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Price Competition Strategy with a Common Retailer for a Fuzzy Supply Chain

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Abstract

This paper considers supply chain models with two competitive manufacturers acting as the leaders and a retailer acting as follower under a fuzzy decision environment. The parameters of demand function and manufacturing cost are treated as fuzzy variables. Two manufacturers are assumed to pursue Cournot competitive behavior and the optimum policy of the expected value and chance-constrained programming models are derived. Finally, a numerical example is provided to illustrate the results of proposed models. It is shown that in fuzzy models, the confidence level of the profits for supply chain members affects the final optimal solutions.

Keywords: *Supply Chain, Price competition, Game theory, Fuzzy variable*

1. Introduction

In today's highly competitive market, more and more firms realize that price is important behavior and competing firms often play a price war to attract customers. In supply chain competition, manufacturers compete with each other in determining their retail prices and order quantities to maximize their profits.

There is a large body of literature that deals with price competition in supply chain. Choi [1] used the linear and constant elasticity demand functions to study the price competition in a two-manufacture and one-retailer supply chain with two Stagckelberg and one Nash games. Ingene and Parry [2] considered the coordination of the supply chain with two retailers competing in price. Yang and Zhou [3] investigated two duopolistic retailers' three kinds of competitive behaviors: Cournot, Collusion and Stackelberg. Xiao and Qi [4] studied the coordination model of cost and demand disruptions for a supply chain with two competing retailers. Yao *et al.*, [5] investigated a revenue sharing contract for coordinating a supply chain comprising one manufacturer and two competing retailers. They showed that the intensity of competition between the retailers leaded to a higher system efficiency, but it would hurt the retailers themselves. Anderson and Bao [6] considered *n* supply chains price competing with a linear demand function. Farahat and Perakis [7] studied the efficiency of price competition among multi-product firms in differentiated oligopolies. Zhao and Chen [8] investigated a coordination mechanism of a supply chain that consists of one supplier and duopoly retailers from the perspective of operating uncertainty. Choi and Fredj [9] studied pricing strategies in a market channel composed of one national brand manufacturer and two retailers. Wang *et al.*, [10] studied a markup contract for coordinating a supply chain comprising two competitive manufacturers and a common dominant retailer.

Besides price, a few papers extended retailer competition in price to competing in service and quality. Iyer [11] investigated how manufacturer should coordinate the supply chain with one manufacturer and two retailers competing in price and service. Bernstein and Federgruen

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[12] developed a general equilibrium model of oligopoly retailers competing in price and service under demand uncertainty. Banker *et al.*, [13] and Matsubayashi [14] investigated a price and quality competition under a duopolistic setting, where the consumers' demand was modeled as a linear function of price and quality levels and the cost as a quadratic function of the quality level. Shaffer and Zhang [15] explored the competitive effects of one-to-one promotions in a model with two competing firms where the firms differed in size and consumers had heterogeneous band loyalty. Wu [16] studied the price and service competition between new and remanufactured products in a two-manufacture and one-retailer supply chain.

Most of the existing literatures discussed the retailer competition models under a crisp environment, such as a probabilistic market demand and known production cost. However, in real world, especially for new products, the relevant precise date or probabilities are not possible to get due to lack of history data. Moreover, in today's highly competitive market, shorter and shorter product life cycles make the useful statistical data less and less available. Thus, the fuzzy set theory, rather than the traditional probability theory is well suited to the supply chain models problem.

In this paper, we will concentrate on the price competition problem between two competitive manufacturers who sell their products to a common retailer under a fuzzy decision environment. We also perform sensitivity analysis of the confidence level of the profits for supply chain members of the models.

2. Preliminaries

We start this section by giving some concepts and properties of fuzzy variables, which will be used in the rest of the paper. Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), \mathrm{Pos})$ (for the concept of the possibility space, see Nahmias [17]), where Θ is a universe, $P(\Theta)$ is the power set of Θ and Pos is a possibility measure defined on $P(\Theta)$.

Definition 1 (Liu [18]) A fuzzy variable ξ is said to be nonnegative, if $\mathrm{Pos}\{\xi < 0\} = 0$.

Definition 2 (Liu [18]) Let ξ be a fuzzy variable and $\alpha \in (0,1]$. Then $\xi_\alpha^L = \inf\{r \mid \mathrm{Pos}\{\xi \le r\} \ge \alpha\}$ and $\xi_\alpha^R = \sup\{r \mid \mathrm{Pos}\{\xi \ge r\} \ge \alpha\}$ are called the α-pessimistic value and the α-optimistic value of ξ.

