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New Design Approach for Sliding Mode Control of Plants with a Time Delay

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Abstract

Sliding mode control (SMC) has demonstraed good robustness, and thus, it is widely used in many control systems. However, because it is not suitable for a plant with a time delay because it is not considered a time delay control method. In this study, to solve the above problem, we propose an advanced SMC technique for plants having time delays. In our method, we introduce a simple plant predictor to the SMC method. Our method is easy to design. In simulation studies, the results show that the proposed method is applicable to plants having time delays. In an experiment using a pneumatic actuator, we confirm the practicality of the proposed method.

Keywords: *sliding mode control, time delay, plant predictor, pneumatic actuator*

1. Introduction

In control engineering, it is often problematic to maintain control because of an estimated error of the controlled equipment due to either a decrease or changes in the environment of the equipment used. For several years, researchers have attempted to solve this problem by studying robust control, and various control methods have been proposed. Sliding mode control (SMC) is one such robust control method that is based on the concept of a variable structure control system.

SMC has a switching surface, which is called a switching hyperplane, and it is used to stabilize a system [1-3]. Furthermore, the sliding mode controller can ensure good robustness by having both linearity and nonlinearity [4-6]. However, in SMC, there exists a high-frequency vibration known as chattering. This phenomenon is noticeable in systems having time delays [7]. As the time delay increases, the ability to maintain control becomes more difficult. Because time delays are often in real systems, it is important to solve this problem in practical applications of SMC.

In this study, we control a system having a time delay by adding a predictor to SMC. In the simulation study, we confirm the effectiveness of our proposed method. We also performed an actual experiments using a pneumatic actuator, and showed the practicality of the proposed method.

2. Controlled System

For this study, the state space equation of a controlled plant, which is controllable and observable, is expressed by Equation (1). This plant includes a time delay. In

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Equation (1), $y(t)$ is the plant output, $u(t)$ is the plant input, τ is the time delay, $x(t)$ is a state vector, and $x(0) = 0$.

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t-\tau) \\ y(t) = \boldsymbol{c}\boldsymbol{x}(t) \end{cases}, \tag{1}$$

where $\boldsymbol{A} \in \mathbf{R}^{n\times n}$, $\boldsymbol{b} \in \mathbf{R}^{n\times 1}$, $\boldsymbol{c} \in \mathbf{R}^{1\times n}$, $x(t) \in \mathbf{R}^{n\times 1}$

3. Servo System

The state space equations are designed on the basis of an extended system as follows.

$$\begin{cases} \dot{\boldsymbol{x}}_{se}(t) = \boldsymbol{A}_{se}\boldsymbol{x}_{se}(t) + \boldsymbol{b}_{se}u_{se}(t-\tau) \\ y_{se}(t) = \boldsymbol{c}_{se}\boldsymbol{x}_{se}(t) \end{cases}, \tag{2}$$

where

$$\boldsymbol{A}_{se} = \begin{bmatrix} A & 0 \\ -c & 0 \end{bmatrix}, \boldsymbol{b}_{se} = \begin{bmatrix} b \\ 0 \end{bmatrix}, \boldsymbol{c}_{se} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \textit{ and } \boldsymbol{x}_{se}(t) = \begin{bmatrix} x(t) \\ z(t) \end{bmatrix},$$
$$\boldsymbol{A}_{se} \in \mathbf{R}^{(n+1)\times(n+1)}, \boldsymbol{b}_{se} \in \mathbf{R}^{(n+1)\times 1}, \boldsymbol{c}_{se} \in \mathbf{R}^{1\times(n+1)}, \boldsymbol{x}_{se}(t) \in \mathbf{R}^{(n+1)\times 1}.$$

and

$$z(t) = \int (r(t) - y(t))\,dt, \dot{z}(t) = r(t) - y(t),$$

where $r(t)$ is a reference signal.

The feedback gain $F_1 \in \mathbf{R}^{1\times n}$ and servo gain $F_2 \in \mathbf{R}^{1\times 1}$ are provided by the optimal control method required to minimize the cost function:

$$J = \int_0^\infty \left\{ \boldsymbol{x}_{se}(t)^{\mathrm{T}} \boldsymbol{Q}_{se} \boldsymbol{x}_{se}(t) + r_{se}u(t)^2 \right\} dt, \tag{3}$$

where $Q_{se} \in \mathbf{R}^{(n+1)\times(n+1)}$ and $r_{se} \in \mathbf{R}^{1\times 1} \geq 0$. The feedback gain F_1 and servo gain F_2 are rewritten as $\boldsymbol{F} = [F_1\ F_2]$, where $\boldsymbol{F}$ is given by Equation (4).

