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Study on Sliding Mode Control with Reaching Law for DC Motor

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Abstract

The DC motor is a key element for servo system, and the control quality of control system would be seriously affected by the nonlinear, the parameter uncertain, the inaccurately modeling and some uncertain existing in servo system. So it is very difficult to satisfy control requirements by using the traditional control method. In this paper, the sliding mode control algorithm with reaching law is proposed. By using the reaching law to weaken the chattering phenomenon existed in the SMC, also by using the SMC method can enhance the anti-interference ability of the servo system. In order to test and verify the effectiveness of the proposed SMC with reaching law for the DC motor, the designed controller was dynamically simulated by using the MATLAB. It is shown from the simulation results that the proposed controller offers several advantages such as good chattering inhibition capability, fast response, good disturbance rejection capability, good position tracking capability and so forth. It is also revealed that the proposed control strategy is valid for the DC servo system from simulation results. The results also provide the certain theoretical basis and application value for engineering practice.

Keywords: *DC Motor, Sliding Mode Control, Reaching Law, Chattering*

1. Introduction

DC motor is an important executive element in mechatronics system because of its small size, light weight, good servo performance, fast response and good stability. These advantages make it widely used in servo systems, factory automation, defense industry, household appliances and instruments field. However, with the rapid development of electronic control and microelectronics technology, the dynamic controlling quality for the DC motor require more higher than before [1, 2].

Recently, sliding mode control has been used in electro-hydraulic servo control widely [3], due to its simple algorithm, good robustness against parameter uncertainties and external disturbance, high reliability, fast response, and so on. However, the chattering phenomenon existed in the Sliding Mode Control (SMC), which was caused by the time lag switch, spatial lag switch, inertia, uncertainties and other factors, has seriously impacted the application of the Sliding Mode Control [4].

Due to the chattering phenomenon, the high-frequency unmodeled in the system would be excited easily, and the effectiveness of the system also would be affected. In order to reduce the chattering phenomenon in the SMC, some ways have been proposed such as the quasi-sliding mode control, Sliding mode control with reaching law, Fuzzy sliding mode control,

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Dynamic sliding mode control, Adaptive sliding mode control, Integral sliding mode control, *etc*. [5-7].

In this paper, a sliding mode control with improved reaching law is proposed to reduce the chattering. The sliding mode controller with an improved reaching law has been designed, and it is applied in a DC motor servo system.

The remainder of the paper is organized as follows. The mathematical model of the DC motor is established in the Section 2. In Section 3, the SMC controller with an improved reaching law is discussed and designed. In Section 4, we make numerical simulations made for the DC motor transmission system using the proposed control method. Conclusions are drawn in Section 5.

2. Mathematical Model

In the modern industry, DC motor is widely used as the Executive Termination of the servo system. Based on the working principle of magnetic brushless DC motor, the mathematical model is established in this paper [6]. The DC motor's equivalent circuit is shown in Figure 1.

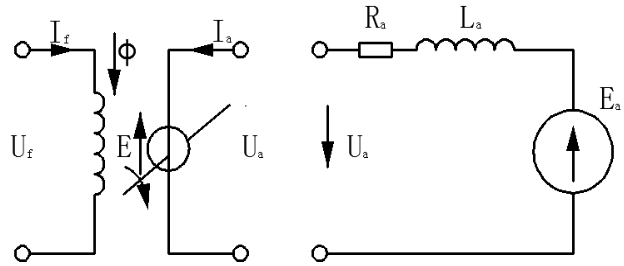

Figure 1. The DC motor's equivalent circuit

Based on the DC motor's equivalent circuit, the voltage balance equation of the DC motor armature circuit is as follows:

$$U_a = E_a + R_a I_a = C_E \Phi_n + I_a R_a = K_E n + R_a I_a \tag{1}$$

The dynamic equation of DC motor is given:

$$u_a = K_E n + R_a i_a + L_a \frac{di_a}{dt} \tag{2}$$

In the above equations R_a is the resistance of loop, I_a is the current, E_a is the induction electromotive force, U_a is the voltage of the circuit, C_E is the electromotive force constant, n is the speed of motor, K_E is the electromotive force which is produced by unit speed.