Example 1 Let $\xi = (a, b, c)$ be a triangular fuzzy variable, then its α-pessimistic value and α-optimistic value are respectively

$\xi_\alpha^L = b\alpha + a(1-\alpha)$ and $\xi_\alpha^R = b\alpha + c(1-\alpha)$

Proposition 1 (Liu and Liu [18] and Zhao et al [19]) Let ξ and η be two nonnegative independent fuzzy variables. Then

(a) for any $\alpha \in (0,1]$, $(\xi+\eta)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$ and $(\xi+\eta)_\alpha^R = \xi_\alpha^R + \eta_\alpha^R$;

(b) if $\lambda > 0$, for any $\alpha \in (0,1]$, $(\lambda\xi)_\alpha^L = \lambda\xi_\alpha^L$ and $(\lambda\xi)_\alpha^R = \lambda\xi_\alpha^R$;

(c) for any $\alpha \in (0,1]$, $(\xi\cdot\eta)_\alpha^L = \xi_\alpha^L\cdot\eta_\alpha^L$ and $(\xi\cdot\eta)_\alpha^R = \xi_\alpha^R\cdot\eta_\alpha^R$;

(d) for any $\alpha \in (0,1]$, $(\xi-\eta)_\alpha^L = \xi_\alpha^L - \eta_\alpha^R$ and $(\xi-\eta)_\alpha^R = \xi_\alpha^R - \eta_\alpha^L$.

Proposition 2 (Liu and Liu [20]) Let ξ be a fuzzy variable with the finite expected value $E[\xi]$, Then we have

$$E[\xi] = \frac{1}{2}\int_0^1 (\xi_\alpha^L + \xi_\alpha^R)\,\mathrm{d}\alpha$$

Proposition 3 (Liu and Liu [18]) Let ξ and η be two independent fuzzy variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]$$

Definition 7 Let ξ and η be two nonnegative independent fuzzy variables, $\xi > \eta$ if and only if for any $\alpha \in (0,1]$, $\xi_\alpha^L > \eta_\alpha^L$ and $\xi_\alpha^R > \eta_\alpha^R$.

Definition 8 Let ξ and η be two nonnegative independent fuzzy variables, if $\xi > \eta$, then $E[\xi]>E[\eta]$.

3. Problem Descriptions

This paper will consider a two-echelon supply chain consisting of two competitive manufacturers selling their products to a common retailer, who in turn retails it to the customer. We assume each manufacturer produces only one product, which is a substitute to the other, and the quality is similar. To simply the model, we assume their manufacturing fuzzy costs are also the same, denoted by $\tilde{c}$. Let $w_i (i = 1,2)$ denote the wholesale price per unit charged to the retailers by the manufacturer i, p the sale price charged to customers by retailer and m_i the profit margin on product i.

We assume the demand for each product is a linear function of its own price, and the competitor's price, which is given by

$$\tilde{q}_i = \tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j , \ i, j = 1,2, \ \ j \neq i \tag{1}$$

where $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$ are positive and independent fuzzy variables. The parameter $\tilde{D}$ represents the market and the parameter $\tilde{\beta}$ represents the measure of sensitivity of product-i's sales to changes of the product-j's price. The parameter $\tilde{\gamma}$ represents the degree of substitutability between products and reflects the impacts of the marketing mix decision of retailer on customer demand. The parameters $\tilde{\beta}$ and $\tilde{\gamma}$ are assumed to satisfy $\tilde{\beta} > \tilde{\gamma}$ and $\tilde{\beta}_\alpha^L > \tilde{\gamma}_\alpha^R$. By definition, the demand $\tilde{q}_i$ is also a fuzzy variable. Since we do not have negative demand in the real world we assume $\mathrm{Pos}\{\tilde{D} - \tilde{\beta} p_i + \tilde{\gamma} p_j < 0\} = 0$. The quantity ordered by product i can be expressed as $q_i = E[\tilde{q}_i]$. $\tilde{q}_i$ is called a fuzzy liner demand function in this paper. Let the cost $\tilde{c}$ be a positive fuzzy variables and be independent of parameters $\tilde{D}$, $\tilde{\beta}$ and $\tilde{\gamma}$.

