# Travel Time Model of a New Compact Storage System

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### Abstract

*In this paper, we do research on a new compact storage system which is an extension of conventional automated storage and retrieval system (AS/RS). The new system consists of the S/R machine which travels simultaneously in the horizontal and vertical dimension. At the same time, there is a conveyor system which is in charge of motion of depth dimension which means there are several storage cells near the aisle. With consideration of operating characteristic, a new travel time model is formulated for the single command cycle. In order to analyze performance and optimal rack dimension, we present the minimum expected travel time and optimal rack dimension. In the end, we verify the travel time model and optimal rack dimension design by a practical example.*

*Key words: Compact storage system, travel time model, AS/RS, rack dimension design*

## 1. Introduction

Automated storage/retrieval systems (AS/RS) are of strong current interest due to such benefits as lower building and land cost, labor savings, reduced inventory levels, and an improved throughput level, among others [1]. This system is made up with an automated crane, which can simultaneously move in the horizontal and the vertical direction. On account of dozens of benefits, AS/RS is being widely used in logistics industry [2]. However, with steady increase on initial investment, land and employment cost, a conventional AS/RS cannot fulfill demand of enterprises, especially in consideration of factors on economy and productivity. As a result, a new structure of AS/RS is put forward to cater to the trend. This new system, with concept of compact storage system, consists of a conventional automated stacker crane for reaching and accessing storage cells in the horizontal and vertical direction. In the depth direction, a powered convey including several storage cells replaces single pallet location in conventional AS/RS. Obviously, in the same condition of space, there are more storage cells and less use of aisles. The compact storage systems have gained popularity for storing standard products in several industries throughout the world. But, a limited range of literature can be found in the field of layout and throughput for compact storage systems. Literature review on conventional AS/RS based on travel-time modeling can provide a research perspective on compact storage systems, though.

The first precise travel-time model for conventional AS/RS is proposed by Bozer and White, which believes optimal rack shape should be square in time and gives typical study views for operation optimization [3]. Bozer comes up with travel-time models based on constant velocities for horizontal and vertical travel. Consequently, Hwang H and Lee Seong B [4] give some amendments for travel-time models considering the operating characteristics. Since then, numerous articles appears on special views of travel-time models, such as multi-shuttle[5], turn-over based or two-class based storage strategy [6], separate input and

output point. In recent years, Gu *et al.*, [7] and De Koster *et al.*, [8] both present related comprehensive reviews of research on warehouse design and performance evaluation. By means of helpful reference of conventional AS/RS, small amounts of papers on compact storage system can be found. Van den Berg and Gademann [2] present a simulation study of a special kind of automated storage/retrieval system and examine a wide variety of control policies. Then, Hu *et al.*, present travel time analysis of a new automated storage and retrieval system, which has one vertical platform and $N$ horizontal platforms to serve $N$ tiers of an AS/RS rack [9]. With regard to two different compact storage system variants for the depth movement of pallets: one with gravity conveyors and one with powered conveyors, De Koster [10] and Yu *et al.*, [11] present optimal rank design based on random storage strategy and two-class storage strategy, respectively. In both papers, they assume storage/retrieval machine (S/R machine) is capable of moving at constant speeds.

In this paper, we firstly describe figures of a new compact storage system. Then, a travel-time model considering operating characteristics is presented. After that, we extend the optimal rack shape in three dimensions condition. Finally, we verify our study results with a practical case.

## 2. A New Compact Storage System

As is illustrated in Figure 1, there is similar automated stacker crane in every aisle in horizontal and vertical dimension. However, in the depth dimension, there are more than one unit load cells between each two aisles, where single pallet is conveyed by a powered conveyor from input side to output end one by one. It means there are amount of storage units in one slot(lane) of a tier. Motion in every dimension is independent. The input and output station (I/O station) is located at the ground level on one corner of the rack.
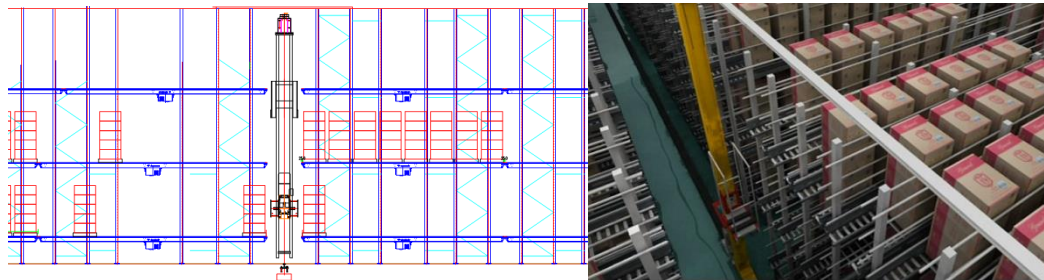