Balanced equation of electrodynamics is as follows:

$$T_e = Bn + T_L + J\frac{dn}{dt} = C_E \Phi i_a = K_T i_a \tag{3}$$

In equation (3), T_e is the instantaneous electromagnetic torque, T_L is the load torque, B is damping coefficient, J is the moment of inertia, K_T is torque constant.

Assuming that the initial condition is zero and the motor load is fixed. After the above equations being transformed via using the Laplace way, the transfer function of DC motor can be expressed:

$$G(s)=\frac{n(s)}{U(s)}=\frac{K_T}{L_aJs^2+\left(R_aJ+L_aB\right)s+R_aB+K_TK_E} \tag{4}$$

3. Design of Sliding Model Controller

From the paper [4, 5], in the sliding model control, the time lag switch, space d lag switch, inertia and other factors resulted in the chattering phenomenon, because of it, the system can not strictly slide along the sliding mode for the balance. Due to the chattering phenomenon, the high-frequency unmodeled in the system would be excited easily and the effectiveness of the system also would be affected. So it is important for us to take effective method to weaken or inhibit the chattering phenomenon, and which is regarded as a hot issue and an important research direction in the sliding model control [7-9].

3.1 Reaching Law

W. B. Gao, an expert who studied the SMC in china, has proposed the reaching law approach to reduce or inhibit the chattering of SMC in the premise of ensuring the condition of sliding existence $S\dot{S}<0$ has been meet [8]. He has given four reaching laws (constant reaching law, exponential approach law, power reaching law and general reaching law), and the exponential approach is applied widely [4]:

Constant reaching law

$$\dot{s}=-\varepsilon signs \qquad \varepsilon>0 \tag{5}$$

Exponential reaching law

$$\dot{s}=-\varepsilon signs-qs \qquad \varepsilon>0;q>0 \tag{6}$$

Power reaching law

$$\dot{s}=-q\,|\,s\,|^{\alpha}\operatorname{sgn}s \qquad 1>\alpha>0;q>0 \tag{7}$$

General reaching law

$$\dot{s}=-\varepsilon signs-f(s) \qquad \varepsilon>0 \tag{8}$$

where ε is reaching velocity.

When we use the exponential reaching law method to reduce or inhibit the chattering in the SMC, the velocity of s is related to the parameter ε. And the reaching speed reduces to zero gradually, which make the moving point move to switch surface with a small speed. By adjusting the parameters ε and q of the exponential reaching law we can guarantee dynamic quality of the process of sliding mode reaching, and weaken the chattering existed in the SMC method. The discrete form of exponential reaching law's switching zone is zonal. So which can't be close to the origin ultimately but a chattering near the origin during the process moving. So as to reduce the chattering phenomenon existed in the SMC with the exponential reaching law, K. B. Park and K. W. Tong proposed a variable rate reaching law for SMC. The reaching law is discrete form as follows [10-13]:

$$S(k+1)-S(k)=-\varepsilon T\left\|X\right\|_1 \operatorname{sgn}(S(k)) \qquad (9)$$

Where $\|X\|_1$ -- 1 norm of x .

Reaching speed of the variable rate reaching law is $\varepsilon\|X\|_1$, which is proportional to the $\|X\|_1$. The switching zone gets through the origin with two rays, in the middle of the two rays the S = 0 can be get by it, it can be stabilized at the origin. However, $\|X\|_1$ will get a large value and the SMC has a big chattering after the system entered to switching zone. In order to overcome the problem of the variable rate reaching law and exponential approach law, we have proposed a novel reaching law, which is expressed as follows:

$$S(k+1)=(1-Tq)S(k)-\varepsilon T \tan sig(\|X\|_1)\operatorname{sgn}(S(k)) \qquad (10)$$

Where $\tan sig(\|X\|_1)=2sig(\|X\|_1)-1=\frac{1-e^{-\|X\|_1}}{1+e^{-\|X\|_1}}$.