We assume there is no brand discrimination for the retailer, who requires the same profit margin $m_1 = m_2 = m$ from both products. So the retail prices of the two products are

$$p_i = w_i + m , \ i=1, 2 \tag{2}$$

Then the fuzzy demand for each product can be rewritten as:

$$\tilde{q}_i = \tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) , \ \ i, j = 1,2, \ \ j \neq i \tag{3}$$

The fuzzy profit of manufacturer i and retailer can be expressed respectively as

$$\tilde{\pi}_{M_i} = (w_i - \tilde{c})\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right) \tag{4}$$

$$\tilde{\pi}_R = \sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right), \ i, j = 1,2, \ \ j \neq i \tag{5}$$

4. Fuzzy Two-echelon Supply Chain Models in Price Competition

In this section, we will develop the fuzzy two-echelon supply chain models with two competitive manufacturers and a retailer, which can tell both the manufacture and the retailers how to make their decisions when the duopolistic manufacturers pursuing the Manufacturer-Stackelberg game. In this condition, each manufacture sets the wholesale price using the reaction function of the retailer, conditional on the observed wholesale price of the competitor's product. The retailer sets the profit margin so as to maximize total fuzzy expected profit from both brands given the respective wholesale prices, hence, the fuzzy optimal model in this condition can be formulate as below.

$$\begin{cases} \max\limits_{w_i} E\left[\tilde{\pi}_{M_i}(w_i, m^*)\right] = E\left[(w_i - \tilde{c})\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right] \\ \text{s.t.} \\ \operatorname{Pos}\{w_i - \tilde{c} < 0\} = 0 \\ m^* = \arg\max E\left[\tilde{\pi}_R(m)\right] \\ \quad \begin{cases} \max\limits_{m} E\left[\tilde{\pi}_R(m)\right] = E\left[\sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right] \\ \text{s.t.} \\ \operatorname{Pos}\left\{\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) < 0\right\} = 0 \end{cases} \\ i, \ j = 1, 2, \quad j \neq i. \end{cases} \tag{6}$$

Theorem 1 Let $E[\tilde{\pi}_R(m)]$ be the fuzzy expected profit for retailer. The wholesale price w_i chosen by the manufacturer $i\,(i = 1, 2)$ is fixed. If $\operatorname{Pos}\{\tilde{D} - \tilde{\beta} A_1 + \tilde{\gamma} A_2 < 0\} = 0$ and $\operatorname{Pos}\{\tilde{D} - \tilde{\beta} A_2 + \tilde{\gamma} A_1 < 0\} = 0$, then the reaction function of the retailer is

$$m^*(w_1, w_2) = \frac{E\left[\tilde{D}\right]}{2\left(E[\beta] - E[\gamma]\right)} - \frac{w_1 + w_2}{4}$$

where $A_1 = \dfrac{E\left[\tilde{D}\right]}{2\left(E[\beta] - E[\gamma]\right)} + \dfrac{3w_1 - w_2}{4}$, $A_2 = \dfrac{E\left[\tilde{D}\right]}{2\left(E[\beta] - E[\gamma]\right)} + \dfrac{3w_2 - w_1}{4}$.

Proof: The fuzzy profit of retailer is

$$E[\tilde{\pi}_R(m)] = -2\left(E\left[\tilde{\beta}\right] - E[\tilde{\gamma}]\right)m^2 + \left(2E\left[\tilde{D}\right] - \left(E\left[\tilde{\beta}\right] - E[\tilde{\gamma}]\right)(w_1 + w_2)\right)m \tag{7}$$

Notice that the second-order derivatives $\dfrac{\partial^2 E[\tilde{\pi}_R(m)]}{\partial m^2} = -4\left(E\left[\tilde{\beta}\right] - E[\tilde{\gamma}]\right) < 0$ since $\tilde{\beta}$ and $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta} > \tilde{\gamma}$. Consequently, $E[\tilde{\pi}_R(m)]$ is a concave function of *m*. Hence, for any given w_i, the optimal profit margin of retailer can be obtained by solving $\dfrac{\partial E\left[\tilde{\pi}_R(m)\right]}{\partial m} = 0$, which give

$$m = \frac{E\left[\tilde{D}\right]}{2\left(E[\beta] - E[\gamma]\right)} - \frac{w_1 + w_2}{4} \tag{8}$$

The poof of Theorem 1 is completed. □

Theorem 2 Let $E\left[\tilde{\pi}_{M_i}\left(w_i, m^*(w_i)\right)\right]$ be fuzzy expected profit for manufacturer $i\,(i=1,2)$. If $\mathrm{Pos}\left\{w_i^* - \tilde{c} < 0\right\} = 0$ and $\mathrm{Pos}\left\{\tilde{D} - \tilde{\beta}(w_i^* + m^*) + \tilde{\gamma}(w_j^* + m^*) < 0\right\} = 0 \ (i, j = 1,2,\ j \neq i)$, then the optimal strategy in the MS(Manufacturer-Stackelberg) case is

$$m^* = \frac{E\left[\tilde{D}\right]\left(3E\left[\tilde{\beta}\right] + E\left[\tilde{\gamma}\right]\right) - \left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)\left(3E\left[\tilde{\beta}\tilde{c}\right] + \frac{1}{2}\int_0^1\left(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha\right)}{2\left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)\left(5E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)}$$

$$w_1^* = w_2^* = \frac{2E\left[\tilde{D}\right] + 3E\left[\tilde{\beta}\tilde{c}\right] + \frac{1}{2}\int_0^1\left(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha}{5E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]}$$

