Recursive Solution of Position Determination Problem using Time Difference of Arrival Method

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Abstract

Recently, the time difference of arrival (TDOA) based position determination technique is broadly used in many kinds of real time locating systems. In contrast with other position determination methods, TDOA based localization technique has the strength that it is unnecessary to synchronize the time between each receivers. With at least three receivers which we know their position, we can obtain TDOA solution. In a position determination problem including TDOA method, the accuracy of the estimated position is one of the significant factors. In this paper, in order to compensate measurement error due to time delay of signal propagation, the recursive least square (RLS) method is used. The RLS method is an iterated technique of least square (LS) method. The accuracy and calculation speed of estimated solution are improved through this algorithm. Simulation results show the performance of the proposed algorithm.

Keywords: Position Determination Problem, Time Difference of Arrival, Recursive Least Square, Measurement Error Compensation

1. Introduction

The position determinate method is used in many types of modern location based services such as wireless communication, social network service, navigation and augmented reality platform. The global positioning system (GPS) is the most widely used location system. However, GPS has a limitation to estimate accurate position of target since GPS signal can be disturbed easily by jamming. Moreover, due to the limited condition of GPS which necessarily uses a satellite signal, it is difficult to estimate a precise position of target in downtown or indoor environment [1-3]. Recently, position determination technique using TDOA method has been used broadly as it uses time difference of signal arrival from target to fixed receivers. The strength of TDOA based localization method is that it is unnecessary to synchronize between receivers. Due to this characteristic, TDOA has been used extensively in real time locating systems [4-6]. We can obtain the TDOA signal by two methods. Firstly, we need to measure time of arrival (TOA) of signal between a target and each receiver. By subtracting TOA measurements from reference one, we can acquire the TDOA measurements. In order to apply this method, however, every receiver needs time synchronization. The second one is cross-correlation method of each received signal to derive TDOA signals [7].

In TDOA based position determination technique, the estimation of target's accurate position is also a significant problem. To estimate the target's accurate location, Chan [8] proposed a time of arrival (TOA) and TDOA based localization algorithm through

an approximate maximum likelihood method. Chan showed the performance of the proposed algorithm compared with Cramer-Rao lower bound (CRLB). Chiang [9] suggested Kalman tracking algorithm for heterogeneous wireless location systems for improving the accuracy of estimation. Zhu [10] proposed a particle filter to improve underground mine geolocation through UWB ranging.

In this paper, we propose a TDOA based position determination of target using the RLS algorithm to compensate for the measurement errors from TDOA data. The RLS algorithm is an iterated technique of LS algorithm. This proposed algorithm provides a rapid calculation speed even when we get additional TDOA data from each receiver. On the basis of this strength, we can treat much more TDOA data for the estimation of an target's position. As the iteration process is increased, we obtain more precise position of target.

This paper is organized as follows. Section 2 analyzes TDOA based position determination equation. In order to apply LS algorithm to a geolocation problem, a specific form of position determination equation is needed to formulate. In Section 3, we first obtain an LS solution of derived position determination equation. Then we explain a measurement noise compensation through RLS algorithm. The effectiveness of proposed algorithm is confirmed through some simulations in Section 4. Finally conclusion appears in Section 5.

2. Position Determination by TDOA

In this section, we derive the TDOA based position determination equation for applying RLS algorithm. If there are fixed receivers of which the location is supposed to be known, we can get TDOA data on the basis of specific receiver [11]. TDOA method needs at least three receivers for applying RLS algorithm in two dimensional environments. When the TDOA data is available, we can derive the geolocation formulation of a target. We express the unknown position of a target which we try to estimate as $\mathbf{p} = [x, y]^T$ and the known positions of receivers are represented as $\mathbf{r}_i = [x_i, y_i]^T$, $i = \{1, 2, \cdots, M\}$ in two dimensional coordinates. We can obtain the TDOA measurement values as follows

$$
\begin{aligned}
d_i^0 &= \|\mathbf{p} - \mathbf{r}_i\|, \\
d_{i1}^0 &= ct_{i1} = c(t_i - t_1)
\end{aligned}
\tag{1}
$$