3.2 The chattering analysis of exponential reaching law control

The discrete reaching law based on the exponential is given as:

$$s(k+1)=(1-qT)s(k)-\varepsilon T \operatorname{sgn}(s(k)) \qquad (11)$$

$$s(k+1)=(1-qT)s(k)-\varepsilon T\frac{s(k)}{|s(k)|}=(1-qT-\frac{\varepsilon T}{|s(k)|})s(k)=ps(k) \qquad (12)$$

Where the sampling time T is very small, $T<<1.0$.

From the (12) we can obtain:

$$|p|=\frac{|s(k+1)|}{|s(k)|}, \quad p=1-qT-\frac{\varepsilon T}{|s(k)|}<1.0 \qquad (13)$$

Aim at the (12), we can discuss according to the following three conditions:

Conditions ONE:

$|s(k)|>\frac{\varepsilon T}{2-qT}$, then

$$p>1-qT-\frac{\varepsilon T(2-qT)}{\varepsilon T}$$

$$p>-1$$

So $|p|<1$, $|s(k+1)|<|s(k)|$, $|s(k)|$ is decreasing.

Conditions TWO:

$|s(k)|<\frac{\varepsilon T}{2-qT}$, then

$$p < 1 - qT - \frac{\varepsilon T(2-qT)}{\varepsilon T}$$

$$p < -1$$

So $|p| > 1$, $|s(k+1)| > |s(k)|$, $|s(k)|$ is increasing.

Conditions THREE:

$$|s(k)| = \frac{\varepsilon T}{2-qT}, \text{ then}$$

$$p = 1 - qT - \frac{\varepsilon T(2-qT)}{\varepsilon T} = -1$$

So $|p| = 1$, $|s(k+1)| = |s(k)|$, $|s(k)|$ goes in the state of oscillation.

From the above analysis, we can obtain that the sufficient condition of the $s(k)$ decreasing is:

$$|s(k)| > \frac{\varepsilon T}{2-qT} \tag{14}$$

In the process of sliding mode, the $|s(k)|$ infinitely close to $\frac{\varepsilon T}{2-qT}$. Once the $|s(k)| = \frac{\varepsilon T}{2-qT}$ is met, the system entered the oscillation state. For the any initial value $s(0) \neq 0$. when $k \to \infty$, $|s(k)| \to \frac{\varepsilon T}{2-qT}$, and $|s(k)| = \frac{\varepsilon T}{2-qT}$, then $s(k+1) = -s(k)$.

So $k \to \infty$, the sliding mode motion of steady-state oscillation amplitude is given:

$$h = \frac{\varepsilon T}{2-qT} \tag{15}$$

We can obtain that the convergence degree of $|s(k)|$ is influenced by ε, q, T, especially ε, T. When the ε, T are small enough, the $|s(k)|$ can become smaller.

In the discrete reaching law (6), the effect of the parameter ε is very large. When the ε is decreased, the chattering of the system can reduced. But the ε is too small to affect the system to reach the approach speed of switching surface. At the same time due to factors such as technology, equipment, the sampling period T could not be selected as smaller. Therefore, the ideal ε should be time-varying, namely when the system begin moving, the ε is selected as bigger. The ε should be reduced gradually with the increase of time.

From the (14) , when the $|s(k)| > \frac{\varepsilon T}{2 - qT}$ is existed, the $|s(k)|$ only will decrease, and which requires $qT + \frac{\varepsilon T}{|s(k)|} < 2$, namely the ε should be satisfied.

$$\varepsilon < \frac{1}{T}(2 - Tq)|s(k)| \tag{16}$$

Select

$$\varepsilon = |s(k)|/2 \tag{17}$$

Obvious, if sampling time T meets

$$T < \frac{4}{1 + 2q} \tag{18}$$

So (13) can be satisfied.

From (6) and (12), the modified reaching law is given as:

$$s(k+1) - s(k) = -qTs(k) - \frac{|s(k)|}{2} T \operatorname{sgn}(s(k)) \tag{19}$$

Aim at the discrete system $x(k+1) = Ax(k) + Bu(k)$, assuming $CB \neq 0$.