Proof: The fuzzy profit of the manufacturer $i\ (i=1,2)$ is

$$\begin{aligned}
E\left[\tilde{\pi}_{M_i}\right] &= \frac{1}{2}\int_0^1\left(\left(w_i - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^L \mathrm{d}\alpha \\
&\quad + \frac{1}{2}\int_0^1\left(\left(w_i - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^R \mathrm{d}\alpha \\
&= \frac{1}{2}\int_0^1\left(\left(w_i - \tilde{c}_\alpha^R\right)\left(\tilde{D}_\alpha^L - \tilde{\beta}_\alpha^R(w_i + m) + \tilde{\gamma}_\alpha^L(w_j + m)\right)\right)\mathrm{d}\alpha \\
&\quad + \frac{1}{2}\int_0^1\left(\left(w_i - \tilde{c}_\alpha^L\right)\left(\tilde{D}_\alpha^R - \tilde{\beta}_\alpha^L(w_i + m) + \tilde{\gamma}_\alpha^R(w_j + m)\right)\right)\mathrm{d}\alpha \\
&= E\left[\tilde{D}\right]w_i - E\left[\tilde{\beta}\right]\left(w_i^2 + w_i m\right) + E\left[\tilde{\gamma}\right]\left(w_i w_j + w_i m\right) + E\left[\tilde{\beta}\tilde{c}\right](w_i + m) \\
&\quad - \frac{1}{2}\left(w_j + m\right)\int_0^1\left(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha - \frac{1}{2}\int_0^1\left(\tilde{D}_\alpha^L\tilde{c}_\alpha^R + \tilde{D}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha \quad i, j = 1,2,\ j \neq i
\end{aligned} \tag{9}$$

Substituting $m = \dfrac{E\left[\tilde{D}\right]}{2\left(E[\beta] - E[\gamma]\right)} - \dfrac{w_1 + w_2}{4}$ into (9), we can get $\dfrac{\partial^2 E\left[\tilde{\pi}_{M_i}\right]}{\partial w_i^2} = -\frac{1}{2}\left(3E\left[\tilde{\beta}\right] + E\left[\tilde{\gamma}\right]\right) < 0$, since $\tilde{\beta}$ and $\tilde{\gamma}$ are positive fuzzy variables. Consequently, $E\left[\tilde{\pi}_{M_i}\right]$ is a concave function of w_i. Hence, the optimal wholesale price of manufacturer *i* can be obtained by solving $\dfrac{\partial E\left[\tilde{\pi}_{M_1}\right]}{\partial w_1} = 0$ and $\dfrac{\partial E\left[\tilde{\pi}_{M_2}\right]}{\partial w_2} = 0$, which give

$$w_1^* = w_2^* = \frac{2E\left[\tilde{D}\right] + 3E\left[\tilde{\beta}\tilde{c}\right] + \frac{1}{2}\int_0^1\left(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha}{5E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]} \tag{10}$$

Substituting w_1^* and w_2^* into (8), we can get

$$m^* = \frac{E\left[\tilde{D}\right]\left(3E\left[\tilde{\beta}\right] + E\left[\tilde{\gamma}\right]\right) - \left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)\left(3E\left[\tilde{\beta}\tilde{c}\right] + \frac{1}{2}\int_0^1\left(\tilde{\gamma}_\alpha^L\tilde{c}_\alpha^R + \tilde{\gamma}_\alpha^R\tilde{c}_\alpha^L\right)\mathrm{d}\alpha\right)}{2\left(E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)\left(5E\left[\tilde{\beta}\right] - E\left[\tilde{\gamma}\right]\right)} \tag{11}$$

The poof of Theorem 2 is completed.

Combining (7) with (9), (10) and (11) will easily yield the optimal fuzzy expected profits for retailer and two competitive manufacturers.