**Figure 1. A New Compact Storage System**

When one pallet needs to be retrieved, the S/R machine travels to the corresponding tier. Simultaneously, the conveyor carrying the first unit load in the tier moves to the retrieval position. When both S/R machine and conveyor stop and get prepared for next action, the fork of S/R machine get the unit load from the retrieval position. Finally the S/R machine carries the unit load to the I/O station.

This structure of compact storage system does not precisely abide by the principal of first-in-first-out (FIFO) since the initial stock is located in the rear end of the lane and the latest stock, however, is retrieved at the first place. Whereas, the FIFO rule does not lead to any negative effect on warehouse operation if all stocks in one lane are parallel under a high turn-over rate. Besides, the innovation of AS/RS rack design maximizes the total number of unit load cells and minimizes the total initial construction investment as a benefit of fewer amounts of stacker cranes and useless aisle room.

**Table 1. Differences between Traditional AS/RS and the New Compact Storage System**

|  | Traditional AS/RS | New Compact Storage System |
|---|---|---|
| Investment | High | Low |
| Flexibility | High | Low |
| Total storage | Low | High |
| Space use ratio | Low | High |

Compared with traditional AS/RS, the new compact storage system offers us many advantages. To explain it explicitly, major differences between them are summarized in Table 1 and further explanations are presented as follows:

- Because of the decrease of total number of stacker cranes, the initial investment on these devices drops sharply. As every single stacker crane is almost the highest valued machine in an automated warehouse, the total investment on purchase of equipment is lower in the new compact storage system.
- As there is more than one storage unit in one lane in the new compact storage system, the total kinds of stocks show an evident decrease depending on number of lanes. As a result, the new compact storage system behaves worse under condition of larger ranges of categories.
- By decreasing the total number of aisles, it is definitely beneficial to mass storage and the quantity of storage units increases compared with conventional AS/RS.
- Using the new AS/RS structure, the storage density is undoubtedly higher. Consequently, the space use ratio grows in the same limited storage room.

## 3. Assumptions and Travel-time Model

### 3.1. Assumptions

To study more precisely in this case, we not only refer to some assumptions which are widely used in previous papers but also take practical operating characteristics into consideration. The fundamental assumptions are made as follows:

- The rack is considered to be a continuous rectangular pick face. Thus, the stacker crane can reach any location in the aisle. The I/O point is located at the lower left-hand corner.
- The mode of stacker crane is single-command basis as there is different performance time window for either inbound or outbound task in the practical warehouse operation.
- Randomized storage is applied in the travel-time model, which means any point within the pick face is equally likely to store or retrieve one unit load.
- The pick-up and deposit time at I/O station for a given load is known and constant. As every unit load is a standard pallet which includes fixed number of cartons, it takes constant time to palletize or unpalletize cartons.
- Job sequence is continuous and there is no prior information for specific task.
- The length of the rack is invariably greater than the height, which is based on empirical results and is a widespread principal in warehouse design.

Apart from these widespread used assumptions in previous research, the following specific assumptions in real world for this new compact storage system are equally brought into consideration:

- The acceleration/deceleration effect of the stacker crane in the horizontal and vertical dimension is considered into our travel-time model and the acceleration/deceleration rate

is also equal. Besides, throughout period of fulfilling job sequence, stacker crane cannot run faster than the maximum velocity and every task operation experiences both acceleration and deceleration period. It means every single storage or retrieval operation has specific speed profit including acceleration/deceleration rate and the maximum velocity restriction.

● The conveyor that is in charge of orthogonal depth motion is assumed to move in a constant velocity.

## 3.2. Travel-time Model

To start firstly, some basic notations are used to standardize the pick face and vehicle movement as follows:

$H$    is the length of the rack in the horizontal dimension.

$V$    is the height of the rack in the vertical dimension.

$D$    is the width of the rack in the depth dimension.

$a_h$    is the acceleration/deceleration rate of horizontal movement.

$a_v$    is the acceleration/deceleration rate of vertical movement.

$v_d$    is the constant velocity of depth movement.

