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Fuzzy PID Based Trajectory Tracking Control of Mobile Robot and its Simulation in Simulink

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Abstract

The precise control of mobile robot is an important issue in robotics field. In this paper, the motion model of mobile robot is established by mechanism analysis. Then, a fuzzy PID controller is designed for trajectory tracking of mobile robot. The controller consists of a PID controller and a fuzzy inference unit with two inputs and three outputs to tune the parameters of PID controller according to the error and error rate. Finally, the model of a four-wheel mobile robot, fuzzy PID and traditional PID controller are all simulated in Simulink. Simulation experiments are reached in different conditions. The result shows that the mobile robot with the fuzzy PID controller can track the desired trail by about 3 seconds in advance and the overshoot of the system will decrease by 40 percent, comparing to the mobile robot with the traditional PID controller. The advantages of fuzzy PID controller for trajectory tracking control of mobile robot are major in its rapidity, stability, anti-interference and tracking precision.

Keywords: *Mobile Robot, Trajectory Tracking, Fuzzy PID Control, Simulation in Simulink*

1. Introduction

Robot is one of the most popular fields of automation techniques and artificial intelligence techniques. It also represents a new level of manufacturing technology development. Meanwhile, the development of robots asks for higher standard of automation control technology, intelligent technology, sensor technology and manufacturing technology. From birth to now, robot technology has made a considerable progress and development. The development of the robot which can move in work space is incredibly fast and has become a branch of robotics field which is developing prosperously. According to the moving space range, robots can be divided into underwater robots, ground robots, flying robots and space robots, etc. According to driving methods, it can be divided into wheeled mobile robots, tracked mobile robot, and leg mobile robot (including humanoid robots), *etc.* [1].

Mobile robot technology is a combination of sensing technology, controlling technology, information processing technology, machining processing technology, electronic technology, computer technology and many other technologies [2]. In the process of development of robotics, controlling has always been an important issue. The performance of controllers will be directly related to the level of the robot's working ability. Control problems of mobile robot can be divided into two categories: stabilization control and tracking control. Tracking control is an important and practical issue in mobile robot motion control. It can be divided into trajectory tracking control and path tracking control. In the trajectory tracking control, the desired trajectory required by mobile robot is given in graph based on time relation. While

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in path tracking control, the desired trajectory is described by geometric parameters which can be easily achieved (such as the arc of the path). Trajectory tracking control is necessary when the robot is asked to arrive at a certain location in a certain time. Path tracking control is appropriate when the robot is asked to track the path given by geometric parameters at a certain speed [3].

Among several control methods, PID control is the most common one and used widely, also in robot field. By using PID control, both stability control and tracking control can be achieved. But static error exits in tracking control, and its performance is related to scaling factor Kp [4].

Wu designed a PD controller that the velocity is the control objective to control a guidance mobile robot [5]; Qijun Chen analyzed the stability and robustness of three commonly used robot tracking algorithm based on PD, and compared their control performance [6]. In addition, there are control methods combining PID control and some other control methods. For example, Xinxin proposed an adaptive control method which combined the feedforward and PID feedback, making use of the properties of operator dynamic model and can be applied to tracking control robot [7]. While studying the trajectory tracking control for the uncertain robot and using the combination of PD controller and feedforward control, Daiying designed a Robust Adaptive Controller to ensure stability of the overall situation [8]. Jafarov designed a PID controller with a new variable structure for mechanical hand system of stabilization model with disturbance of parameters [9].

Mobile robot based on PID controller uses the course angle error as the input of the controller, and output of the controller is the robot's driving angle. But in fact, the robot’s course angle is also affected by its velocity, rotational inertia, centre-of-gravity position and cornering coefficient of the front and rear wheel (caused by the wheel which is not strictly perpendicular to the ground). And the difference between diameter of driving wheels with friction and the number of changes and other factors which are difficult to determine in the practical road conditions make it difficult to obtain global coordination of PID controller. The parameters gained from the small turning angle experiment are inappropriate when using in a large turning angle experiment, and vice versa [10]. At the same time, due to various environmental factors, setting the suitable parameters of PID controller is extremely hard to reach. What’s more, it’s less intelligent [11].