The chance-constrained programming, which was introduced by Liu and Iwamura [21-22], plays an important role in modeling fuzzy decision systems. Its basic ideal is to optimize some critical value with a given confidence level subject to some chance constraints (Gao and Liu [23]). Motivated by this ideal, we formulate the following maximax chance-constrained programming model for the two-echelon supply chain in the MS case.

$$\begin{cases} \max\limits_{w_i} \bar{\tilde{\pi}}_{M_i} \\ \text{s.t.} \\ \text{Pos}\left\{ (w_i - \tilde{c})\left(\tilde{D} - \tilde{\beta}(w_i + m^*) + \tilde{\gamma}(w_j + m^*)\right) \geq \bar{\tilde{\pi}}_{M_i} \right\} \geq \alpha \\ \text{Pos}\{ w_i - \tilde{c} < 0 \} = 0 \\ m^* = \arg\max \bar{\tilde{\pi}}_R \\ \qquad \begin{cases} \max\limits_{m} \bar{\tilde{\pi}}_R \\ \text{s.t.} \\ \text{Pos}\left\{ \sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right) \geq \bar{\tilde{\pi}}_R \right\} \geq \alpha \\ \text{Pos}\left\{ \tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) < 0 \right\} = 0 \end{cases} \\ i,\ j = 1, 2,\quad j \neq i. \end{cases} \tag{12}$$

where α is a predetermined confidence level of the profits for the manufacture i and the retailer. For each fixed feasible m, $\bar{\tilde{\pi}}_R$ should be the maximum value of the profit function for retailer, which $\tilde{\pi}_R(m)$ achieves with at least possibility α, and $\bar{\tilde{\pi}}_{M_i}$ should be maximum value of the profit function for manufacture i, which $\tilde{\pi}_{M_i}(w_i, m^*(w_i))$ achieves with at least possibility α. Clearly, the model (12) can be transformed into the following model (13) in which the manufacture i and the retailer try to maximize their optimal α-optimistic profits $\left(\tilde{\pi}_{M_i}(w_i, m^*(w_i))\right)_\alpha^R$ and $\left(\tilde{\pi}_R(m)\right)_\alpha^R$ by selecting the best pricing strategies, respectively

$$\begin{cases} \max\limits_{w_i} \left(\tilde{\pi}_{M_i}(w_i, m^*)\right)_\alpha^R = \left((w_i - \tilde{c})\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^R \\ \text{s.t.} \\ \text{Pos}\{ w_i - \tilde{c} < 0 \} = 0 \\ m^* = \arg\max \left(\tilde{\pi}_R(m)\right)_\alpha^R \\ \qquad \begin{cases} \max\limits_{m} \left(\tilde{\pi}_R(m)\right)_\alpha^R = \left(\sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^R \\ \text{s.t.} \\ \text{Pos}\left\{ \tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) < 0 \right\} = 0 \end{cases} \\ i,\ j = 1, 2,\quad j \neq i. \end{cases} \tag{13}$$

Theorem 3 Let $(\tilde{\pi}_R(m))_\alpha^R$ be the α-optimistic value of the profit for retailer. The wholesale price w_i chosen by the manufacturer i $(i = 1, 2)$ is fixed. If $\text{Pos}\{\tilde{D} - \tilde{\beta} A_3 + \tilde{\gamma} A_4 < 0\} = 0$ and $\text{Pos}\{\tilde{D} - \tilde{\beta} A_4 + \tilde{\gamma} A_3 < 0\} = 0$, then the reaction function of the retailer is

$$m^*(w_1,w_2)=\frac{\tilde{D}_\alpha^R}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}-\frac{w_1+w_2}{4}$$

where $A_3=\frac{\tilde{D}_\alpha^R}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}+\frac{3w_1-w_2}{4}$, $A_4=\frac{\tilde{D}_\alpha^R}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}+\frac{3w_2-w_1}{4}$.

Proof: The fuzzy profit of retailer is

$$\left(\tilde{\pi}_R(m)\right)_\alpha^R=-2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)m^2+\left(2\tilde{D}_\alpha^R-\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)(w_1+w_2)\right)m \tag{14}$$

Notice that the second-order derivatives $\frac{\partial^2\left(\tilde{\pi}_R(m)\right)_\alpha^R}{\partial m^2}=-2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)<0$, since $\tilde{\beta}$ and $\tilde{\gamma}$ are positive fuzzy variables and $\tilde{\beta}_\alpha^L>\tilde{\gamma}_\alpha^R$. Consequently, for any $\alpha\in(0,1]$, $\left(\tilde{\pi}_R(m)\right)_\alpha^R$ is a concave function of *m*. Hence, for any given w_1 and w_2, the optimal profit margin of retailer can be obtained by solving $\frac{\partial\left(\tilde{\pi}_R(m)\right)_\alpha^R}{\partial m}=0$, which give

$$m^*(w_1,w_2)=\frac{\tilde{D}_\alpha^R}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}-\frac{w_1+w_2}{4} \tag{15}$$

The poof of Theorem 3 is completed.