where d_i^0 means the range from target to i-th receiver and c is the propagation speed of signal. t_{i1} means the measured TDOA between i-th receiver and the first receiver. In this paper, the first receiver is reference receiver which computes the time difference of signal arrival. d_i^0 means the calculated range difference between i-th receiver and the first receiver. In a real case, the measured TDOA data contains noise due to nonline-of-sight (NLOS) problem [12]. TDOA data can be as following

$$
\begin{aligned}
\mathbf{t} &= col\{t_{k1}^0 + \Delta t_{k1}, \quad k = 2, \cdots, M\} = \mathbf{t_0} + \Delta\mathbf{t}, \\
\Delta\mathbf{t} &= col\{\Delta t_{k1}, k = 2, \cdots, M\}, \quad E[\Delta\mathbf{t}] = 0
\end{aligned}
\tag{2}
$$

Given $M-1$ noisy TDOA measurements, the unknown position of a target can be obtained by the following equations

$$\mathbf{Ap} = \mathbf{h} \tag{3}$$

with

$$\mathbf{h} = \mathbf{b} + \boldsymbol{\rho}d_1^0, \quad \mathbf{A} = \begin{bmatrix} \mathbf{r}_2^T \\ \vdots \\ \mathbf{r}_M^T \end{bmatrix}, \quad \boldsymbol{\rho} = -\begin{bmatrix} d_{21}^0 \\ \vdots \\ d_{M1}^0 \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} \langle \mathbf{r}_2, \mathbf{r}_2 \rangle \\ \vdots \\ \langle \mathbf{r}_M, \mathbf{r}_M \rangle \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} \|\mathbf{r}_2\|^2 - (d_{21}^0)^2 \\ \vdots \\ \|\mathbf{r}_M\|^2 - (d_{M1}^0)^2 \end{bmatrix} = \frac{1}{2}(\mathbf{q} - \boldsymbol{\rho} \square \boldsymbol{\rho}) \tag{4}$$

In equation (3), the vector $\mathbf{p}$ is then the solution of the system equations. $\square$ denotes Hadamard process which multiplies vector elements with same array. The measurement error in TDOA data of real environment causes that it has a difference between the estimated position and the true position of a target. In order to solve this problem, we propose the RLS algorithm in Section 3 [13].
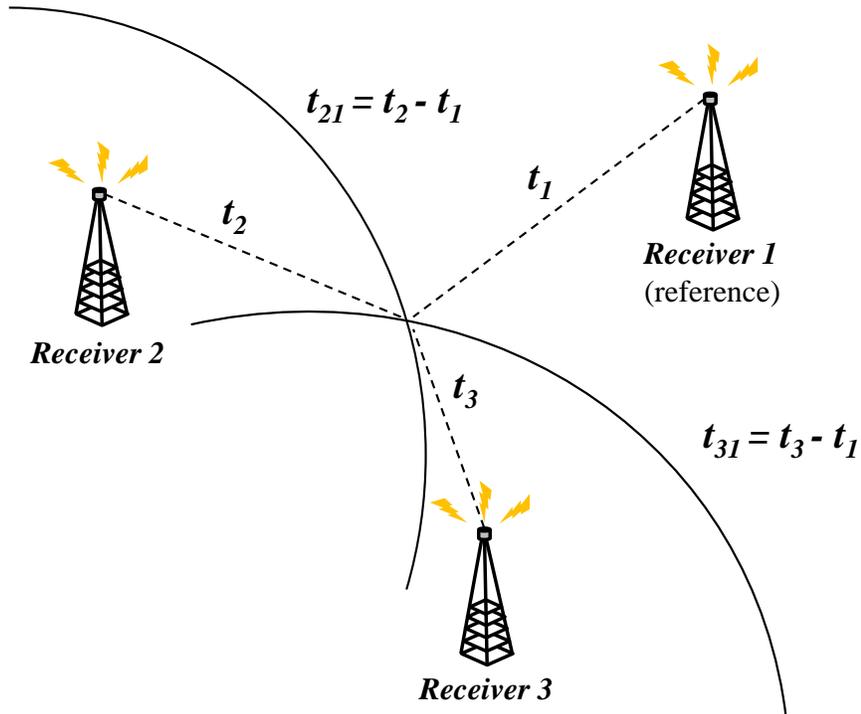

Figure 1. Estimated Position of Target Using RLS Method

3. RLS Estimator for TDOA localization

The RLS algorithm is an iterated method of LS algorithm. In order to apply the RLS algorithm to TDOA based position estimation, we need to obtain the LS solution. In this section, we derive the LS solution of the TDOA formulation which is obtained in Section 2. Based on LS solution, the RLS criterion is applied to a geolocation problem. When we get additional TDOA data set in each receiver, it is unnecessary to compute all over again by using the RLS technique [14-16].