The control law of discrete reaching law (16) is given:

$$u(k) = -(CB)^{-1}[CAx(k) - (1 - qT)s(k) + \frac{|s(k)|}{2} T \operatorname{sgn}(s(k))] \tag{20}$$

Where $s(k) = Cx(k)$.

3.3 Stability analysis

THEOREMS 1: The designed controller should ensure the exist condition of sliding mode [14-16].

PROOF: Consider the following function as the Lyapunov function to prove the Stability of the proposed algorithm.

Define the yapunov function as:

$$V(k) = \frac{1}{2} S(k)^2 \tag{21}$$

Then:

$$V(k+1) = \frac{1}{2} S(k+1)^2 \tag{22}$$

$$\begin{aligned} V(k+1) - V(k) &= \frac{1}{2}[S(k+1)^2 - S(k)^2] \\ &= \frac{1}{2}[S(k+1) - S(k)][S(k+1) - S(k)] \end{aligned} \tag{23}$$

$$\because [s(k+1)-s(k)]\operatorname{sgn}(s(k)) = [-qTs(k) - \frac{|s(k)|}{2} T \operatorname{sgn}(s(k))]\operatorname{sgn}(s(k))$$
$$= -(q+0.5)T\,|s(k)| < 0 \tag{24}$$

$$[s(k+1)+s(k)]\operatorname{sgn}(s(k)) = [(2-qT)s(k) - \frac{|s(k)|}{2} T \operatorname{sgn}(s(k))]\operatorname{sgn}(s(k))$$
$$= (2-0.5T-qT)\,|s(k)| > 0 \tag{25}$$

For the former case, We can obtain the following equation:

$$V(k+1)-V(k) = \frac{1}{2}[S(k+1)^2 - S(k)^2]$$
$$= \frac{1}{2}\{[S(k+1)-S(k)]\operatorname{sgn}(s(k))\}\{[S(k+1)-S(k)]\operatorname{sgn}(s(k))\} \tag{26}$$

By equation (24), (25), we can get:

$$V(k+1)-V(k) < 0 \tag{27}$$
$$\therefore\ V(k+1) < V(k) \tag{28}$$

From the equation (27), we can get that the proposed sliding reaching law meets the existence and reaching of discrete sliding mode, and the designed control system is stable.

4. Simulation

In this section, sliding mode control with reaching law is made simulation for DC motor by using the MATLAB [17]. The transfer function of the DC motor is:

$$G(s) = \frac{103}{s^2+15s}$$
(29)

The simplified discrete equation is given as following:

$$x(k) = Ax(k-1) + B(u(k-1)+d(k-1)) \tag{30}$$

Where $A = \begin{bmatrix} 1.0 & 0.001 \\ 0.0 & 0.97 \end{bmatrix}$, $B = \begin{bmatrix} 0.000825 \\ 0.256470 \end{bmatrix}$.

Where the sampling time is 1ms, the ideal position is selected as $x(k) = \sin(t)$, the interference signal is selected as $d(k) = 1.5\sin(t)$. The parameter of controller is selected as $c = 10$, $\varepsilon = 15$, $q = 30$. The steady-state oscillation amplitude of the sliding mode motion is $h = \frac{\varepsilon T}{2-qT} = 0.0076$.

Simulation results are shown in the Figure 2 to Figure 7. These simulation results demonstrate that SMC control principle with reaching law has better tracking performance comparing the SMC control principle without reaching law. The Figure 2-Figure 3 shows that the DC motor has better position tracking performance. The Figure 4-Figuare 5 shows that the DC motor has better speed tracking performance.

