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Use of Sensitivity Method for the Detection and the Localization of Linear System Defect

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Abstract

The methods of the fault diagnosis and degradations used in the different industrial sectors are various and consider the specifity of the materials forming their industrial processes. For some relatively simple processes, the relations between the causes and their effects are Biunivocal and the diagnosis with reverse reasoning is easy. Contrarily for the complexes processes, the situation is slightly different and is impossible to precede a deductive reasoning. The diagnosis is therefore only possible by using a different and a complementary knowledge. The sensitivity method for the detection and localization of the linear system defects is used in this paper.

Keywords: *- Sensitivity, linear system, fault, faulty and defect*

1. Introduction

Fault detection is a very wide field of research. Indeed, in industrial technology fields such as transport of energy [1, 2] and aerospace [3], defects must absolutely be detected.

In this paper, we present a diagnosis method based on the sensitivity theory [4]. This method can detect and localize the defects that may exist in a linear system.

Once the model of the faulty linear system is obtained [5-8], the sensitivity matrix of the initial system is computed and the tolerance intervals of all the system are defined, we can detect and localize the faults that affect the system. This method has been applied to second and third order linear systems.

2. Method Principle

For a second order system, the localization problem doesn't arise really, since, having two equations with two unknown parameters, we can determine their values and check their appurtenance to tolerance intervals. However, for higher order system this assumption is not guaranteed [9-11].

For a third order system with depending coefficients, the variation of two unknown parameters should verify a system of three equations. This kind of systems not always have solutions, determining of both parameters remains almost impossible. In such this case, we use the sensitivity which enables to determine from the variations of the transfer function coefficient, the parameter causing the fault [5, 12-14], knowing the influence of its variation on these coefficients. The principle of the sensitivity method is described by Figure 1.

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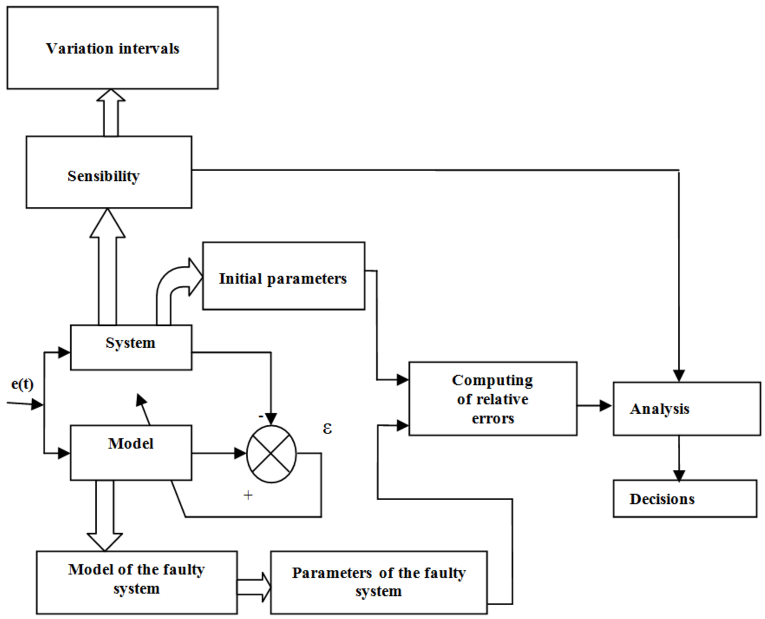

Figure 1. Fault detection and localization using the sensitivity technique

3. Theory of the Sensitivity of System Performances with Respect to Parameter Variations

H(p,x) the transfer function of a given system depending on a real parameter *x* which has a nominal value x_n and the corresponding transfer function H(p,xn) is noted H(p). In the following, we focus on the variation of the real x around its nominal value x_n. We show that H(p,x) is a homographic function of x such as :

$$H(p,x)=\frac{C(p)+x\,D(p)}{A(p)+x\,B(p)} \qquad (1)$$

the coefficients of polynomials A(p), B(p), C(p) and D(p) are real, independent of *x*.

3.1. Classical Sensitivity

Assume that *x* is the only parameter which varies. The classical sensitivity of a transfer function H(p,x) with respect to *x* is:

$$S_x^H(p,x)=\frac{dH/H}{dx/x}=\frac{d(\log H)}{d(\log x)} \qquad (2)$$

3.1.1. Properties

If $H(p,x)=\dfrac{P(p,x)}{Q(p,x)}$ we have:

$$S_x^H(p,x) = x\left[\frac{1}{P(p,x)}\frac{\partial P}{\partial x} - \frac{1}{Q(p,x)}\frac{\partial Q}{\partial x}\right] \quad (3)$$

Let’s consider the frequential study $(p = j\omega)$:

$$\mathrm{Log}\left[H(j\omega,x)\right] = \mathrm{Log}\left|H(j\omega,x)\right| + j\mathrm{Arg}\left(H(j\omega,x)\right) \quad (4)$$

$$\begin{aligned} S_x^H(p,x) &= \frac{d(\log H)}{d(\log x)} \\ &= x\frac{d\left[\mathrm{Log}\left|H(j\omega,x)\right|\right]}{dx} + jx\frac{d\left[\mathrm{Arg}\left(H(j\omega,x)\right)\right]}{dx} \end{aligned} \quad (5)$$