$$\boldsymbol{F} = -r_{se}^{-1} \boldsymbol{B}_{se}^{\mathrm{T}} \boldsymbol{P}_{se} \tag{4}$$

The matrix $\boldsymbol{P}_{se}$ is the solution of the Riccati equation and is given by Equation (5).

$$\boldsymbol{P}_{se}\boldsymbol{A}_{se} + \boldsymbol{A}_{se}^{\mathrm{T}}\boldsymbol{P}_{se} - \boldsymbol{P}_{se}\boldsymbol{B}_{se}r_{se}^{-1}\boldsymbol{B}_{se}^{\mathrm{T}}\boldsymbol{P}_{se} + \boldsymbol{Q}_{se} = \boldsymbol{0}, \boldsymbol{P}_{se} \in \mathbf{R}^{n\times n} \tag{5}$$

4. SMC

SMC [8] is designed by broadly the switching hyperplane and sliding mode controller. The SMC system is expressed as follows:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \\ \sigma(t) = \boldsymbol{S}\boldsymbol{x}(t) \end{cases}, S \in \mathbf{R}^{n\times 1}, \tag{6}$$

Where $\sigma(t)$ is a switching function and $\boldsymbol{S}$ is the gradient of the switching hyperplane.

4.1 Switching Hyperplane

In this section, we explain the design of the switching hyperplane. The gradient $\boldsymbol{S}$ is required when designing a switching hyperplane, and is determined by solving the Riccati equation, as given by the following:

$$\boldsymbol{P}_s\boldsymbol{A}_\varepsilon + \boldsymbol{A}_\varepsilon^{\mathrm{T}}\boldsymbol{P}_s - \boldsymbol{P}_s\boldsymbol{B}r_s^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_s + \boldsymbol{Q}_s = 0, \boldsymbol{S} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}_s\,, \qquad (7)$$

where $\boldsymbol{A}_\varepsilon = \boldsymbol{A} + \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon} \in \mathbf{R}^{n\times n} \geq 0,\ \boldsymbol{P}_s \in \mathbf{R}^{n\times n}$, and $r_s \in \mathbf{R}^{1\times 1}$

On the basis of $\boldsymbol{S}(s\boldsymbol{I} - \boldsymbol{A}^{\mathrm{T}} + \boldsymbol{BS})^{-1}\boldsymbol{B}$ becomes strictly positive real; thus, its zero-point is stable.

4.2 Sliding Mode Controller

The control input $u(t)$ in the sliding mode controller consists of a linear input $u_l(t)$ and a nonlinear input $u_{nl}(t)$. The control input $u(t)$ is expressed as follows:

$$u(t) = u_l(t) + u_{nl}(t) = -(\boldsymbol{SB})^{-1}\boldsymbol{SA}x(t) - k\frac{\sigma(t)}{|\sigma(t)|}, \qquad (8)$$

where $u_l(t)$ is an equivalent controlled input and $k \in \mathbf{R}^{1\times 1}$ is the nonlinear input gain.

We determined the nonlinear input gain k by using the stability theorem of Lyapunov[9].

4.3 Prevention of the chattering phenomenon

In fact, if the sliding mode controller is designed using $u_{nl}(t)$ in Equation (8), chattering occurs. When $\sigma(t) = 0$ for a nonlinear input, $|\sigma(t)|$ in the denominator becomes 0 and quick switching of the input occurs. To avoid chattering, we rewrite Equation (8) as follows:

$$u(t) = u_l(t) + u_{nl}(t) = -(\boldsymbol{SB})^{-1}\boldsymbol{SA}\boldsymbol{x}(t) - k\frac{\sigma(t)}{|\sigma(t)| + \delta}, \delta \in \mathbf{R}^{1\times 1}, \delta > 0\,. \qquad (9)$$

5. Plant Predictor

In this section, we explain the design method for a plant predictor in continuous time [10]. The state space equations in discrete time are obtained by discretizing Equation (1) using the zero-order hold. We calculate the plant predictor for the following equations:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k-d) \\ \qquad y(k) = \boldsymbol{c}\boldsymbol{x}(k) \end{cases}, \qquad (10)$$

where d represents the time delay. In Equation (10), we apply the substitution $k = k + 1$, and obtain the state of the plant at a sampling time of $k + 2$, while we obtain the state of the plant at a sampling time $k + d$ by repeating this calculation, as shown in Equation (11).