3.1. Least Square Method

The LS method is a procedure to determine what the best fit line is to the data. It means the overall solution which minimizes the sum of the squares of the errors resulted from every single equation. Equation (3) is overdetermined system since there are more equations than unknowns. Therefore, we can derive general LS solution of this equation as follows

$$\hat{\mathbf{p}}_{\mathbf{LS}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{h} \tag{5}$$

In equation (5), $\hat{\mathbf{p}}_{\mathbf{LS}}$ is the estimated location of target through the LS algorithm. The parameter $\mathbf{A}$ is an $M \times 2$ matrix, therefore, the inverse matrix of parameter $\mathbf{A}^T\mathbf{A}$ exists. Using this equation, we derive an RLS solution in Section 3.2.

3.2. Recursive Least Square Method

In order to apply RLS criterion, we need to express system equation in a different form. Equation (3) can be rewritten as

$$\begin{bmatrix} \overline{\mathbf{A}}_k \\ \mathbf{A}_{k+1} \end{bmatrix} \mathbf{p} = \begin{bmatrix} \overline{\mathbf{h}}_k \\ \mathbf{h}_{k+1} \end{bmatrix} \tag{6}$$

In equation (6), $\overline{\mathbf{A}}_k = [\mathbf{A}_1 \ \mathbf{A}_2 \ \cdots \ \mathbf{A}_k]^T$ and $\overline{\mathbf{h}}_k = [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_k]^T$ represent the values which contain k TDOA data. $\mathbf{A}_{k+1}$ and $\mathbf{h}_{k+1}$ are $(k+1)$-th TDOA parameters. Using $(k+1)$ TDOA data, the position of a target is obtained through the LS algorithm as follows

$$\hat{\mathbf{p}}_{k+1} = (\overline{\mathbf{A}}_{k+1}^T\overline{\mathbf{A}}_{k+1})^{-1}\overline{\mathbf{A}}_{k+1}^T\overline{\mathbf{h}}_{k+1} \tag{7}$$

With the definition of $\mathbf{S}_{k+1} = (\overline{\mathbf{A}}_{k+1}^T\overline{\mathbf{A}}_{k+1})^{-1}$ for a simple notation, the following equation is satisfied.

$$\mathbf{S}_{k+1}^{-1} = \mathbf{S}_k^{-1} + \mathbf{A}_{k+1}^T\mathbf{A}_{k+1} \tag{8}$$

Using the equation (8), the equation (7) is rewritten as below

$$\hat{\mathbf{p}}_{k+1} = \mathbf{S}_{k+1}[\overline{\mathbf{A}}_k^T \overline{\mathbf{h}}_k + \mathbf{A}_{k+1}^T \mathbf{h}_{k+1}]$$
$$= \mathbf{S}_{k+1}[\mathbf{S}_k^{-1}\mathbf{S}_k \overline{\mathbf{A}}_k^T \overline{\mathbf{h}}_k + \mathbf{A}_{k+1}^T \mathbf{h}_{k+1}]$$

(9)

Since the term $\overline{\mathbf{S}}_k \overline{\mathbf{A}}_k^T \overline{\mathbf{h}}_k$ in equation (9) can be denoted as $\hat{\mathbf{p}}_k$, the equation (9) can be rewritten as follows

$$\hat{\mathbf{p}}_{k+1} = \mathbf{S}_{k+1}[\mathbf{S}_k^{-1}\hat{\mathbf{p}}_k + \mathbf{A}_{k+1}^T \mathbf{h}_{k+1}]$$
$$= \mathbf{S}_{k+1}[(\mathbf{S}_{k+1}^{-1} - \mathbf{A}_{k+1}^T \mathbf{A}_{k+1})\hat{\mathbf{p}}_k + \mathbf{A}_{k+1}^T \mathbf{h}_{k+1}]$$
$$= \hat{\mathbf{p}}_k - \mathbf{S}_{k+1}\mathbf{A}_{k+1}^T \mathbf{A}_{k+1}\hat{\mathbf{p}}_k + \mathbf{S}_{k+1}\mathbf{A}_{k+1}^T \mathbf{h}_{k+1}$$
$$= \hat{\mathbf{p}}_k + \mathbf{S}_{k+1}\mathbf{A}_{k+1}^T(\mathbf{h}_{k+1} - \mathbf{A}_{k+1}\hat{\mathbf{p}}_k)$$