We deduce the following equalities:

$$\mathrm{Re}\left(S_x^H\right) = x\frac{d\left[\mathrm{Log}\left|H\right|\right]}{dx} = \frac{x}{dx}\frac{d\left|H\right|}{\left|H\right|} \quad (6)$$

$$\mathrm{Im}\left(S_x^H\right) = x\frac{d\left(\mathrm{Arg}(H)\right)}{dx} \quad (7)$$

For a small variation δx of x we have then:

$$\frac{\delta\left|H\right|}{\left|H\right|} = \mathrm{Re}\left(S_x^H\right)\frac{\delta x}{x} \quad (8)$$

and

$$\delta\left(\mathrm{Arg}(H)\right) = \mathrm{Im}\left(S_x^H\right)\frac{\delta x}{x} \quad (9)$$

3.2. Generalization

When H(p) depends of many parameters x_1, x_2, …, x_n, we have:

$$dH = \frac{\partial H}{\partial x_1}\left(p, x_1, x_2, ..., x_n\right)\ dx_1 + \frac{\partial H}{\partial x_2} dx_2 + \dots + \frac{\partial H}{\partial x_n} dx_n \quad (10)$$

$$\frac{dH}{H} = d\left(\text{Log}H\right) = \sum_{i=1}^{n} S_{xi}^{H} \frac{dx_i}{x_i} \quad (11)$$

We define the vector S^H as:

$$S_{xi}^{H} = \frac{\partial\left(\text{Log}H\right)}{\partial\left(\text{Log}x_i\right)} \quad (12)$$

and the column vector Δ the term of which are d(Log x_i), we have:

$$\frac{dH}{H} = (S^H)^T.\Delta \quad (13)$$

Where $(S^H)^T$ it the transpose of S^H.

3.3. Sensitivity of poles and zeros of H(p,x) with respect to x

3.3.1. Case of one root (pole or zero)

Let p_j a pole of a zero of a transfer function H(p,x). The variation of x to x+dx transforms the pole p_j to p_j+dp_j. The sensitivity of p_j with respect to *x* is given by:

$$S_{x}^{p_j} = \frac{dp_j}{dx/x} \quad (14)$$

Since the performances of any system depend the position of its poles and zeros (roots), the classical sensitivity is therefore related to the root sensitivity.

Let study the case of poles which are assumed to be all simple. A similar reasoning can be applied in the case of the zeros.

Let Q(p,x) the denominator of H(p,x); it can be written as:

$$Q(p,x) = A(p) + x\,B(p)$$

Let p_j a root of Q(p,x), we have:

$$A(p_j) + x\,B(p_j) = 0 \tag{15}$$

For the variation of x to x+dx, the pole p_j moves to t $p_j^{'} = p_j + dp_j$ and the relation (15) become:

$$A(p_j^{'}) + (x + dx)B(p_j^{'}) = 0 \tag{16}$$

If $F(p,x) = \dfrac{x\,B(p)}{Q(p,x)}$, the poles of F(p,x) are the roots of Q(p,x) and we have:

$$1 + \frac{dx}{x}\,F(p_j^{'}, x) = 0 \tag{17}$$

Then F(p,x) becomes:

$$F(p,x) = \sum_i \frac{K_i}{p + p_i} \tag{18}$$

Where K_i is the residual of F(p,x) relatively to the pole p_i. We have:

$$1 + \frac{dx}{x}\left[\sum_i \frac{K_i}{p_j^{'} + p_i}\right] = 0 \tag{19}$$

By neglecting the terms which goes to zero with dx, the relation (19) is written

$$1 + \frac{dx}{x}\left[\frac{K_j}{p_j^{'} + p_j}\right] = 1 + \frac{dx}{x}\left[\frac{K_j}{dp_j}\right] \tag{20}$$

Where

$$S_x^{p_j} = \frac{x dp_j}{dx} = -K_j \tag{21}$$

If the numerator of H(p,x) is $P(p,x) = C(p) + x\,D(p)$ and if z_i is a simple root of P(p,x) we have:

$$S_x^{z_i} = -K_i$$

Where K_i is the residual relative to z_i.