$T_{max\_h}$ is the horizontal travel time required to the farthest column from the I/O station.

$T_{max\_v}$    is the vertical travel time required to the highest tier from the I/O station.

$T_{max\_d}$   is the depth travel time required to the farthest bay from the aisle.

$t_h$    is the horizontal travel time required to the specific location from the I/O station.

$t_v$    is the vertical travel time required to the specific location from the I/O station.

$t_d$    is the time to prepare a outbound unit load or a inbound storage cell.

$t_{x,y}$   is the travel time when a S/R crane to the specific location from the I/O station, thus

$$t_{x,y} = \max(t_h, t_v)$$

As is mentioned as above, stock keeping unit (SKU) in one lane is shared with the same property. Thus, let $x_0$ denote the horizontal travel distance required to one unit load location from the I/O station. Likewise, let $y_0$ represent the vertical travel distance required to the same unit load position from the I/O station and $z_0$ represent the depth travel distance required to the prepared position by the conveyor. This is, every location in one aisle can be represented by $(x_0, y_0, z_0)$, where $0 \leq x_0 \leq H$, $0 \leq y_0 \leq V$ and $0 \leq z_0 \leq D$. To standardize the coordinate, we let $x = x_0 / H$, $y = y_0 / V$ and $z = z_0 / D$. Therefore, the storage (or retrieval) point in our travel-time model is represented by $(x, y, z)$, where $0 \leq x \leq 1, 0 \leq y \leq 1$ and $0 \leq z \leq 1$.

Firstly, if the maximum speed is not yet reached and the acceleration/deceleration rate is equal to $a_s$, the relationship between travel-time ($t$ or $f(s)$) and travel-distance ($s$) is given by $t = f(s) = 2s^{\frac{1}{2}} / a_s^{\frac{1}{2}}$. To be simplified, it is transformed into $t = f(s) = \alpha s^{\beta}$ as acceleration/deceleration rate is constant. Then, for each dimension, $T_{max\_h} = \alpha_h H^{\beta_h}$ and $T_{max\_v} = \alpha_v V^{\beta_v}$, where $\alpha_h, \beta_h, \alpha_v, \beta_v$ denote respective variant in above formula for horizontal and vertical dimension. Besides, the parameter $\beta$ is a positive number that is not greater than 1[12]. Consequently, a specific location, $(x_0, y_0)$, can be represented as follows:

$$t_h = T_{max\_h}(x_0 / H)^{\beta_h} \quad \text{for } 0 \leq x_0 \leq H, \ 0 < \beta_h \leq 1, \tag{1}$$

And

$$t_v = T_{\max\_v}(y_0/V)^{\beta_v} \quad \text{for} \ \ 0 \le y_0 \le V, \ \ 0 < \beta_v \le 1, \tag{2}$$

Using the same method of standardization, then

$$t_h = T_{\max\_h} x^{\beta_h} \quad \text{for} \ \ 0 \le x \le 1, \ \ 0 < \beta_h \le 1, \tag{3}$$

And

$$t_v = T_{\max\_v} y^{\beta_v} \quad \text{for} \ \ 0 \le y \le 1, \ \ 0 < \beta_v \le 1, \tag{4}$$

Now let $T_{\max\_2D} = \max(T_{\max\_h}, T_{\max\_v})$, $b = \min(T_{\max\_h}/T_{\max\_2D}, T_{\max\_v}/T_{\max\_2D})$, which implies $0 \le b \le 1$ [3]. Without loss of generality, assume that $T_{\max\_2D} = T_{\max\_h}$ and $b = T_{\max\_v}/T_{\max\_2D}$. To normalize the equation (3) and equation (4), we can see

$$t_h = x^{\beta_h} \quad \text{for} \ \ 0 \le x \le 1, \ \ 0 < \beta_h \le 1, \tag{5}$$

And

$$t_v = b y^{\beta_v} \quad \text{for} \ \ 0 \le y \le 1, \ \ 0 < \beta_v \le 1, \tag{6}$$

Apparently, a real-world storage operation consists of three basic components, which is also applied in a retrieval operation. A typical retrieval operation is formed as follows:

- Expected time needed for traveling from I/O station to retrieval position and successfully get the pallet back into the S/R machine from the retrieval position, $E(F)$. This is, $E(F)$ includes time needed for waiting for the preparation of conveyor, if necessarily, and the minimum waiting time can decrease to zero.
- Expected time needed for traveling back to I/O station, the dwell point, $E(B)$.
- Expected time needed for palletizing/depalletizing SKUs, $E(P)$, which is considered as a constant and neglected in this travel-time model.

