

# Nonlinear Robust Adaptive Control of Electro-hydraulic Position Servo System

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## Abstract

*For the influence of time varying parameter on servo accuracy of electro-hydraulic position servo system, nonlinear robust adaptive controller with the ability of restraining the parameter perturbation for symmetry valves-control symmetry cylinder system is put forward which is based on the Lyapunov stability theory, The nonlinear state equation of symmetry valves-control symmetry cylinder system was derived, and the controller design process was given. the control strategy was verified on Co-simulation of MATLAB & AMESim and semi-physical simulation platform respectively, the simulation and experimental results show that the tracking performance of electro-hydraulic position servo with nonlinear robust adaptive controller meet the double ten index, and verify efficiency of the method presented through simulations.*

**Keywords:** *Electro-hydraulic position servo, nonlinear control robust control, adaptive control this section*

## 1. Introduction

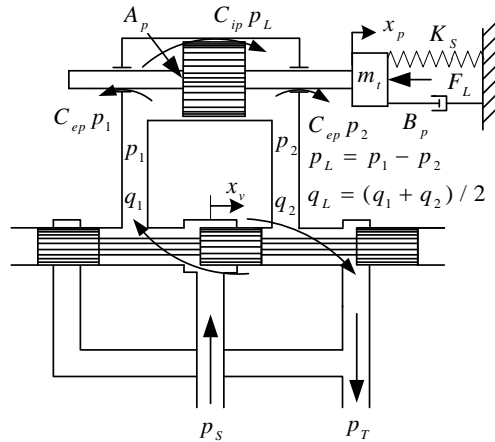
Electro-hydraulic position servo system is a typical nonlinear time-varying system, according to the characteristics of the electro-hydraulic position servo system, researchers combined with advanced control algorithms and had achieved good control effect in different applications. Some scholars made the flow equation of a servo valve under certain conditions and then carried out the electro-hydraulic position servo system control [1-4], with further research, nonlinear control methods have been widely applied in electro-hydraulic servo system such as sliding mode control method [5-8], state feedback linearization method [9] and backstepping method [10-11].

The mature theory which the flow equation of a servo valve under certain conditions is linearized could be applied directly to the electro-hydraulic servo system, but when conditions are more complex, this method would bring greater control error, sliding mode control was easy to produce jitter although it had a strong anti-interference ability, feedback linearization method needed to know the exact dynamics model of the system [12], and the application usually required a combination with other methods. Backstepping method had a clear design idea, but the controller was more complex, and had large amount of conclusions, so it could not be directly applied to the electro-hydraulic servo system, it required the use of a certain simplified method [13]. For the characteristics of existing methods, this paper presented nonlinear robust adaptive controller of electro-hydraulic position servo system combining with electrical characteristics of hydraulic servo system, which could effectively suppress the parameters influence on system performance. The controller had a simple structure, and it was easy to design and implement and so on. The simulation and

experimental results showed that this controller could change the parameters of the system when the system was less than 10% of the amplitude attenuation and phase lag was less than  $10^\circ$ , obtaining a better control performance.

## 2. Electro-hydraulic Position Servo System Modeling

The diagram of symmetry valves control symmetry cylinder system was shown in figure 1, where  $p_s$  is the hydraulic supply pressure,  $p_r$  is the return oil pressure,  $x_p$  is piston displacement,  $x_v$  is servo valve spool displacement,  $q_1$  is flow into chamber 1,  $q_2$  is flow into chamber 2,  $q_L$  is load flow defined as  $q_L = (q_1 + q_2) / 2$ ,  $p_1$  and  $p_2$  are pressure in the two chambers,  $p_L$  is load pressure defined as  $p_L = p_1 - p_2$ ,  $m_i$  is equivalent mass of piston rod,  $B_p$  is equivalent impedance coefficient,  $K_s$  is spring stiffness,  $F_L$  is external interference force of hydraulic cylinder.



**Figure 1. Principle Diagram of Valves-Control Cylinder System**

The pressure—flow equation of symmetry servo valve is described by

$$q_L = C_d w x_v \sqrt{\frac{1}{\rho} \left[ p_s - \frac{x_v}{|x_v|} p_L \right]} \quad (1)$$

Where  $C_d$  is flow coefficient,  $w$  is area gradient of servo valve,  $\rho$  is oil density.