(10)

where $\mathbf{S}_{k+1}$ represents $(\overline{\mathbf{A}}_k^T \overline{\mathbf{A}}_k + \mathbf{A}_{k+1}^T \mathbf{A}_{k+1})^{-1}$ due to the equation (8). Therefore, the equation (10) can be rewritten as

$$\hat{\mathbf{p}}_{k+1} = \hat{\mathbf{p}}_k + (\overline{\mathbf{A}}_k^T \overline{\mathbf{A}}_k + \mathbf{A}_{k+1}^T \mathbf{A}_{k+1})^{-1}\mathbf{A}_{k+1}^T(\mathbf{h}_{k+1} - \mathbf{A}_{k+1}\hat{\mathbf{p}}_k)$$

(11)

Equation (11) is the RLS solution of a position determination problem. We can derive the position of a target more rapidly through equation (11) when we receive additional TDOA data sets. In equation (11), $\hat{\mathbf{p}}_k$ denotes the estimated position of target which is calculated at previous iteration by using k TDOA data. $\overline{\mathbf{A}}_k$ is the parameter based on information of k data. These values were calculated by the previous k-th iteration. $\mathbf{A}_{k+1}$ and $\mathbf{h}_{k+1}$ are additional measured data. We can rapidly estimate the location of a target with previously calculated TDOA solution and additionally received information.

4. Simulation Results

In this section, we demonstrate the performance of a proposed position determination algorithm using RLS method through some simulations. In our simulation, the location of a target is estimated by using four receivers. The locations of each receivers are (0, 0) km, (100, 0) km, (0, 100) km and (100, 100) km, respectively. In two dimension environment, at least three TDOA data are needed to obtain estimated position of target. If only three TDOA data are available, however, it is unnecessary to apply LS solution since the parameter $\mathbf{A}$ in equation (3) has an inverse function. In a real case, each TDOA signal contains measurement noises. We assume that the measurement noises follow the Gaussian distribution with a variance of 0.1. We assume the signal propagation speed c is 1 for simplifying the calculation. Figure 2 describes that the estimated trajectory using a proposed algorithm follows the true trajectory. The thin line means the true trajectory of a target, the thick line represents the estimated

trajectory using RLS method and the upward triangles express the receivers. As shown in Figure 2, the estimated trajectory of a target is almost same with true trajectory.
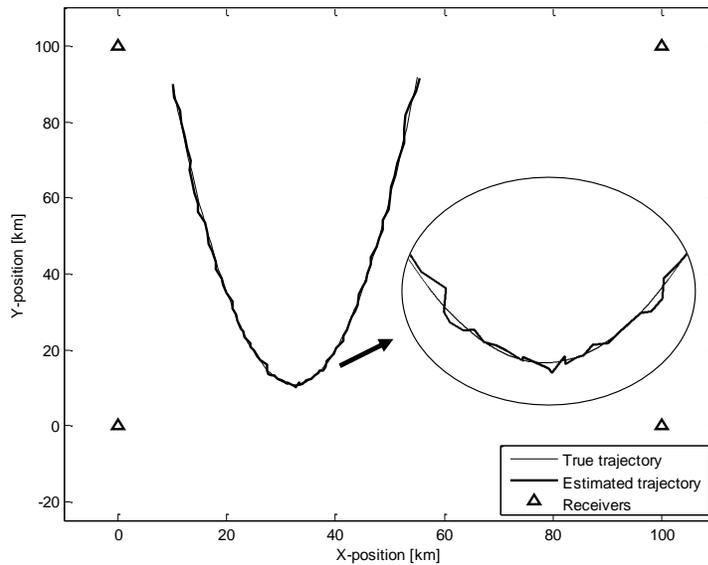