Fuzzy control is useful for the control object which has a complex mathematic model, nonlinearity, lag and coupling. Generally fuzzy relation is fuzzificated concept of relationship in mathematics [12].

While fuzzy controller can well adapt to the nonlinear and time variability of controlled object and exhibits good robust properties, its stable control accuracy is poor and not delicate enough. It is difficult to achieve high control precision, especially near the equilibrium point. Meanwhile, it is lack of integral control and hard to eliminate the system static error. To deal with these shortcomings, the basic fuzzy controller is often combined with PID controller in practice in order to use their own character and make the effect more perfect to meet the needs of a variety of industries.

A fuzzy control based PID parameter auto setting controller is developed in this paper. According to the feedback error, parameters of the PID controller such as Kp, Ki, Kd can be tuned. The simulation results show that the controller can reduce the difficulty of PID parameter tuning, improve the accuracy of trajectory tracking control, and improve the adaptability, rapidity and anti-jamming performance of the controlled system. The availability of the method can be ensured by simulation and experimental results.

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2. Kinematics Model of Mobile Robots

In this study, it is assumed that the robot was moving on ideal plane, the ground can be considered as regular, the robot is a rigid body that the deformation of it can be ignored; wheels and the ground can meet the pure rolling conditions without relative sliding.

The position of the robot on a Cartesian plane can be illustrated by Figure 1. Point P is the reference point of the robot body, the coordinate system of the body is defined as XvPYv, position and orientation of the robot can be defined through [xp yp θ]T, where (xp , yp) is the coordinate position of the robot's center in the Cartesian plane, and θ is its course angle [13]. The matrix which used to transform rectangular plane coordinate system to robot's coordinate system is as formula (1).

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

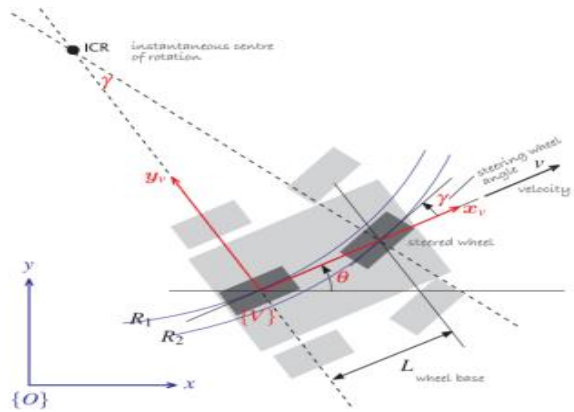

Figure 1. Kinematics Models of Mobile Robots

As the Figure 1 shows, γ is the driving angle, R is the turning radius, L is the distance between the front and rear line, v is the rate [14]. Thus, kinematics models of mobile robots can be established as formula (2) shown below:

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= \frac{v}{L}\tan\gamma \end{aligned} \tag{2}$$

3. Design of Fuzzy PID Controller

3.1. Structure of the Fuzzy self-tuning PID Controller

Combining fuzzy auto tuning PID control and fuzzy control, achieving nonlinear function between error *e* and error rate *ec* and parameters Δk_p、Δk_i、Δk_d through fuzzy inference

system, the PID controller can tune its parameters adaptively when error and error rate change. The principle of this control system is shown in Figure 2.

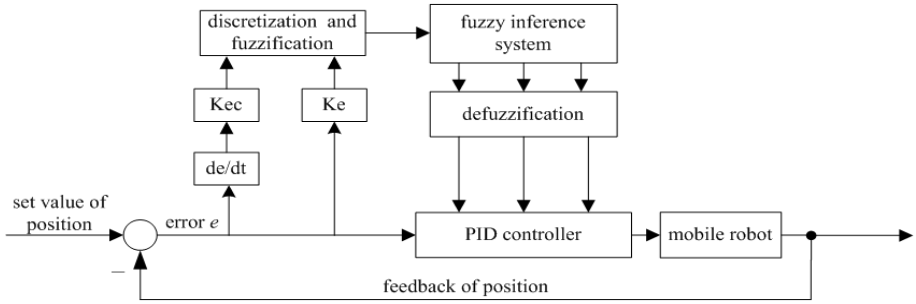

Figure 2. The Principle Diagram of the Mobile Robot Controller

During the operation of control system, continuously detecting e and ec, modify parameters of PID in real time according to the fuzzy control rules, the controlled object can achieve a good dynamic and static performance [15].