$$\begin{aligned}\tilde{\boldsymbol{x}}(k+2) &= \boldsymbol{A}\boldsymbol{x}(k+1)+\boldsymbol{b}u(k-d+1) \\ &= \boldsymbol{A}^2\boldsymbol{x}(k)+\boldsymbol{A}\boldsymbol{b}u(k-d)+\boldsymbol{b}u(k-d+1) \\ &\vdots \\ \tilde{\boldsymbol{x}}(k+d) &= \boldsymbol{A}^d\boldsymbol{x}(k)+\boldsymbol{A}^{d-1}\boldsymbol{b}u(k-d)+\cdots+\boldsymbol{b}u(k-1) \\ &= \boldsymbol{A}^d\boldsymbol{x}(k)+\sum_{i=1}^{d}\boldsymbol{A}^{d-i}\boldsymbol{b}u(k-d+i-1)\end{aligned} \tag{21}$$

The predicted output of the plant is given by

$$\tilde{y}(k+d)=\boldsymbol{c}\tilde{\boldsymbol{x}}(k+d). \tag{32}$$

Finally, we convert Equation (11) in continuous time. Because these predicted state variables are not observed directly, an observer estimates the state variables of the plant. $\boldsymbol{L}_o \in \mathbf{R}^{1\times n}$ in Equation (13) represents the observer gain.

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\boldsymbol{A}\hat{\boldsymbol{x}}(t)+\boldsymbol{b}u(t-\tau)+\boldsymbol{L}_o(y(t)-\hat{y}(t)) \\ \hat{y}(t)=\boldsymbol{c}\hat{\boldsymbol{x}}(t)\end{cases} \tag{43}$$

6. Proposed Method

Figure 1 shows a block diagram of the proposed method. The proposed method consists of SMC and a plant predictor.

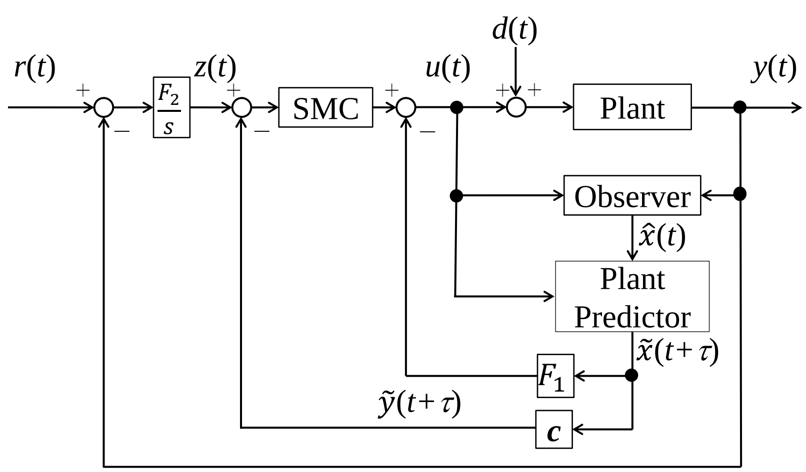

Figure 1. Block diagram of the proposed method

7. Simulation Results

In this section, we conduct two case simulations to confirm that the proposed method is effective for a plant having a time delay.

7.1. Case 1

In Case 1, we assume a first-order plant having a time delay as given in Equation (14). Equation (14) expresses the transfer function of a pneumatic actuator. In this simulation, the sampling time T_s is 10 [ms] a step input of 5.0/s is introduced at t = 0 [s], and the step disturbance of −0.5/s is introduced at t = 50 [s]. We add a 1.5 times error in the transfer function of the controlled plant to confirm the robustness of the proposed method.

$$G(s) = \frac{1 \times error}{0.229s} e^{-0.211s} \quad \textbf{(54)}$$

The parameters used for the proposed method in this simulation are shown in Table 1. Figure 2 shows the results of the simulation for a first-order plant having a time delay(Case 1).

Table 1. Parameters used in the simulations

Parameters	Case 1	Case 2
Q_{se}	0.01	1000
r_{se}	1	1
$\boldsymbol{F}_1$	0.214	[2.922 0.103]
F_2	0.100	31.623
$\boldsymbol{Q}_s$	100	[100 0.01]
r_s	1	1
$\boldsymbol{S}$	10.231	[10.268 0.238]
$\boldsymbol{\varepsilon}$	1	[1 0;0 1]
k	5	60
δ	100	100
$\boldsymbol{L}_o$	436.681	[250 31264]