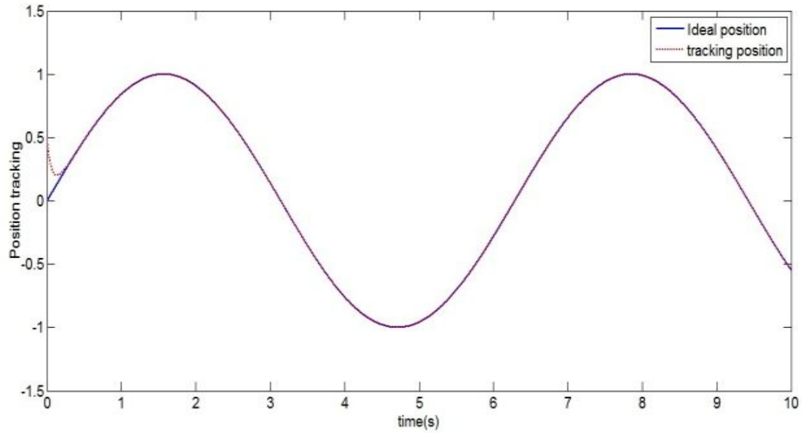

Figure 2. Position tracking of SMC

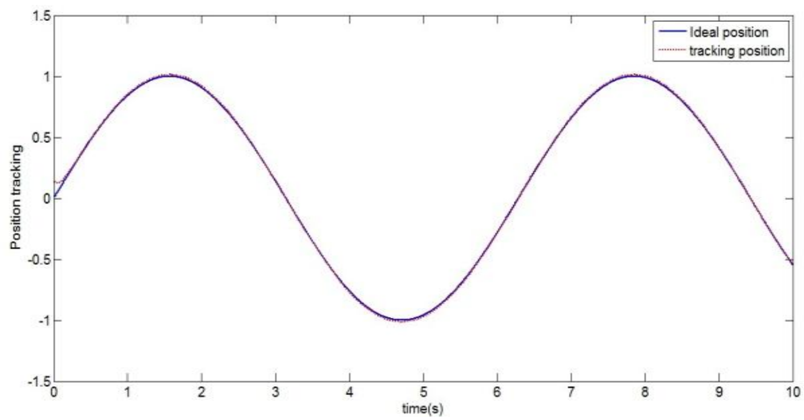

Figure 3. Position tracking of SMC with reaching law

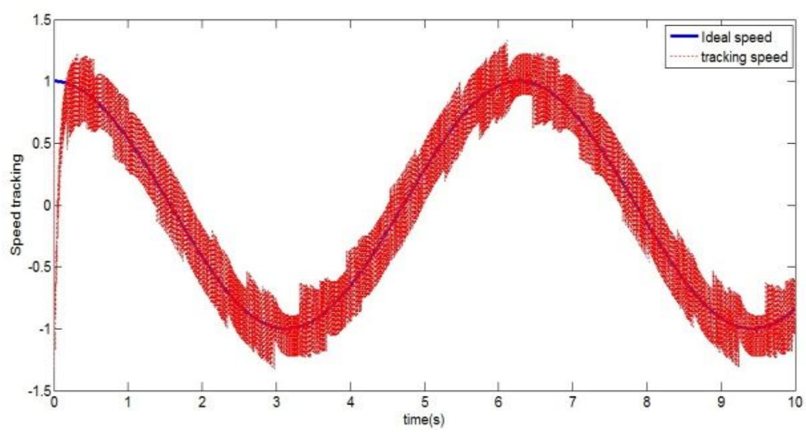

Figure 4. Speed tracking of SMC

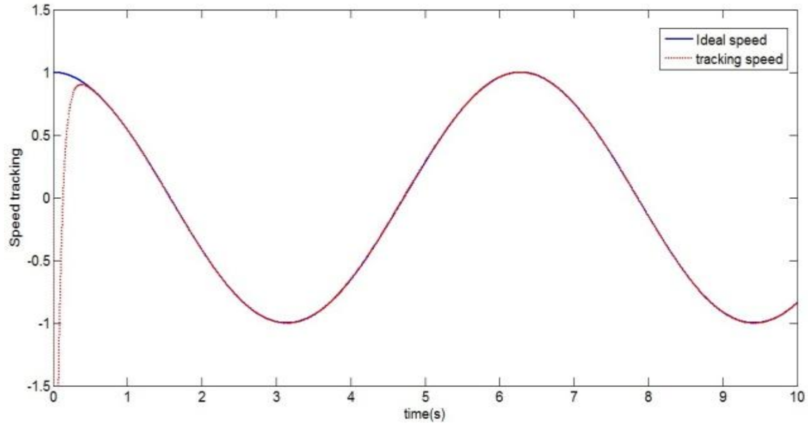

Figure 5. Speed tracking of SMC with reaching law

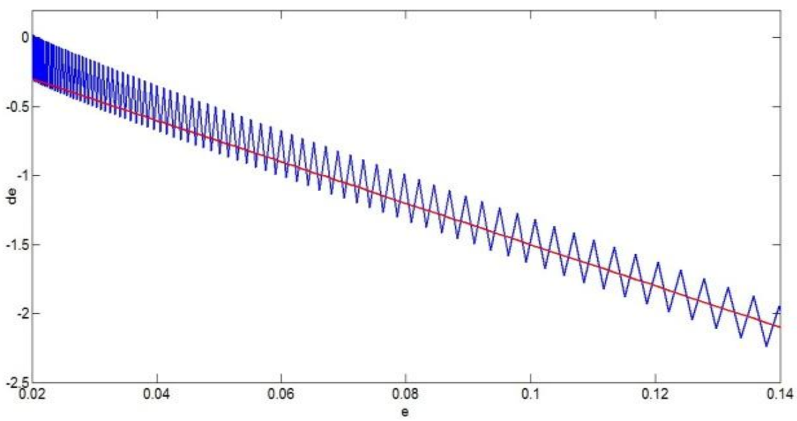

Figure 6. The phase trajectory of SMC

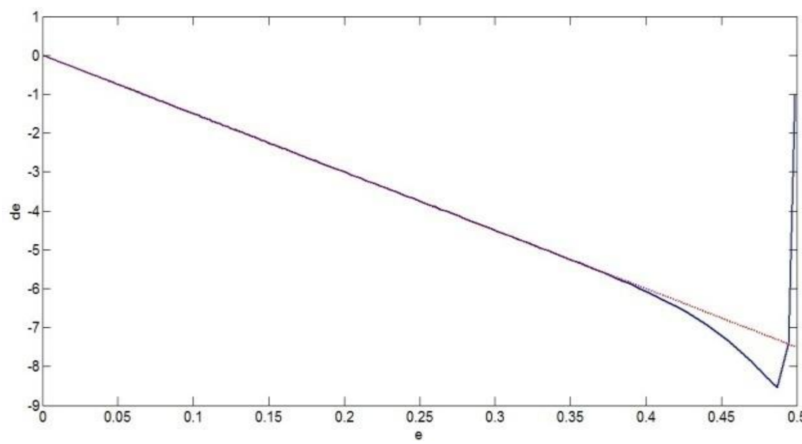

Figure 7. The phase trajectory of SMC with reaching law

By the simulation results we can get the conclusions that the proposed SMC with reaching law can better inhibit the chatting and the method makes the system stability and control quality been further improved. The results also provide the certain theoretical basis and application value for engineering practice.

5. Conclusion

In the work reported here, we investigated the suitability of variable structure methods for the DC motor systems. Aim at the high order, nonlinear, parameter uncertain system, the inaccurately modeling and some uncertain existing in the DC motor, which seriously affected the control quality of control system. So it is very difficult to gain good performance with traditional control method.

The sliding mode control algorithm with an improved reaching law was proposed in this paper. The improved reaching law was used to weaken the chattering phenomenon existed in the sliding mode control. And which make the servo system have strong ability of anti-interference and the ability of weakening the chattering problem existed in the sliding mode control. The designed controller was dynamically simulated by using the MATLAB. The results shown that the ability of anti-interference and the ability of weakening are enhanced by using the proposed sliding mode control algorithm with an improved reaching law, and the method makes the system stability and control quality been further improved. The results also provide the certain theoretical basis and application value for engineering practice.

Compared with the general sliding mode control scheme, the main advantages of the adopted nonlinear control design approach applied to the DC motor system are as follows:

(1) It has a good performance on velocity tracking, and the proposed control strategy is valid for the DC motor System.

(2) The external disturbance and parameter perturbation of the DC motor system were predicted and compensated effectively.

(3) The problem of chattering was inhibited obviously and the ability of anti-interference and anti-parameters perturbation, stability and control quality of the system was improved.

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