Combined with the control objectives of the motion control system, the design of PID parameter auto setting controller's structure is shown in Figure 3. The inputs of the controller are the deviation between the mobile robot's position and the target position *e* and the deviation rate *ec*. The outputs of the controller are the parameter variables Δk_p、Δk_i、Δk_d of PID controller at the next time. With using the PID controller, the velocity *v* and driving angular γ of the mobile robot can be controlled.

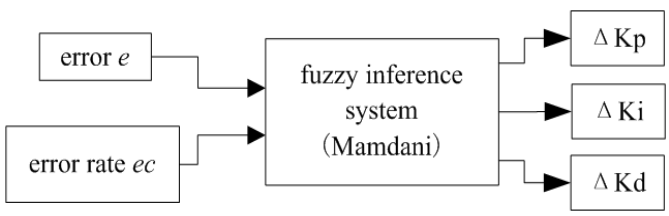

Figure 3. Structure of the Controller Designed for the System

Δk_p、Δk_i、Δk_d are correction parameter. The parameters of the PID controller Δk_p、Δk_i、Δk_d can begotten from the formula (3):

$$\begin{aligned} k_p &= k_p' + \Delta k_p \\ k_i &= k_i' + \Delta k_i \\ k_d &= k_d' + \Delta k_d \end{aligned} \tag{3}$$

Thus, based on the incremental PID control algorithm, we can achieve the transfer function of active setting PID controller by formula (4).

$$\begin{aligned} u(k) = u(k-1) + (k_p' + \Delta k_p)[e(k) - e(k-1)] + \\ (k_i' + \Delta k_i)e(k) + (k_d' + \Delta k_d)[e(k) - 2e(k-1) + e(k-2)] \end{aligned} \tag{4}$$

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3.2. Membership Design of Fuzzy Controller

In this system, input and output of the fuzzy controller variables are the same: {NB (negative big), NM (negative middle), NS (negative small), ZO (zero), PS (positive small), PM (positive middle) , PB (positive big)}. The basic domain of error e is {-180,180}, of error rate ec is {-30,30}, of Δk_p is {0,6}, of Δk_i is{0,1.8 }, of Δk_d is {0,6}.

After formulate language variable and domain, we must determine membership of the fuzzy language variables. Gaussian membership functions and triangular membership function are commonly used as Membership functions. Considering the design should be simple and meet real-time requirements, the system adopts triangular membership function. Figure 4 shows the membership function scatter gram of input variable *e*.

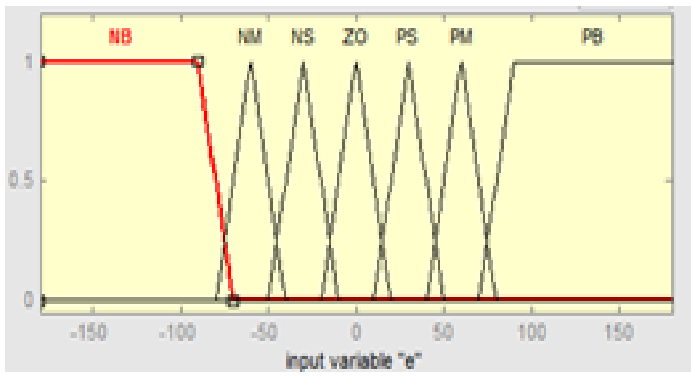

Figure 4. Membership Function Scatter Gram of input Variable e

3.3. Design of Fuzzy Rules

The most important part of designing a fuzzy controller is to summarize the technology and practical operating experiences of the engineering staffs and obtaining fuzzy rule table of the three parameters Kp, Ki, Kd which are tuned separately [16].