Hence, the total expected travel time, $E$, can be expressed as followed:

$$E = E(F) + E(B) \tag{7}$$

Firstly, by using the same approach for randomized storage as shown in Bozer and White (1984), we derive $E(B)$ that is similar to the case of 2-dimensional rack. Then, we obtain $E(F)$ according to the constant time in depth dimension, $t_d$.

Assuming motions in horizontal and vertical dimension are independent and the coordinate locations are uniformly distributed because of randomized storage, let $G(m)$ denote the probability that travel time to $(x, y)$, say $t_{x,y}$, is not greater than $m$. According to probability theory,

$$G(m) = P(t_{x,y} \le m) = P(t_h \le m)P(t_v \le m) \tag{8}$$

Consequently, we define $\theta_h = 1/\beta_h$ and $\theta_v = 1/\beta_v$ for simplifying next equation. Combined with equation (5) and equation (6), the each probability is

$$P(t_h \le m) = P(x^{\beta_h} \le m) = P(x \le m^{\theta_h}) = m^{\theta_h} \quad for \quad 0 < m \le 1 \tag{9}$$

And

$$P(t_v \le m) = P(by^{\beta_v} \le m) = P(y \le (m/b)^{\theta_v}) = \begin{cases} (m/b)^{\theta_v} & for \quad 0 < m \le b \\ 1 & for \quad b < m \le 1 \end{cases} \tag{10}$$

Hence,

$$G(m) = \begin{cases} m^{\theta_h + \theta_v}/b^{\theta_v} & for \quad 0 < m \le b \\ m^{\theta_h} & for \quad b < m \le 1 \end{cases} \tag{11}$$

Therefore, by definition of the probability density function we can derive

$$E(B) = T_{\max\_2D} \int\limits_{m=0}^{1} m\, G'(m)\, dm$$

$$= T_{\max\_h} \left\{ \int\limits_{m=0}^{b} \frac{(\theta_h + \theta_v) m^{\theta_h + \theta_v}}{b^{\theta_v}} dm + \int\limits_{m=b}^{1} \theta_h m^{\theta_h} dm \right\} \qquad (12)$$

$$= T_{\max\_h} \left\{ \frac{\theta_h}{\theta_h + 1} + \left( \frac{1}{\theta_h + 1} - \frac{1}{\theta_h + \theta_v + 1} \right) b^{\theta_h + 1} \right\}$$

To think about $E(F)$, we should pay attention to the shape of the rack. In conventional circumstance, the length of the rack is usually greater than the height. By adding the depth dimension, there are three different cases on the basis of length (in time) from largest to smallest:

● $T_{\max\_h} > T_{\max\_v} > T_{\max\_d}$, which means the depth dimension ranks the lowest position (RACK_HVD)

● $T_{\max\_h} > T_{\max\_d} > T_{\max\_v}$, which means the depth dimension is intermediate (RACK_HDV)

● $T_{\max\_d} > T_{\max\_h} > T_{\max\_v}$, which means the depth dimension is the longest sides of the rack (RACK_DHV).

Let $T_{\max\_3D} = \max(T_{\max\_h}, T_{\max\_v}, T_{\max\_d})$, $h = (T_{\max\_h} / T_{\max\_3D})$, $v = (T_{\max\_v} / T_{\max\_3D})$, $d = (T_{\max\_d} / T_{\max\_3D})$ respectively denote the shape factors of three dimension. Then the rack can be considered as a normalized one, $h \times v \times d$, which one of the shape factor is equal to 1.

Taking RACK_HVD as an example, we can see $h = 1$ and $h \geq v \geq d$. Therefore, we apply again the similar approach established firstly by Bozer and White (1984). Let $G(n)$ denote the probability that S/R machine's traveling time to $(x, y, z)$ and conveyor gets unit load prepared, say $t_{x,y,z}$, is not greater than $n$.

$$G(n) = P(t_{x,y,z} \leq n) = P(t_h \leq n) P(t_v \leq n) P(t_d \leq n) \qquad (13)$$