Theorem 4 Let $\left(\tilde{\pi}_{M_i}(w_i,m^*)\right)_\alpha^R$ be the *α*-optimistic value of the profit for manufacturer *i* $(i=1,2)$. If $\mathrm{Pos}\left\{w_i^*-\tilde{c}<0\right\}=0$ and $\mathrm{Pos}\left\{\tilde{D}-\tilde{\beta}(w_i^*+m^*)+\tilde{\gamma}(w_j^*+m^*)<0\right\}=0$ $(i,j=1,2,\,j\neq i)$, then the optimal strategies in this case are

$$m^*=\frac{\left(3\tilde{\beta}_\alpha^L+\tilde{\gamma}_\alpha^R\right)\left(\tilde{D}_\alpha^R-\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L\right)}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)\left(5\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}$$

$$w_1^*=w_2^*=\frac{2\tilde{D}_\alpha^R+\left(3\tilde{\beta}_\alpha^L+\tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L}{5\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R}$$

Proof: The *α*-optimistic value of the profit for manufacturer $i\,(i=1,2)$ is

$$\left(\tilde{\pi}_{M_i}(w_i,m^*)\right)_\alpha^R=\left((w_i-\tilde{c})\left(\tilde{D}-\tilde{\beta}(w_i+m^*)+\tilde{\gamma}(w_j+m^*)\right)\right)_\alpha^R$$

$$=\left(w_i-\tilde{c}_\alpha^L\right)\left(\tilde{D}_\alpha^R-\tilde{\beta}_\alpha^L(w_i+m^*)+\tilde{\gamma}_\alpha^R(w_j+m^*)\right)\quad i,j=1,2,\,j\neq i \tag{16}$$

Substituting $m^*(w_1,w_2)=\frac{\tilde{D}_\alpha^R}{2\left(\tilde{\beta}_\alpha^L-\tilde{\gamma}_\alpha^R\right)}-\frac{w_1+w_2}{4}$ into (16), we can get $\frac{\partial^2\left(\tilde{\pi}_{M_i}(w_i,m^*)\right)_\alpha^R}{\partial w_i^2}=$ $-\frac{1}{4}\left(3\tilde{\beta}_\alpha^L+\tilde{\gamma}_\alpha^R\right)<0$, since $\tilde{\beta}$ and $\tilde{\gamma}$ are positive fuzzy variables. Consequently, $\left(\tilde{\pi}_{M_i}(w_i,m^*)\right)_\alpha^R$ is a concave function of w_i. Hence, the optimal wholesale price of manufacturer *i* can be obtained by solving $\frac{\partial\left(\tilde{\pi}_{M_1}(w_i, m^*)\right)_\alpha^R}{\partial w_1} = 0$ and $\frac{\partial\left(\tilde{\pi}_{M_2}(w_i, m^*)\right)_\alpha^R}{\partial w_2} = 0$, which give

$$w_1^* = w_2^* = \frac{2\tilde{D}_\alpha^R + \left(3\tilde{\beta}_\alpha^L + \tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L}{5\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R} \tag{17}$$

Substituting w_1^* and w_2^* into (15), we can get

$$m^* = \frac{\left(3\tilde{\beta}_\alpha^L + \tilde{\gamma}_\alpha^R\right)\left(\tilde{D}_\alpha^R - \left(\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L\right)}{2\left(\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)\left(5\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)} \tag{18}$$

The poof of Theorem 4 is completed.

Combining (14) with (16), (17) and (18) will easily yield the optimal α-optimistic value of profits for two competition manufacturers and retailer, which is given by

$$\left(\tilde{\pi}_{M_i}(w_i^*, m^*)\right)_\alpha^R = \frac{\left(3\tilde{\beta}_\alpha^L + \tilde{\gamma}_\alpha^R\right)\left(\tilde{D}_\alpha^R - \left(\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L\right)^2}{\left(5\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)^2},\ i = 1, 2 \tag{19}$$

$$\left(\tilde{\pi}_R(m^*)\right)_\alpha^R = \frac{\left(3\tilde{\beta}_\alpha^L + \tilde{\gamma}_\alpha^R\right)^2\left(\tilde{D}_\alpha^R - \left(\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)\tilde{c}_\alpha^L\right)^2}{2\left(\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)\left(5\tilde{\beta}_\alpha^L - \tilde{\gamma}_\alpha^R\right)^2} \tag{20}$$