Table 1. Engineering Staffs and Obtaining Fuzzy Rule Table

e	*ec*						
	NB	NM	NS	ZO	PS	PM	PB
NB	PB/NB/PS	PB/NB/NS	PM/NM/NB	PM/NM/NB	PS/NS/NB	ZO/ZO/NM	ZO/ZO/PS
NM	PB/NB/PS	PB/NB/NS	PM/NM/NB	PS/NS/NM	PS/NS/NM	ZO/ZO/NS	NZ/ZO/ZO
NS	PB/NB/ZO	PB/NM/NS	PS/NS/NM	PS/NS/NM	ZO/ZO/NS	NS/PS/NS	NS/PS/ZO
ZO	PM/NM/ZO	PM/NM/NS	PS/NS/NS	ZO/ZO/NS	NS/PS/NS	NS/PM/ZO	NM/PM/ZO
PS	PS/NM/ZO	PS/NM/ZO	ZO/ZO/ZO	NS/PS/ZO	NS/PS/ZO	NM/PM/ZO	NM/PB/ZO
PM	PS/ZO/PB	ZO/ZO/PS	NS/PS/PS	NM/PM/PS	NM/PM/PS	NM/PM/PS	NB/PB/PB
PB	ZO/ZO/PB	ZO/ZO/PM	NM/PM/PM	NM/PM/PM	NM/PM/PS	NB/PB/PS	PB/PB/PB

The format of the fuzzy reasoning is: if the error e is NB, the error change rate ec is NB, then Δk_p is PB, Δk_i is NB, Δk_d is PS.

3.4. Defuzzification

What we gained from the fuzzy PID controller is only a fuzzy set. In fact, we must use an exact value to control the controlled objective. Thus, the fuzzy set needs to be defuzzified. The method of defuzzification adopted by the system is area centroid [17].

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4. Simulink Model

4.1. Control Objective Modeling

Establish a model of the control objective in Simulink, input includes velocity v and driving angular γ , output includes the position of the mobile robot (x, y) and the azimuth angle θ. In order to make it easier to set up the control system, encapsulate the model in the mobile robot module, the structure is shown in Figure 5.

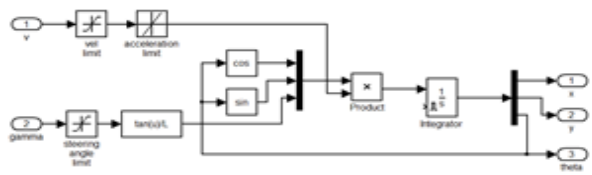

Figure 5. Simulink Model of Mobile Robots

4.2. Trajectory Modeling

The input of the system is a real-time moving point; mobile robot can reach the track by tracing this point. The target trajectory will be set into circle, ellipse and straight line. The size of the circles and ellipses, position of the straight line, speed of the trajectory points can be set arbitrarily.

Model of the trajectory in Simulink is shown in Figure 6.

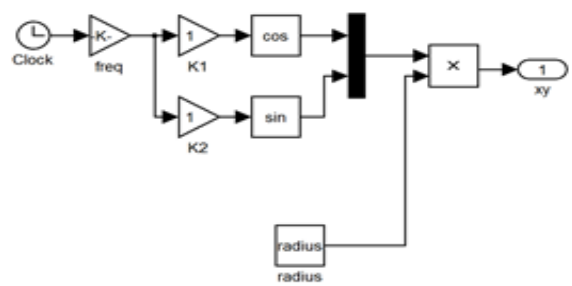

Figure 6. Model of Target Trajectory in Simulink

By changing values of K1 and K2, different trajectory shapes such as ellipse and straight line can be set.

In target tracking, the robot should keep a distance with the moving point, which is set to 0.01in this system. And the following formula (5) is defined as the speed deviation:

$$e = \sqrt{(x^* - x)^2 + (y^* - y)^2} - d^* \tag{5}$$

(x^* , y^* is the coordinate of the moving point in trajectory.)