Define the sign function  $\text{sign}()$  as:

$$\text{sign}(x_v) = \begin{cases} 1 & x_v \geq 0 \\ 0 & x_v < 0 \end{cases} \quad (2)$$

The equation (1) can be rewritten by

$$q_L = C_d w x_v [\text{sign}(x_v) \sqrt{p_s - p_L} + \text{sign}(-x_v) \sqrt{p_s + p_L}] \sqrt{\frac{1}{\rho}} \quad (3)$$

The flow continuity equation of hydraulic cylinder is described by

$$q_L = A_p \dot{x}_p + C_{ip} p_L + (V_t / 4 \beta e) \dot{p}_L \quad (4)$$

Where:  $A_p$  is effective area of cylinder,  $C_{ip}$  is total flow-pressure coefficient defined as  $C_{ip} = C_{ip} + C_{ep} / 2$ ,  $C_{ip}$  is internal leakage coefficient,  $C_{ep}$  is external leakage coefficient,  $V_t$  is total effective volume of hydraulic cylinder ( $\text{m}^3$ ),  $\beta e$  is bulk modulus of oil (Pa).

The force balanced equation of hydraulic cylinder is described by [14-16]

$$A_p p_L = m_t \ddot{x}_p + B_p \dot{x}_p + K_s x_p + F_L \quad (5)$$

Define the state variables as  $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^T = [x_p \quad \dot{x}_p \quad \ddot{x}_p]^T$ , define  $x_v = K_a u$ , the state space equation of the electro-hydraulic position servo system as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a u + \psi_2 \theta \end{cases} \quad (6)$$

Where

$$\psi_2 = [x_1 \quad x_2 \quad x_3 \quad 1] \quad , \quad a = \frac{1}{K_a m_t} C_d w [\text{sign}(x_v) \sqrt{p_s - p_L} + \text{sign}(-x_v) \sqrt{p_s + p_L}] \sqrt{\frac{1}{\rho}}$$

$$\theta = \begin{bmatrix} -\frac{4C_{ip} K_s \beta e A_p}{V_t m_t} & \frac{K_s V_t - 4C_{ip} B_p \beta e A_p}{V_t m_t} & -\frac{B_p V_t + 4m_t C_{ip} \beta e A_p}{V_t m_t} & \frac{F_L V_t - 4C_{ip} F_L \beta e A_p}{V_t m_t} \end{bmatrix}^T$$

The parameters  $\beta e$ ,  $C_{ip}$ ,  $B_p$ ,  $K_s$  and  $F_L$  are time varying parameters, and the controller should overcome the impact of time varying parameters to the system.

### 3. Nonlinear Robust Adaptive Controller

Define  $e_p$  and  $x_d$  as tracking error and the demand output signal respectively, the tracking error can be described as

$$e_p = x_1 - x_d \quad (7)$$

Define  $\beta_1 = -e_p + \dot{x}_d$  and  $y = x_2 - \beta_1$ , the equation (6) and (7) can be rewritten as

$$\begin{cases} \dot{e}_p = y - e_p \\ \dot{y} = x_3 - \dot{\beta}_1 \\ \dot{x}_3 = au + \psi_2 \theta \end{cases} \quad (8)$$

According to subsystem  $(e_p, y)$ , the Lyapunov function is selected as

$$V_1 = \frac{1}{2}e_p^2 + \frac{1}{2}y^2 \quad (9)$$

The positive definite function is

$$W_1 = \frac{1}{2}(e_p^2 + y^2) \quad (10)$$

Define the virtual controllers  $x_3 = -e_p - y + \dot{\beta}_1$ , the time derivative of  $V_1$  along subsystem  $(e_p, y)$  is given by

$$\dot{V}_1 = -e_p^2 - y^2 \leq -\frac{1}{2}(e_p^2 + y^2) = -W_1 \quad (11)$$

Thus, the subsystem  $(e_p, y)$  is stable, according to La Sall-Youshizawa theorem [17],  $t \rightarrow \infty$ ,  $e_p \rightarrow 0$ ,  $y \rightarrow 0$ . Namely, the subsystem  $(e_p, y)$  is asymptotically stable.

Define  $Y = x_3 + e_p + y - \dot{\beta}_1$  and further define the follow Lyapunov function as

$$V_2 = \frac{1}{2}e_p^2 + \frac{1}{2}y^2 + \frac{1}{2}Y^2 + \frac{1}{2}(\theta - \hat{\theta})\Gamma(\theta - \hat{\theta}) \quad (12)$$

Where  $\hat{\theta}$  is the estimated value of  $\theta$ ,  $\Gamma = \text{diag}\{\rho_{11} \ \rho_{12} \ \rho_{13} \ \rho_{14}\}$  and  $\rho_{1i} > 0, i = 1, 2, 3, 4$ .