The minimax chance-constrained programming model for the two-echelon supply chain in MS case can also be formulated as bellow

$$\begin{cases} \max\limits_{w_i}\ \min\limits_{\bar{\tilde{\pi}}_{M_i}} \bar{\tilde{\pi}}_{M_i} \\ \text{s.t.} \\ \mathrm{Pos}\left\{\left(w_i - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta}(w_i + m^*) + \tilde{\gamma}(w_j + m^*)\right) \le \bar{\tilde{\pi}}_{M_i}\right\} \ge \alpha \\ \mathrm{Pos}\left\{w_i - \tilde{c} < 0\right\} = 0 \\ m^* = \arg\max \min\limits_{\bar{\tilde{\pi}}_R} \bar{\tilde{\pi}}_R \\ \qquad \begin{cases} \max\limits_{m}\ \min\limits_{\bar{\tilde{\pi}}_R} \bar{\tilde{\pi}}_R \\ \text{s.t.} \\ \mathrm{Pos}\left\{\sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right) \le \bar{\tilde{\pi}}_R\right\} \ge \alpha \\ \mathrm{Pos}\left\{\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) < 0\right\} = 0 \end{cases} \\ i,\ j = 1, 2,\ \ j \ne i. \end{cases} \tag{21}$$

where α is a predetermined confidence level of the profits for the manufacture *i* and the retailer. For each fixed feasible *m*, $\bar{\tilde{\pi}}_R$ should be the minimum value of the profit function for retailer, which $\bar{\tilde{\pi}}_R(m)$ achieves with at least possibility α, and $\bar{\tilde{\pi}}_{M_i}$ should be minimum value of the profit function for manufacture *i*, which $\bar{\tilde{\pi}}_{M_i}\left(w_i, m^*\right)$ achieves with at least possibility α. It is clear that the model (21) can be transformed into the following model (22) in which the manufacture i and the retailer try to maximize their optimal α-pessimistic profits $\left(\tilde{\pi}_{M_i}\left(w_i, m^*\right)\right)_\alpha^L$ and $\left(\tilde{\pi}_R\left(m\right)\right)_\alpha^L$ by selecting the best pricing strategies, respectively

$$\begin{cases} \max\limits_{w_i} \left(\tilde{\pi}_{M_i}(w_i, m^*)\right)_\alpha^L = \left(\left(w_i - \tilde{c}\right)\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^L \\ \text{s.t.} \\ \text{Pos}\left\{w_i - \tilde{c} < 0\right\} = 0 \\ m^* = \arg\max \left(\tilde{\pi}_R(m)\right)_\alpha^L \\ \quad \begin{cases} \max\limits_{m} \left(\tilde{\pi}_R(m)\right)_\alpha^L = \left(\sum_{i=1}^{2} m\left(\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m)\right)\right)_\alpha^L \\ \text{s.t.} \\ \text{Pos}\left\{\tilde{D} - \tilde{\beta}(w_i + m) + \tilde{\gamma}(w_j + m) < 0\right\} = 0 \end{cases} \\ i, j = 1, 2, \quad j \neq i. \end{cases} \tag{22}$$

Theorem 5 Let $\left(\tilde{\pi}_R(m)\right)_\alpha^L$ and $\left(\tilde{\pi}_{M_i}(w_i, m^*)\right)_\alpha^L$ be the α-pessimistic value of the profit for retailer and manufacturer i.If $\text{Pos}\left\{w_i^* - \tilde{c} < 0\right\} = 0$ and $\text{Pos}\left\{\tilde{D} - \tilde{\beta}(w_i^* + m^*) + \tilde{\gamma}(w_j^* + m^*) < 0\right\} = 0$ $(i, j = 1, 2, j \neq i)$, then the optimal strategies in this case are

$$m^* = \frac{\left(3\tilde{\beta}_\alpha^R + \tilde{\gamma}_\alpha^L\right)\left(\tilde{D}_\alpha^L - \left(\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)\tilde{c}_\alpha^R\right)}{2\left(\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)\left(5\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)}$$

$$w_1^* = w_2^* = \frac{2\tilde{D}_\alpha^L + \left(3\tilde{\beta}_\alpha^R + \tilde{\gamma}_\alpha^L\right)\tilde{c}_\alpha^R}{5\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L}$$

Proof: Similar to the proof of Theorem 4.

