \in [r_j^{\min}, r_j^{\max}]$ ,  $j = 1, 2, 3, \dots, n$ ,  $CN_{t,j}^{\min}$  and  $CN_{t,j}^{\max}$  are the minimum and maximum values of  $j$ -th element, they are from the  $t$ -th generation population's first 70% fitness species. When  $S_t / S < S_r$ , it will change the population search range:

$$r_j^{\min} = w CN_{t,j}^{\min} + (1-w)r_{t,j}^{\min} \quad (14)$$

$$r_j^{\max} = w CN_{t,j}^{\max} + (1-w)r_{t,j}^{\max} \quad (15)$$

The factor  $F$  is set as dynamic variation factors, and it will decrease with the increasing of iteration. It is described like equation (16).  $ci$  is the maximum number of iterations, and  $i$  is the current iteration number.  $F_{\max}$  and  $F_{\min}$  are the maximum and minimum values of the adaptation.

$$F = F_{\max} - \frac{F_{\max} - F_{\min}}{ci} \quad (16)$$

The next generation of the species is as follows:

$$CG_i^{t+1} = \begin{cases} UG_i^{t+1}, & f(UG_i^{t+1}) < f(CG_i^t) \\ CG_i^t, & f(UG_i^{t+1}) \geq f(CG_i^t) \end{cases} \quad (17)$$

Mutation and crossover constantly repeat until it is the maximum generation, then the value of G which is needed in ECT can be obtained.

The image is corrected from the physical meaning, and the formula (18) is described as follows:

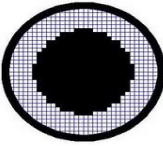
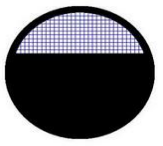
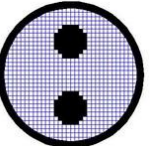
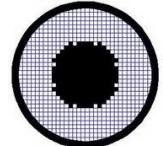

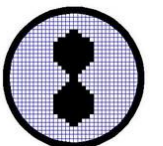
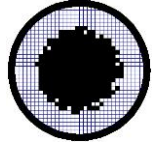
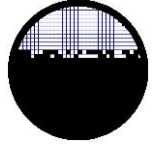
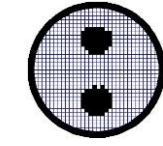
$$G_i^{(k)} = \begin{cases} 0 & \text{if } G_i^{(k)} \leq 0 \\ G_i^{(k)} & \text{if } 0 < G_i^{(k)} \leq 1 \\ 1 & G_i^{(k)} > 1 \end{cases} \quad (18)$$

In the formula,  $G_i^{(k)}$  is the optimal population of i-th component.

#### 4. Results of Simulation and Experiment

In order to verify the effectiveness of the algorithm, simulation test is done with the 12-electrode. In this study, pipeline section is divided into 1024 pixels with  $32 \times 32$  grid, and the effective cross-sectional area of the pipe has 856 pixels. Three typical flow patterns are used in the numerical simulation, and they are the core flow, high-level flow and multi trickle. The comparison and verification will be made between the new algorithm of this study and LBP algorithm, then the results are shown in Table 1 (the dark area is water, and light areas are oil).

**Table 1. Results of Image Reconstruction**

a	b	c
		
LBP		
		
New Algorithm		
		

Simulation applications are performed on computers with Core (TM) 2 Duo CPU, 2 GHz, and 1.00GB memory by using MATLAB 7.0. In order to analyze and evaluate the quality of the reconstructed image, the quality evaluation of the image is defined as follows:

$$\eta = \frac{\sum_{i=1}^n |g_i(img) - g_i(init)|}{\sum_{i=1}^n g_i(init)} \quad (19)$$

$g_{img}$  represents the calculated result,  $g_{init}$  represents the original image vector,  $i$  represents split unit number,  $n$  is the total split unit. As can be seen from the experiment, the new algorithm can make the high-level flow, multi trickle and the core flow all show good imaging effect. Errors of the image are shown in Table 2.

**Table 2. Error (%)**

	a	b	c
LBP	38.42	49.68	76.92
New Algorithm	25.56	34.97	19.62

According to the imaging results in Table 1 and Table 2, the results show that the imaging principle of LBP algorithm is simple, but the imaging effect is poor. Especially, when the error rate of multi trickle is as high as 76.92%, there is a large gap between the reconstructed image and original flow pattern. So the new algorithm has better imaging accuracy and quality than LBP algorithm.

## 5. Conclusion

This study proposes a new algorithm based on population entropy and variable variation factor's adaptive differential evolution. Firstly, the algorithm gets the initial population according to WAGN algorithm. Then it carries the initial population into the new algorithm to mutation, crossover and selection. So it can get the optimal population, and that is the needed pixels value of ECT imaging. For the simple standard flow pattern, the results of numerical experiments show that the new algorithm has higher image quality and smaller error rate than LBP algorithm in the image reconstruction of ECT. The algorithm solves the problem of premature phenomenon in traditional genetic algorithm, and it overcomes the difficulty of the selection of regularization parameter and the design of stable functional in Tikhonov algorithm. The algorithm is different from the traditional binary-coded genetic algorithm, but it encodes with the real number. This algorithm has the advantage of simple programming and global convergence etc. In addition, it is easy to implement. So it provides a new and effective solution for ECT image reconstruction.

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