Furthermore, it is easily seen that the motion of depth dimension is also uniformly distributed. Therefore, we get probability distribution of three dimensions as follows:

$$P(t_h \leq n) = P(x^{\beta_h} \leq n) = P(x \leq n^{\theta_h}) = n^{\theta_h} \qquad for \qquad 0 < n \leq 1 \qquad (14)$$

$$P(t_v \leq n) = P(vy^{\beta_v} \leq n) = P(y \leq (n/v)^{\theta_v}) = \begin{cases} (n/v)^{\theta_v} & for & 0 < n \leq v \\ 1 & for & v < n \leq 1 \end{cases} \qquad (15)$$

$$P(t_d \leq n) = \begin{cases} n/d & for & 0 < n \leq d \\ 1 & for & d < n \leq 1 \end{cases} \qquad (16)$$

Therefore,

$$G(n) = \begin{cases} \dfrac{n^{\theta_h + \theta_v + 1}}{v^{\theta_v} d} & for & 0 < n \leq d \\[3mm] \dfrac{n^{\theta_h + \theta_v}}{v^{\theta_v}} & for & d < n \leq v \\[3mm] n^{\theta_h} & for & v < n \leq 1 \end{cases} \qquad (17)$$

To get the expected time, $E(F)$, we continue to use integral as follows:

$$E(F) = T_{\max\_3D} \int_{n=0}^{1} n G'(n)dn$$

$$= T_{\max\_h}\{\int_{n=0}^{d} \frac{(\theta_h + \theta_v + 1)n^{\theta_h+\theta_v+1}}{v^{\theta_v}d}dn + \int_{n=d}^{v} \frac{(\theta_h + \theta_v)n^{\theta_h+\theta_v}}{v^{\theta_v}}dn + \int_{n=v}^{1} \theta_h n^{\theta_h}dn\}$$

$$= T_{\max\_h}\{\frac{(\theta_h + \theta_v + 1)d^{\theta_h+\theta_v+1}}{(\theta_h + \theta_v + 2)v^{\theta_v}} + \frac{(\theta_h + \theta_v)v^{\theta_h+1}}{\theta_h + \theta_v + 1}$$

$$- \frac{(\theta_h + \theta_v)d^{\theta_h+\theta_v+1}}{(\theta_h + \theta_v + 1)v^{\theta_v}} + \frac{\theta_h}{\theta_h + 1} - \frac{\theta_h v^{\theta_h+1}}{\theta_h + 1}\}$$

(18)

Where $h = 1$ and $h \geq v \geq d$ .

At present, we can reckon the overall travel-time model of RACK_HVD, $E_{RACK\_HVD}$ , as follows:

$$E_{RACK\_HVD} = E(F) + E(B)$$

$$= T_{\max\_h}\{\frac{(\theta_h + \theta_v + 1)d^{\theta_h+\theta_v+1}}{(\theta_h + \theta_v + 2)v^{\theta_v}} + \frac{(\theta_h + \theta_v)v^{\theta_h+1}}{\theta_h + \theta_v + 1} -$$

$$\frac{(\theta_h + \theta_v)d^{\theta_h+\theta_v+1}}{(\theta_h + \theta_v + 1)v^{\theta_v}} + \frac{\theta_h}{\theta_h + 1} - \frac{\theta_h v^{\theta_h+1}}{\theta_h + 1}\} +$$

$$T_{\max\_h}\{\frac{\theta_h}{\theta_h + 1} + (\frac{1}{\theta_h + 1} - \frac{1}{\theta_h + \theta_v + 1})b^{\theta_h+1}\}$$

(19)

Where $h = 1$ and $h \geq v \geq d$ .

Meanwhile, in the similar procedure, we can get the overall travel-time models of another two categories of racks:

$$E_{RACK\_HDV} = E(F) + E(B)$$

$$= T_{\max\_h}\{\frac{(\theta_h + \theta_v + 1)v^{\theta_h+2}}{(\theta_h + \theta_v + 2)d} + \frac{(\theta_h + 1)d^{\theta_h+1}}{\theta_h + 2} -$$

$$\frac{(\theta_h + 1)v^{\theta_h+2}}{(\theta_h + 2)d} + \frac{\theta_h}{\theta_h + 1} - \frac{\theta_h d^{\theta_h+1}}{\theta_h + 1}\} +$$

$$T_{\max\_h}\{\frac{\theta_h}{\theta_h + 1} + (\frac{1}{\theta_h + 1} - \frac{1}{\theta_h + \theta_v + 1})b^{\theta_h+1}\}$$

(20)

Where $h = 1$ and $h \geq d \geq v$ .