The optimal α-pessimistic value of profits for two competition manufacturers and retailer are given by

$$\left(\tilde{\pi}_{M_i}(w_i^*, m^*)\right)_\alpha^L = \frac{\left(3\tilde{\beta}_\alpha^R + \tilde{\gamma}_\alpha^L\right)\left(\tilde{D}_\alpha^L - \left(\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)\tilde{c}_\alpha^R\right)^2}{\left(5\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)^2}, \; i = 1, 2 \tag{23}$$

$$\left(\tilde{\pi}_R(m^*)\right)_\alpha^L = \frac{\left(3\tilde{\beta}_\alpha^R + \tilde{\gamma}_\alpha^L\right)^2\left(\tilde{D}_\alpha^L - \left(\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)\tilde{c}_\alpha^R\right)^2}{2\left(\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)\left(5\tilde{\beta}_\alpha^R - \tilde{\gamma}_\alpha^L\right)^2} \tag{24}$$

Remark 1 when α=1, it is clear the manufacturing cost $\tilde{c}$, the market base $\tilde{D}$, the demand change rate $\tilde{\beta}$ and the degree of substitutability between products $\tilde{\gamma}$ degenerate into crisp real numbers, the main result in Theorems 4 and 5 can degenerate into

$$m^* = \frac{(3\beta + \gamma)(D - (\beta - \gamma)c)}{2(\beta - \gamma)(5\beta - \gamma)} \tag{25}$$

$$w_1^* = w_2^* = \frac{2D + (3\beta + \gamma)c}{5\beta - \gamma} \tag{26}$$

There are just the conventional results in crisp solution.

5. Numerical Example

In this section, we present a numerical example which is aimed at illustrating the computational process of the fuzzy supply chain models established in previous section. We will also perform sensitivity analysis of the parameter α of these models. Here, we consider that $\tilde{D}$ is about 600, $\tilde{\beta}$ is about 20, $\tilde{\gamma}$ is about 5 and $\tilde{c}$ is about 10, respectively. They are all considered as triangular fuzzy variables as $\tilde{D}=(580,600,620)$, $\tilde{\beta}=(19,20,21)$, $\tilde{\gamma}=(4,5,6)$ and $\tilde{c}=(9,10,11)$.

Based on the analysis showed in the section 4, we present the results of the optimal expected values, α-optimistic values and α-pessimistic values for the fuzzy supply chain models above in Table1.

Table 1. Optimal Equilibrium Values of the Parameters for Different α in a Fuzzy Supply Chain

	α	m	w_1	w_2	π_{M_1}	π_{M_2}	π_R
Expected value	—	10.26	19.48	19.48	1485.47	1485.47	3158.03
α-optimistic value	1.00	10.26	19.47	19.47	1458.45	1458.45	3159.97
	0.95	10.41	19.51	19.51	1483.38	1483.38	3230.58
	0.90	10.56	19.55	19.55	1508.59	1508.59	3302.58
	0.85	10.72	19.59	19.59	1534.07	1534.07	3375.99
	0.80	10.87	19.63	19.63	1559.83	1559.83	3450.86
	0.75	11.03	19.67	19.67	1585.88	1585.88	3527.22
α-pessimistic value	1.00	10.26	19.47	19.47	1458.45	1458.45	3159.97
	0.95	10.12	19.44	19.44	1433.79	1433.79	3090.72
	0.90	9.97	19.40	19.40	1409.41	1409.41	3022.81
	0.85	9.83	19.36	19.36	1385.29	1385.29	2956.19
	0.80	9.69	19.33	19.33	1361.44	1361.44	2890.85
	0.75	9.55	19.29	19.29	1337.85	1337.85	2826.76

Based on the results showed in Table 1, we find:

(a) The 3th and 9th rows in Table 1 show the solutions for fuzzy models at α=1, which are just the results in crisp case.

(b) The α-optimistic values of the optimal pricing strategies and optimal profits for the manufacture *i* and the retailer decrease with increasing of the confidence level α. With the increasing of the confidence level α, the α-pessimistic values of the optimal pricing strategies and profits for the manufacturer *i* and the retailer will increase.

6. Conclusion

This paper proposes a fuzzy model for two-echelon supply chain management, where two competitive manufacturers pursue the Manufacturer-Stackelberg game. We provide the pricing strategies for manufacturers and retailer in expected value and chance-constrained programming models. We find that the proposed fuzzy models can be reduced to the crisp models and the confidence level of the profits for the manufactures and the retailer affects the final optimal solutions. The models proposed in this paper are easier to implement and requires less data. It is appropriate when the environment is complex, ambiguous, or there is lack of statistical data. Further work is desirable to test whether our conclusions extend to other forms of fuzzy demand function and with multiple competitive retailers or manufacturers.

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