The time derivative of  $V_2$  along subsystem  $(e_p, y, x_3)$  is given by

$$\begin{aligned} \dot{V}_2 &= -e_p^2 - y^2 + yY - (\theta - \hat{\theta})\Gamma\dot{\hat{\theta}} + Y(\dot{x}_3 + \dot{e}_p + x_3 - \dot{\beta}_1 - \ddot{\beta}_1) \\ &= -e_p^2 - y^2 + yY - (\theta - \hat{\theta})\Gamma\dot{\hat{\theta}} + Y(au + \psi_2\theta + \dot{e}_p + x_3 - \dot{\beta}_1 - \ddot{\beta}_1) \end{aligned} \quad (13)$$

Define  $u = \frac{1}{a}(-x_3 - \psi_2\hat{\theta} - x_2 - e_p) - \frac{1}{a}(\dot{e}_p - \ddot{e}_p + \dot{x}_d + \ddot{x}_d + \ddot{\beta}_1)$ , the equation of (13) can be rewritten as

$$\begin{aligned} \dot{V}_2 &= -e_p^2 - y^2 + yY + Y[\psi_2(\theta - \hat{\theta}) - y] - (\theta - \hat{\theta})\Gamma\dot{\hat{\theta}} \\ &= -e_p^2 - y^2 + (\theta - \hat{\theta})^T \Gamma (\Gamma^{-1}\psi_2^T Y - \dot{\hat{\theta}}) \end{aligned} \quad (14)$$

Define  $\dot{\hat{\theta}} = \Gamma^{-1}\psi_2^T Y$ , then  $\dot{V}_2 = -e_p^2 - y^2$ , namely,  $\dot{V}_2 \leq -W_1$ , according to La Sall-Youshizawa theorem,  $t \rightarrow \infty, x_3 \rightarrow 0$ ,

In the conditions of feedback  $u$  and adaptive law  $\dot{\hat{\theta}} = \Gamma^{-1}\psi_2^T Y$ , the system is stable and the tracking error is zero.

### 3. Simulation and Experiment

The main parameters of the electro-hydraulic position servo system which described by equation (6) are as follows: The servo valve rated pressure is 21Mpa ,the rated flow is 30L/m in , rated current is 40mA, the magnitude of -3dB and phase width of -90° is larger than 100Hz. dimension of hydraulic cylinder are 50mm/22mm/±100mm. The servo amplifier gain is  $K_a=0.008A/V$ , displacement sensor gain is  $K_f=50V/m$ . During the experiment, the oil pressure is 6Mpa, the demand signal is  $0.04\sin(4\pi t)m$ . Double ten index was used to evaluate the control effect on tracking performance of electro-hydraulic position servo system, namely, the amplitude attenuation is less than 10% and the phase lag is less than 10°. The validity of the controller which put forward in this paper is verified by two groups of comparison experiments which have been carried out on the same controlled object with Non-adaptive controller and adaptive robust controller.

#### 3.1. Simulation

As The validity of control strategy was verified on Co-simulation of MATLAB & AMESim platform [18-19], the nonlinear robust adaptive controller and the hydraulic system were designed in Matlab and AMESim respectively, the initial inertial load was 50kg and the initial elastic load was  $6.25 \times 10^4 N/m$ , the latter was 150kg and  $1.25 \times 10^5 N/m$  respectively.

The position tracking curve with non adaptive controller and adaptive controller were shown in Figure 2 and Figure 3 respectively. The abscissa is time, in second, the ordinate is tracking position, in millimeter. Adaptive gain matrix with different system parameters was  $\Gamma = \{10^4 \quad 10^4 \quad 5 \times 10^{-2} \quad 1\}$  and  $\Gamma = \{0 \quad 0 \quad 0 \quad 0\}$  respectively.