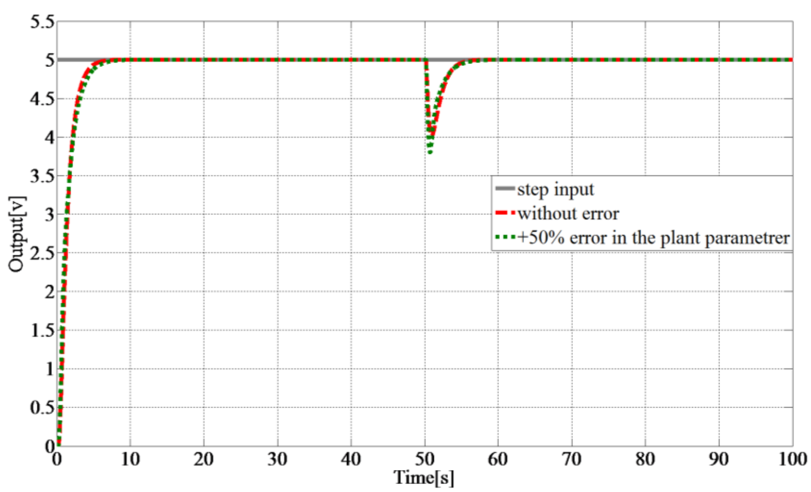

Figure 2. Simulation results for the Case 1

7.2. Case 2

In Case 2, we assume a second-order plant having a time delay as given by Equation (15). Equation (15) expresses the transfer function of a DC motor. In this simulation, the sampling time T_s is 1 [ms], a step input of 1.0/s is introduced at t = 0 [s], and the step disturbance of −0.1/s is introduced at t = 2.5 [s]. We add a 1.5 times error in the controlled plant parameters $\boldsymbol{A}$ and $\boldsymbol{b}$, respectively, to confirm the robustness of the proposed method.

$$G(s)=\frac{339.6}{s(s+10.78)}e^{-0.004s} \tag{65}$$
$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ 0 & -10.78\times error\end{bmatrix},$$
$$\boldsymbol{b}=\begin{bmatrix}0\\ 339.6\times error\end{bmatrix},$$
$$\boldsymbol{c}=\begin{bmatrix}1 & 0\end{bmatrix}.$$

The parameters used for the proposed method in this simulation are shown in Table 1. Figure 3 shows the results of a second-order plant having a time delay(Case 2).

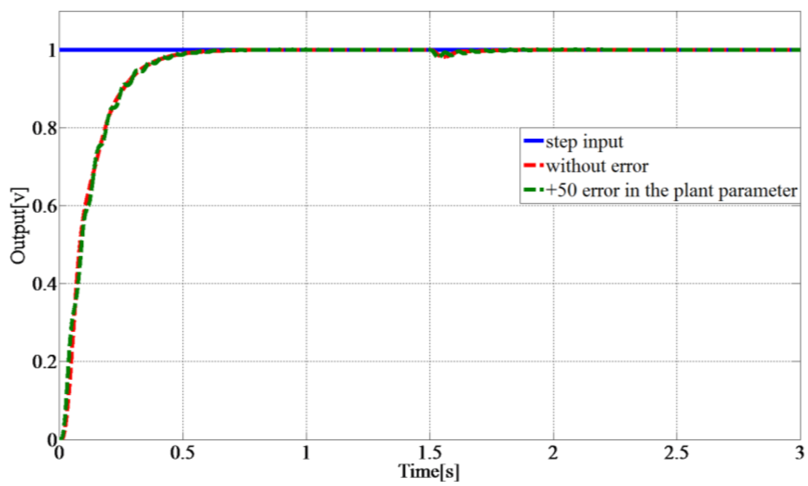

Figure 3. Simulation results for Case 2

The simulation results of Case 1 and Case 2 show that the proposed method is widely applicable to a plant having a time delay. Furthermore, the method demonstrates good robustness for plant parameter variations.

8. Experimental Results

In this section, we experimentally confirm the practicality of the proposed method using a pneumatic actuator. The transfer function of the pneumatic actuator is given in Equation (16). The actuator varies from 0°–90°, and the angle is detected by a sensor and outputs a voltage from −10 to 10 [V], depending on the angle. The air pressure is 0.4 [MPa].

In this experiment, the sampling time T_s is 10 [ms], and the step input 5.0/s is introduced at t = 0 [s]. In addition, the parameters of the proposed method are shown in Table 2. Figure 4 shows the experimental results.

$$G(s) = \frac{1}{0.229s} e^{-0.211s} \qquad (76)$$

Table 2. Parameters used in the experiment

Parameters	Value
Q_{se}	0.05
r_{se}	1
$\boldsymbol{F}_1$	0.320
F_2	0.223
$\boldsymbol{Q}_s$	100
r_s	1
$\boldsymbol{S}$	10.231
$\boldsymbol{\varepsilon}$	1
k	7
δ	100
$\boldsymbol{L}_o$	436.681