$$E_{RACK\_DHV} = E(F) + E(B)$$

$$= T_{\max\_d}\{\frac{(\theta_h + \theta_v + 1)v^{\theta_h + 2}}{(\theta_h + \theta_v + 2)d} + \frac{(\theta_h + 1)h^{\theta_h + 2}}{(\theta_h + 2)d} -$$

$$\frac{(\theta_h + 1)v^{\theta_h + 2}}{(\theta_h + 2)d} + \frac{1 - h^2}{2}\} + \tag{21}$$

$$T_{\max\_h}\{\frac{\theta_h}{\theta_h + 1} + (\frac{1}{\theta_h + 1} - \frac{1}{\theta_h + \theta_v + 1})b^{\theta_h + 1}\}$$

Where $d = 1$ and $d \geq h \geq v$.

## 4. Analysis

To begin with, we should give an important assumption. As is widely known, compact storage system is apparently used in the conditions of limited storage space or costly land resource. For next stage, we define the cubic-time volume parameter, $V_t$. Apparently, space volume, $V_s$, is a constant and $V_s = H * V * D$. By using the theorem of kinematics: $T_{\max\_h} = \alpha_h H^{\beta_h}$, $T_{\max\_v} = \alpha_v V^{\beta_v}$ and $T_{\max\_d} = D / v_d$, we define $V_t = T_{\max\_h}^{\frac{1}{\beta_h}} * T_{\max\_v}^{\frac{1}{\beta_v}} * T_{\max\_d}$, it is obvious that $V_t = V_s * (\alpha^{\frac{1}{\beta_h}} \alpha^{\frac{1}{\beta_v}} / v_d)$ is a constant, similarly. Consequently, as $T_{\max\_h}, T_{\max\_v}$ and $T_{\max\_d}$ can be denoted by $T_{\max\_3D}$ with parameters $h, v, d$ respectively, thus we can conclude $V_t$ is positive correlation with $T_{\max\_3D}$.

According to those above models, for a certain S/R machine or other inbound/outbound machine, parameters $\theta_h$ and $\theta_v$ must be obtained in advance in order to apply the proposed model. In practice, we overlook the constant speed stage and the multi-acceleration/deceleration stage and pay attention to the single acceleration and deceleration condition, as is illustrated in Figure 2.
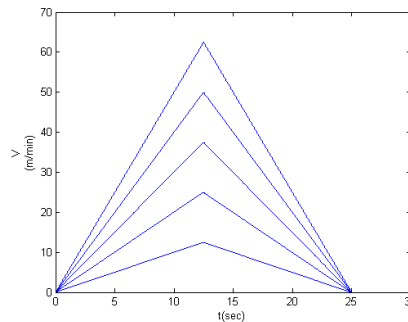


**Figure 2. Operating Characteristic of S/R Machine**

As a result, $\theta_h = \theta_v = 2$. Equations (19), (20) and (21) are converted as following forms:

$$E_{RACK\_HVD} = T_{\max\_h}(\frac{d^5}{30v^2} + \frac{2v^3}{15} + \frac{2}{3}) + T_{\max\_h}(\frac{2}{3} + \frac{2}{15}b^3) \tag{22}$$

Where $h = 1$ and $h \geq v \geq d$.

$$E_{RACK\_HDV} = T_{max\_h}(\frac{v^4}{12d} + \frac{d^3}{12} + \frac{2}{3}) + T_{max\_h}(\frac{2}{3} + \frac{2}{15}b^3) \qquad (23)$$

Where $h = 1$ and $h \geq d \geq v$.

$$E_{RACK\_DHV} = T_{max\_d}(\frac{v^4}{12d} + \frac{3h^4}{4d} + \frac{1 - h^2}{2}) + T_{max\_h}(\frac{2}{3} + \frac{2}{15}b^3) \qquad (24)$$

Where $d = 1$ and $d \geq h \geq v$.

Since we can denote the maximum travel time in each dimension by a constant, $V_t$, in next page we analyze optimal rack shape in these three cases.