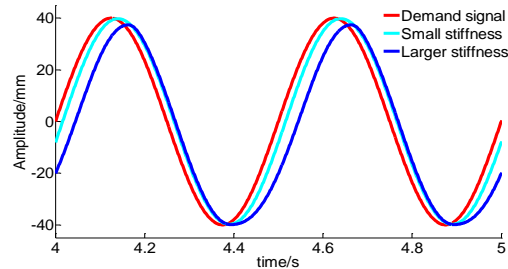
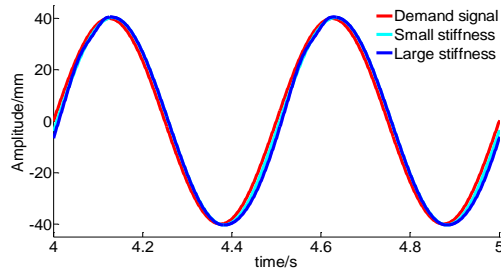


Figure 2. Non-adaptive System Property Curve



**Figure 3. Adaptive System Property Curve**

We can see from Figure 2, the electro-hydraulic position servo system with different inertial load and elastic load has different position tracking performance. In the conditions of initial state, the maximum amplitude attenuation is 2% and the positive and negative travel has the same phase lag and the valve were  $11.52^\circ$ , under the conditions of latter state, the maximum amplitude attenuation is 6.5%, the phase lag in positive and negative travel was  $32.4^\circ$  and  $13.54^\circ$ , respectively. In the conditions of large stiffness, the system has different phase lag in positive and negative travel, it is mainly because of the asymmetric load flow which caused by elastic load.

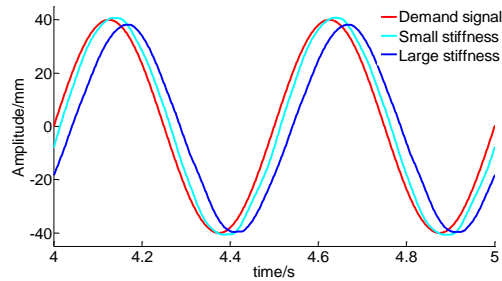
With reference to Figure 3 we know that the nonlinear robust adaptive controller can make the tracking signal amplitude attention and phase lag less than 2% and  $7.2^\circ$  respectively under the conditions of variable parameters, namely, the experiment results satisfies tracking performance. meanwhile, the system has the same phase lag in positive and negative travel and verified the effective of the method presented in this paper.

### 3.2. Experiment

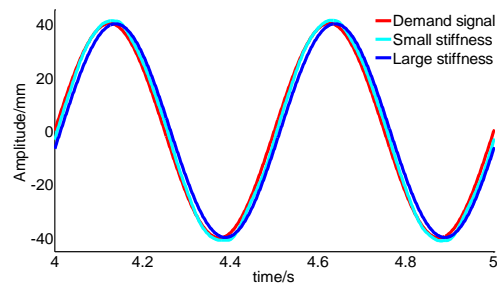
As to further verify the effectiveness of the proposed control strategy, the variable parameters position tracking experiment of electro-hydraulic position servo system was done in laboratory. The Semi-physical simulation platform was shown in Figure 4, the controller was shown in figure a, it was mainly used for acquiring the sensor signal processing the control signal, and finally achieved the matching of different electrical signals. The symmetry valve control symmetry cylinder system was shown in Figure b, and it consist of servo cylinder, servo valve, displacement sensor, force sensor, adjustable inertia load, adjustable elastic load, die block iron and so on. The position tracking curve with Non-adaptive controller and adaptive controller were shown in Figure 5 and Figure 6 respectively,



**Figure 4. Semi-physical Simulation Platform**



**Figure 5. Non-adaptive System Property Curve**



**Figure 6. Adaptive System Property Curve**

With reference to Figure 5 and Figure 6 we can see that the experimental results and the simulation results have the same performance, namely, the nonlinear robust adaptive controller can effectively inhibit the influence which caused by the variety of equivalent load spring stiffness and equivalent load mass on the electro-hydraulic position servo system, and further verify efficiency of the method presented through simulations, further.

#### **4. Conclusions**

1. The nonlinear state equation of electro-hydraulic position servo system was established according to the nonlinear and time varied characteristic of symmetry valve control symmetry hydraulic cylinder.

2. Based on the Lyapunov stability theory, nonlinear robust adaptive controller with the ability of restraining the parameter perturbation was put forward and derived.

3. The validity of control strategy was verified on Co-simulation of MATLAB & AMESim platform, the simulation and experimental results show that the tracking performance of electro-hydraulic position servo with nonlinear robust adaptive controller meet the double ten index, and verify efficiency of the method presented through simulations.

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