The deviation which is azimuth error of the robot is defined as formula (6):

$$\theta^* = \tan^{-1} \frac{y^* - y}{x^* - x} \tag{6}$$

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4.3. PID based Mobile Robot Modeling

After the various parts of the mathematical model and simulation model is set up, put them together to obtain a Simulink simulation model. The deviation of position and azimuth should be feedback to the input of PID controller, the controller's output controls velocity and course angle of robots at next second. Its structure is shown in Figure 7.

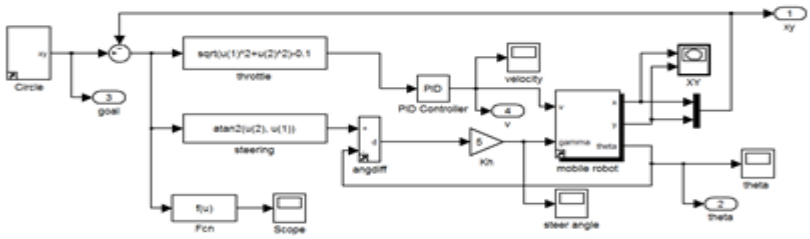


Figure 7. PID based Simulation Model of Mobile Robot

4.4. Fuzzy PID based Mobile Robot Modeling

As for a control system based on fuzzy PID, add fuzzy controller in the front of PID controller. The inputs of fuzzy controller includes feedback of location, azimuth error and error rate, the outputs are the results of parameters Kp, Ki, Kd in real time tuning. After the parameters are tuned, PID controller begins to control the velocity and course angle for the next second angle of the mobile robot. Its structure is shown in Figure 8.

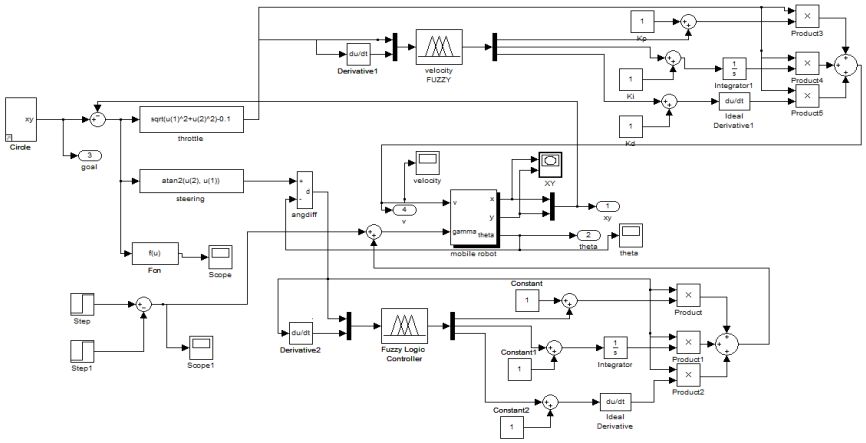


Figure 8. Fuzzy PID based Simulation Model of Mobile Robot

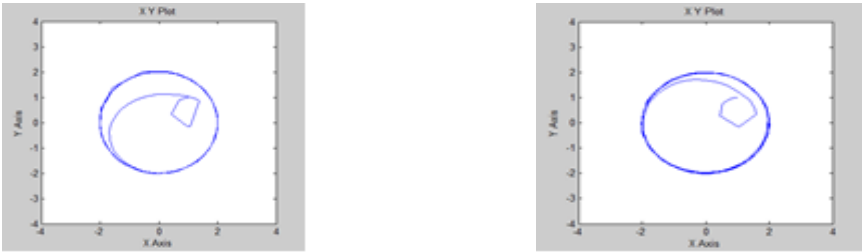
5. Analysis of Simulation and Results in Simulink

5.1. Different Initial Positions

A circle with a radius of 2 is set as tracking trajectory. Set the initial state: the robot's position is point (0, 0), course angle is 0 ° (defined as the 1st condition) and position is (1, 1), course angle is π (defined as the 2nd condition). Observed robot's tracking performance of the two controllers. The tracking trajectory graphs are shown:

(a) PID based (b) Fuzzy PID based

Figure 9. 1st Conditions



(a) PID based (b) Fuzzy PID based

Figure 10. 2nd Conditions

5.2. Compare anti-interference Performance of each Control System

For the two control systems, let us keep the same initial state of the robot (both of them are the 1st condition), and track the circular trajectory. After 10 seconds, artificially add a jamming signal with a strength of 30, width of 1 second in the input of the robot's course angle, and with a strength of 3, width with 1 second in the input of the robot's speed. By simulating robot's tracking performance of the two controllers with jamming signal in speed and course angle, the tracking trajectory graphs are shown:

(a) PID based (b) Fuzzy PID based

Figure 11. Anti-jamming of Course Angle

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(a) PID based (b) Fuzzy PID based

Figure 12. Anti-jamming of Velocity

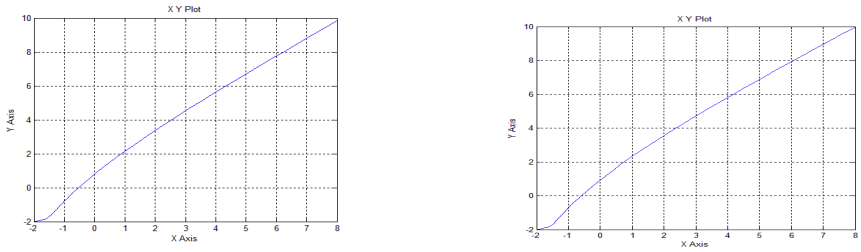
5.3. Different Shape of Tracking Trajectory

5.3.1. Shape of the Tracking Trajectory Is Ellipse: Change the tracking trajectory from circle into ellipse. The formula of ellipse is $x^2/(1.5)^2 + y^2 = 1$. Under the same condition of the initial state, observe trajectory of the two systems:

(a) PID based (b) Fuzzy PID based

Figure 13. Tracking Trajectories in an Ellipse Path

5.3.2. Shape of the Tracking Trajectory is a Straight Line: Change the tracking trajectory from circle into straight line. The formula of ellipse is $y = x + 2$. Under the same condition of the initial state, observe trajectory of the two systems:



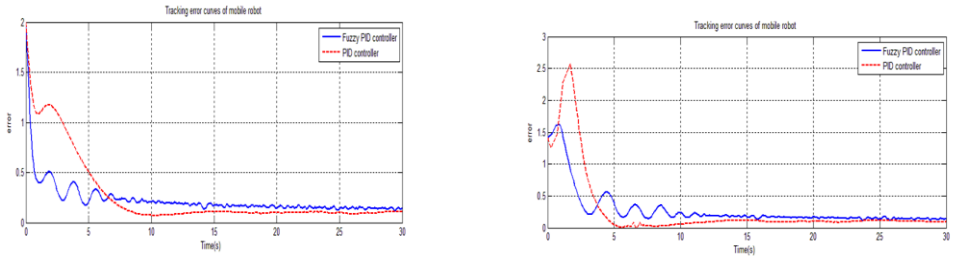
(a) PID based (b) Fuzzy PID based

Figure 14. Tracking Trajectories in a Straight Line Path

5.4. Analysis of the Experimental Results

Suppose that the coordinates of the target locus is (x,y), coordinates of robot's location is (x^*, y^*), the distance $\sqrt{(x-x^*)^2 + (y-y^*)^2}$ between two points is defined as error e. Define time t as abscissa, error e as vertical axis, draw tracking error curves of the two control system in different condition of simulation.