Taking RACK_HVD as an example, the problem of expected travel time turns out to be the following optimization problem:

Minimize $f_{HVD}(d, v, b, T_{max\_h}) = T_{max\_h}(\frac{d^5}{30v^2} + \frac{2v^3}{15} + \frac{2}{3}) + T_{max\_h}(\frac{2}{3} + \frac{2}{15}b^3)$ (25)

Subject to
$$\begin{cases} T_{max\_h} = T_{max\_3D} = \max(T_{max\_h}, T_{max\_v}, T_{max\_d}) \\ 0 < d \leq v \leq h = 1 \\ V_t = T_{max\_h}^2 * T_{max\_v}^2 * T_{max\_d} \\ T_{max\_v} = v * T_{max\_3D} \\ T_{max\_d} = d * T_{max\_3D} \end{cases}$$

Consequently, variables $d, v, b, T_{max\_h}$ can be related with $V_t$ in following equations

$$T_{max\_h} = \sqrt[5]{\frac{V_t}{dv^2}} \qquad (26)$$

And

$$b = \frac{T_{max\_v}}{T_{max\_h}} = v \qquad (27)$$

Therefore, substituting (26) and (27) into (25), the optimal problem is translated into

$$f_{HVD}(d, v) = (\frac{d^5}{30v^2} + \frac{4}{15}v^3 + \frac{4}{3})(dv^2)^{-\frac{1}{5}} * V_t^{\frac{1}{5}} \qquad (28)$$

Where $0 < d \leq v \leq 1$ and $V_t$ is a constant.

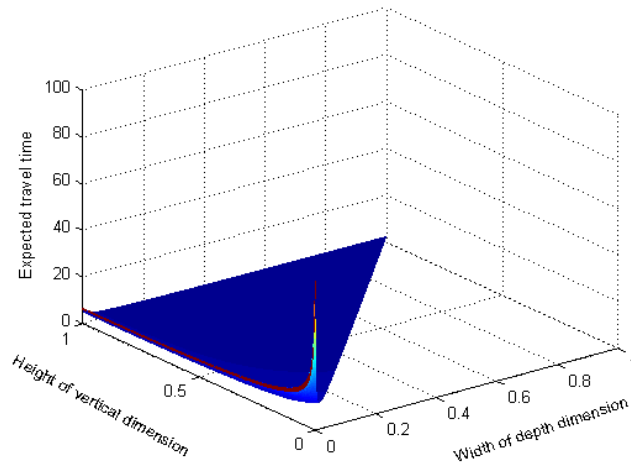We can easily figure out the characteristic of Equation (28) in the Figure 3:

**Figure 3. Effect on Expected Travel Time According to Lengths in Three Dimensions (HVD)**

Since

$$\frac{\partial^2 f_{HVD}(d,v)}{\partial v^2} = (\frac{34}{125}d^{\frac{24}{5}}v^{-\frac{22}{5}} + \frac{416}{375}d^{-\frac{1}{5}}v^{\frac{3}{5}} + \frac{56}{75}d^{-\frac{1}{5}}v^{-\frac{12}{5}}) * V_t^{\frac{1}{5}} > 0 \quad \text{and}$$

$$\begin{vmatrix} \dfrac{\partial^2 f_{HVD}(d,v)}{\partial v^2} & \dfrac{\partial^2 f_{HVD}(d,v)}{\partial v \partial d} \\[2ex] \dfrac{\partial^2 f_{HVD}(d,v)}{\partial d \partial v} & \dfrac{\partial^2 f_{HVD}(d,v)}{\partial d^2} \end{vmatrix} > 0$$

then it is evident that the feasible region is a convex linear set, the optimal solution can be obtained under Karush-Kuhn-Tucker (KKT) conditions as a generalized Lagrange's multiplier method. Because $V_t$ is a constant, $f_{HVD\_e}(d,v) = f_{HVD}(d,v)/V_t^{\frac{1}{5}}$ which is equivalent to $f_{HVD}(d,v)$ is used to describe the similar optimization problem. To obtain this conditional extremum, a Lagrangian function is established as following:

$$L_{HVD} = f_{HVD\_e}(d,v) + \lambda_1 d + \lambda_2(v-d) + \lambda_3(1-v) \tag{29}$$

Where $\lambda_1, \lambda_2$ and $\lambda_3$ are Lagrangian multipliers in broad sense.