Figure 1. Estimated Position of Target Using RLS Method

The effectiveness of iteration process is confirmed in Figure 3 and 4. In Figure 3 and 4, the measurement noises is assumed to follow the Gaussian distribution with a variance of 1 for verifying the performance in proportion to iteration number. Figure 3 denotes each root mean square error (RMSE) of iteration #1, #3, #5 and #10, respectively. As shown in that figure, the more iteration is progressed, the higher performance of estimated position is obtained.
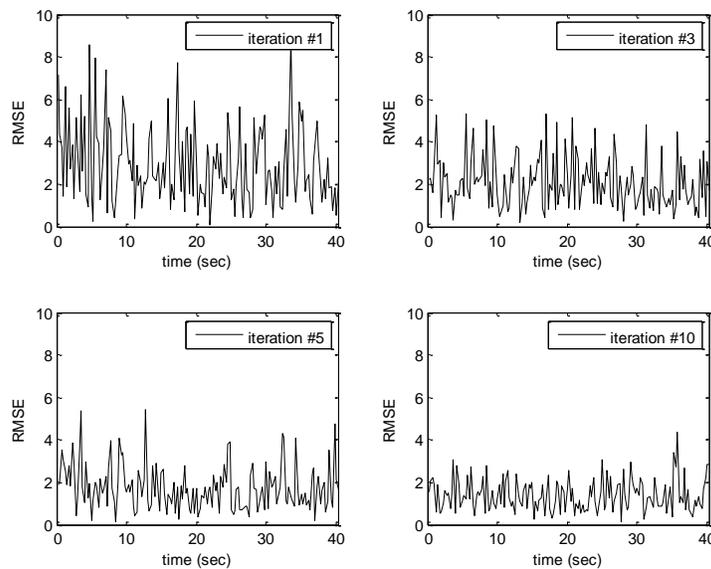

Figure 3. RMSE for Different Iterations

In Figure 4, the change of RMSE is shown depending on iteration increased. The solid line denotes the RMSE between true target position and estimated position using proposed RLS algorithm. As the number of iteration is increased, the RMSE converges to zero. Using RLS algorithm, the estimated position of a target can be easily updated.
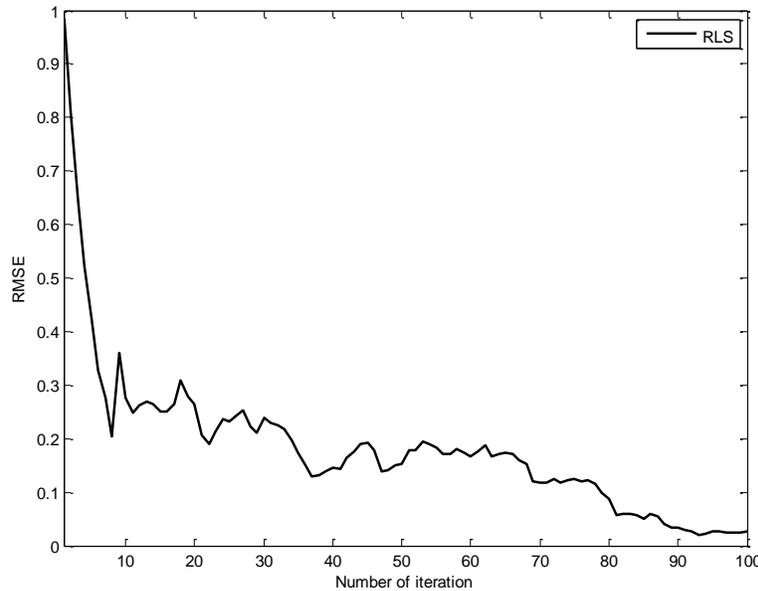

Figure 4. RMSE Depending on Number of Iteration Increase

5. Conclusion

This paper introduces the TDOA based position determination algorithm through RLS method. This proposed algorithm can perform whether the target moves or stops. Moreover, when we obtain additional TDOA data from receivers, the proposed algorithm provides rapid computing speed to estimate target's position. We can treat more TDOA data with this proposed algorithm. In order to determinate the position of target, we use the TDOA signals from at least three receivers. In order to apply the RLS algorithm, we set the position determination formulation with measured TDOA data. We confirm a high performance of proposed RLS algorithm based TDOA localization method through some simulations. As the iteration process is repeated over again, we can obtain much more precise position of a target.

Acknowledgements

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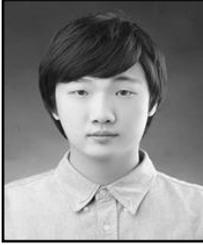
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