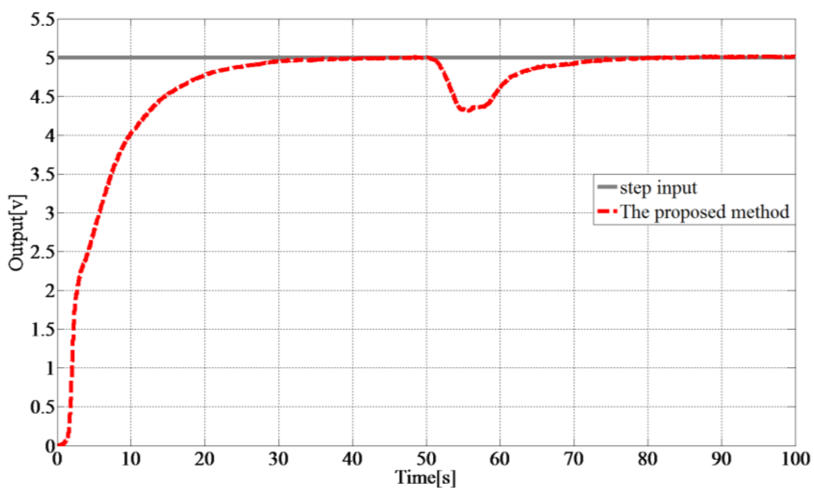

Figure 4. Experimental Results

From the experimental results, we can confirm that our proposed method accurately follows the step input. This indicates that the proposed method can be used for a plant having a long time delay.

9. Conclusion

In this study, we constructed a robust control system corresponding to a system with a time delay by introducing a predictor to control the SMC method. The proposed method was shown to control many process systems such as chemical and thermal processes. Moreover, the system is simple to construct. In the simulation studies, we have shown the effectiveness of the proposed method. Furthermore, we confirmed the practicality of the proposed method experimentally by adapting it to a pneumatic actuator.

References

[1] M. C. Saaj, B. Bandyapadhyay and H. Unbehauen, "A New Algorithm for Discrete-Time Sliding-Mode Control Using Fast Output Sampling Feedback", IEEE Transaction on Industrial Electronics, vol. 49, no. 3, **(2002)** June, pp. 518-523.
[2] M. Uchida, T. Yabumi, Y. Morita and H. Kando, "Positioning Control of Liner Motor via Sliding Mode Control System with Variable Switching Hyperplane", IEEJ Trans. EIS, vol. 126, no. 9, **(2006)**, pp. 1112-1118.
[3] S. Lcc and C. C. Chung, "Sliding Mode Control Design Using Fast Output Sampling", Proceedings of the 42th IEEE, Conference on Decision and Control, **(2003)** December; Maui, Hawaii USA, pp. 3543-3548.
[4] F. Piltan, S. Siamak, M.A. Bairami and I. Nazari, "Gradient Descent Optimal Chattering Free Sliding Mode Fuzzy Control Design: Lyapunov Approach", IJAST, vol. 45, **(2012)** August, pp. 73-90.
[5] F. Plitan, M. R. Rashidian, M. Shamsodini and S. Allahdadi, "Effect of Rule Base on the Fuzzy-Based Tuning Fuzzy Sliding Mode Controller: Applied to 2nd Order Nonlinear System", IJAST, vol. 46, **(2012)** September, pp. 39-70.
[6] F. Piltan, S. Mehrara, R. Bayat and S. Rahmdel, "Design New Control Methodology of Industrial Robot Manipulator: Sliding Mode Baseline Methodology", IJHIT, vol. 5, no. 4, **(2012)** October, pp. 41-54.
[7] Y. Kanai, M. Kasahara and Y. Mori, "Discrete-Time Sliding Mode Control with Varying Switching Hyperplane which is Moving in the Specified Region", IEEJ Trans. EIS, vol. 130, no. 10, **(2010)**, pp. 1833-1839.
[8] T. Ishibasahi, K. Kawaguchi, H. Shibasaki, R. Tanaka, T. Murakami and Y. Ishida, "Sliding Mode Control with a State Predictor for Time Delay Control System", Proceedings of IEEA 2013, **(2013)** March 16-17; Bali, Indonesia.
[9] K. Nonami, H. Nishimura and M. Hirata, "Control System Design by using MATLAB (in Japanese)", Tokyo Denki University, **(1998)**, pp. 163-201.
[10] K. Kawaguchi, H. Shibasaki and Y. Ishida, "The modified Smith predictor with a Plant Predictor in Continuous-Time for an Unstable Plant with Time Delay", IJMO, vol. 3, no. 1, **(2013)**, pp. 98-100.

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