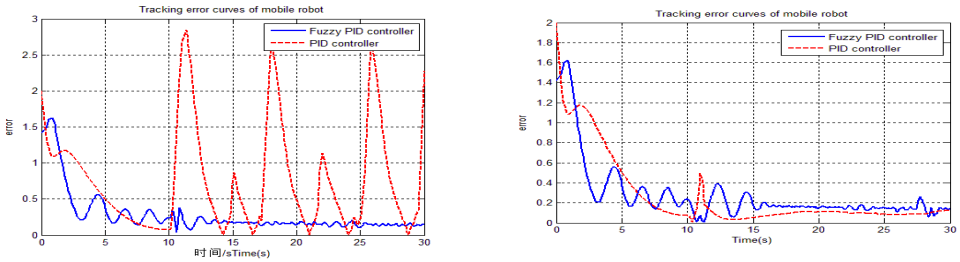
Draw tracking error curves of the mobile robot in different initial state, as shown in Figure 15. From figure (a), the control system can track the target trajectory in 7 seconds after adding fuzzy controller, while it takes 10 seconds for a PID controller to fully track the target trajectory. From figure (b), it is clear that through the analysis that after adding fuzzy controller to the control system, overshoot has reduced by 40%. Therefore, from different initial position and course angle, mobile robot controlled by fuzzy PID control can track target trajectory faster with a smaller overshoot and steady-state error and higher tracking accuracy.



(a) 1st condition (b) 2nd condition

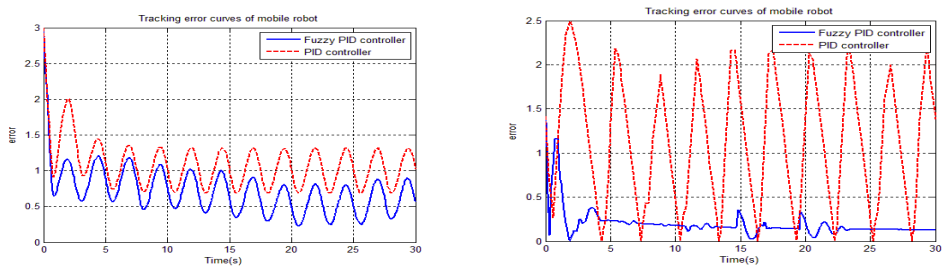
Figure 15. Tracking Error Curves

Similarly, with the interference in course angle and velocity, trajectory tracking errors of the mobile robot can be drawn in Figure 16. According to the two figures, it is clear that the anti-interference performance is very poor when the mobile robot is controlled only by a PID controller, especially in the effect of jamming signals of velocity. At this time, the robot is out of control, and the system is no longer stable. On the contrary, mobile robot has a strong ability to suppress interference and can return to the steady trajectory a short time when using fuzzy PID controller, even if there are interference of velocity and course angle.



(a) Interference of course angle (b) Interference of velocity

Figure 16. Tracking Error Curves with Interference



(a) Ellipse path (b) Straight line path

Figure 17. Tracking Error Curves in a non-Circular Path

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In the experiment of tracking elliptical trajectory and linear trajectory, draw a trajectory tracking error curve of the mobile robot as shown in Figure 17. It is obvious that for non-circular trajectory, tracking effect of the two control systems is really different. Barely using PID controller to control a system will have some trouble in tracing a straight line trajectory, and will have a serious error when tracing ellipse trajectory. But in the control system which is using fuzzy PID controller, mobile robot can still track the non-circular trajectory even if there are errors.

6. Conclusions

From the groups of experiments above, after changing initial state of mobile robot, adding jamming signal in velocity and course angle of mobile robot artificially and changing shapes of the target trajectory, effect of each factor can be achieved by comparing each result. The results show that mobile robot can track the target trajectory faster and more effective with smaller tracking error and higher tracking accuracy, comparing with PID control based mobile robot. Meanwhile, mobile robot which based on fuzzy PID controller has better stability, and improves a lot in anti-interference ability and adaptability to the system, which indicates that fuzzy PID control based mobile robot has many apparent merits in velocity control, stability of system and anti-interference performance in tracking trajectory controlling area.

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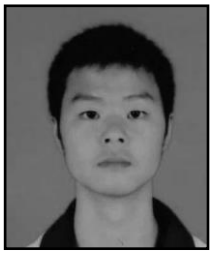

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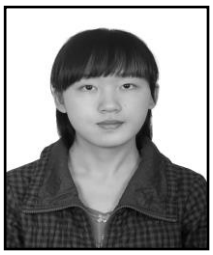

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