Thus, by using partial derivatives we can get a equation set as follows:

$$\begin{cases} -\dfrac{2}{25}d^{\frac{24}{5}}v^{-\frac{17}{5}} + \dfrac{52}{75}d^{-\frac{1}{5}}v^{\frac{8}{5}} - \dfrac{8}{15}d^{-\frac{1}{5}}v^{-\frac{7}{5}} + \lambda_2 - \lambda_3 = 0 \\[2ex] \dfrac{4}{25}d^{\frac{19}{5}}v^{-\frac{12}{5}} - \dfrac{4}{75}d^{-\frac{6}{5}}v^{\frac{13}{5}} - \dfrac{4}{15}d^{-\frac{6}{5}}v^{-\frac{2}{5}} + \lambda_1 - \lambda_2 = 0 \\[2ex] \lambda_1 d = 0 \\[1ex] \lambda_2(v-d) = 0 \\[1ex] \lambda_3(1-v) = 0 \end{cases} \tag{30}$$

Finally, we use Mathematica 9.0.1 to acquire the variable values: $d = v = 1$, $\lambda_1 = 0, \lambda_2 = -\frac{4}{25}, \lambda_3 = -\frac{2}{25}$. Therefore, substituting those values into Equation (28), we gain the minimum travel time $f^*_{HVD}(d,v) = \frac{49}{30} V_t^{\frac{1}{5}}$.

With the similar methods, we obtain the minimum travel time for RACK_HDV and RACK_DHV: $f^*_{HDV}(d,v) = f^*_{DHV}(h,v) = \frac{49}{30} V_t^{\frac{1}{5}}$, while $d = v = h = 1$.

In conclusion, under the limitation of a gross storage capacity $V_t$ for a compact storage system, we formulate that the overall expected travel time will be minimized to $\frac{49}{30} V_t^{\frac{1}{5}}$ when $T_{max\_h} = T_{max\_v} = T_{max\_d}$.

## 5. Case Study

As an illustrating example, assume we have to accomplish the design of a compact storage system. The total requirements are showed in the Table 2 as follows:

**Table 2. The Parameters of a Compact Storage System**

| Elements | Value |
|---|---|
| Total capacity (SKU) | 5000 |
| SKU size in meter (Length * width*height) | 1.2*0.6*1.2 |
| Acceleration rate ( horizontal) | $\leq 2m/s^2$ |
| Acceleration rate ( vertical ) | $\leq 1m/s^2$ |
| Constant speed ( depth ) | 0.4m/s |

The total space capacity is $V_s = 5000*1.2*0.6*1.2 = 4320m^3$, then we get $V_t = V_s * (\alpha^{\frac{1}{\beta_h}} \alpha^{\frac{1}{\beta_v}} / v_d) = 16V / a_h a_v v = 86400 s^5$. Therefore we obtain the optimal travel time $E = \frac{49}{30} V_t^{\frac{1}{5}} \approx 15.86s$ and $T_{max\_h} = T_{max\_v} = T_{max\_d} \approx 9.712s$. On the next step, we can calculate the optimal rack shape: $H = \frac{1}{4} a_h T_{max\_h}^2 = \frac{1}{4} * 2 * 9.712^2 = 47.16m$ (aka 39.3 SKUs), $V = \frac{1}{4} a_v T_{max\_v}^2 = \frac{1}{4} * 1 * 9.712^2 = 23.58m$ (aka 19.7 SKUs), $D = v_d T_{max\_d} = 0.4 * 9.712 = 3.88m$ (aka 6.5 SKU). With the limitation of integer and the basis of SKU size in meter, we get number of storage unit in three dimensions: 39*19*7 (horizontal *vertical*depth). In conclusion, the real storage capacity is 5187 SKU, which is fit for the design requirements.

## 6. Conclusion

In this paper, our research focuses on the travel-time model of a new compact storage system which consists of conveyors and S/R machine. There are three directions of movement which are horizontal, vertical and depth dimensions respectively. With the extension of Bozer and White's method for conventional AS/RS, we obtain the travel time model for the new compact storage system with consideration of operating characteristics. Then, we analyze the travel time model and get the optimal rack shape. For a compact storage system with a limitation of storage capacity $V_t$, the minimum expected travel time is $\frac{49}{30} V_t^{\frac{1}{5}}$ if and only if $T_{\max\_h} = T_{\max\_v} = T_{\max\_d}$.

For further research, we suggest studying on storage assignment (*e.g.*, class-based or duration-of-stay based storage) for this new compact storage system because we made the formula derivation with consideration of random storage policy. In addition, we only find the expected travel time model for the single command cycle. In some conditions, dual command cycle will help increase the throughout rate.

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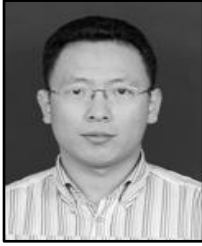
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