Since p_j et z_i are assumed to be simple roots, we have:

$$K_j = \text{résidu}\left[\frac{x\,B(p)}{Q(p,x)}\right]_{p=p_j} = \frac{x\,B(p_j)}{Q^{'}(x,p_j)} \quad (22)$$

and

$$K_i = \text{résidu}\left[\frac{x\,D(p)}{p(p,x)}\right]_{p=z_i} = \frac{x\,D(z_i)}{P^{'}(x,z_i)} \quad (23)$$

In the following we formulate the classical sensitivity in term of root sensitivity:

$$S_x^H(p,x) = x\left[\frac{P^{'}}{P} - \frac{Q^{'}}{Q}\right] \quad (24)$$

Since we have $\frac{P^{'}}{P} = \frac{D(p)}{P(p,x)}$ and $\frac{Q^{'}}{Q} = \frac{B(p)}{Q(p,x)}$

then

$$\begin{aligned} S_x^H(p,x) &= G(p,x) - F(p,x) \\ &= \sum_j \frac{S_x^{p_j}}{p - p_j} - \sum_i \frac{S_x^{z_i}}{p - z_i} \end{aligned} \quad (25)$$

3.4. Generalization of sensitivity matrix

Let P(p) a polynomial with real coefficients and simple roots:

$$P(p) = \sum_{i=0}^{n} a_i\, p^i \quad (26)$$

Without loss of generality we assume that $a_n = 1$; the coefficients a_i are function of m parameters x_1, x_2, …, x_m and $a_i = f_i(x_1, x_2, \dots, x_m)$.

Let changing x_j to $x_j + dx_j$ (j = 1, …, m) and assume that da_i the variation of a_i, (i = 1, …, n-1). If we define Δa the column vector containing the variations da_i and Δx the column vector containing the variations dx_j, we can write:

$$\Delta a = F \,.\, \Delta x \quad (27)$$

Where F is a (n*m)-dimensional matrix the components of which f_{ij} are given by $f_{ij} = \frac{\partial f_i}{\partial x_j}$.

P(p) can be written as:

$$P(p)=\prod_{i=1}^{q}\left(p^2+b_{2i-1}p+b_{2i}\right)\prod_{i=2q+1}^{n}\left(p+b_i\right) \quad (28)$$

The polynomials $\left(p^2+b_{2i-1}p+b_{2i}\right)$ have complex roots and those of the form $\left(p+b_i\right)$ have real roots. The variation of b_j influences the variations of the roots more than those of the a_i. If Δb is column vector with components db_j, we have

$$\Delta a \;=\; D\,.\,\Delta b \quad (29)$$

Where D is a n-dimensional matrix, the coefficients of which are $d_{ij}=\frac{\partial a_i}{\partial b_j}$. If D is a regular matrix we can write:

$$\Delta b \;=\; D^{-1}.\,F.\,\Delta x \quad (30)$$

Let B a diagonal matrix with $B_{ii}=b_i$ and Δb a column vector $\Delta B_i=\frac{db_i}{b_i}$, we have:

$$\Delta b \;=\; B\,.\,\Delta B \quad (31)$$

Similarly we can write (with $\Delta x_j=dx_j/x_j$ and $X_{jj}=x_j$):

$$\Delta x \;=\; X\,.\,\Delta X \quad (32)$$

The relations (30) – (32) leads to:

$$\Delta B \;=\; B^{-1}\,.\,D^{-1}.\,F\,.\,X\,.\,\Delta X \quad (33)$$

We define the sensitivity matrix S as:

$$S \;=\; B^{-1}\,.\,D^{-1}.\,F\,.\,X \quad (34)$$

With $$S_{ij}=\frac{db_i/b_i}{dx_j/x_j}$$

4. Detection and Localization of a Linear System using Sensitivity

In this section we intend to determine, the safe functioning domains of a second order and third order linear systems using the tolerance intervals relative to the parameters of then transfer functions.

The starting from transfer functions describing defect system we determine the fault causes using the sensitivity approach.

4.1. Determination of the safe functioning domain

4.2. Case of a second order system

Let the following second order system given by its transfer function:

$$H_2(p) = \frac{1}{\frac{1}{\omega_0^2}p^2 + \frac{2\xi}{\omega_0}p + 1} \quad (35)$$

Where ω_0 represents the natural pulsation and ξ is the damping factor.

The aim of this section is to determine a domain in which we can ensure a normal functioning of our process. $\omega_0 = 10$ and $\xi = 0.4$. The safe functioning is defined by the tolerance intervals relative to ω_0 and ξ as:

$\Delta\omega_0 = [\omega_0 - 10\%; \omega_0 + 10\%]$ and $\Delta\xi = [\xi - 10\%; \xi + 10\%]$

These intervals should be respected to determine validity domain of the system step response.

This is done by considering all the variations of the parameters (ω_0 and ξ) as summarized in Table 1.

Table 1. Validity domain by considering the variations of the parameters ω_0 and ξ

Case°	Variation of ω_0	Variation of ξ	Expression of H(p)
1	-10%	-10%	$\frac{1}{0.0123p^2 + 0.08p + 1}$
2	-10%	0%	$\frac{1}{0.0123p^2 + 0.088p + 1}$
3	-10%	10%	$\frac{1}{0.0123p^2 + 0.097p + 1}$
4	0%	-10%	$\frac{1}{0.01p^2 + 0.072p + 1}$
5	0%	10%	$\frac{1}{0.01p^2 + 0.088p + 1}$
6	10%	-10%	$\frac{1}{0.0082p^2 + 0.065p + 1}$
7	10%	0%	$\frac{1}{0.0082p^2 + 0.072p + 1}$
8	10%	10%	$\frac{1}{0.0082p^2 + 0.08p + 1}$

The Figure 2 illustrates the functioning domain when a unit step is chose as system input.

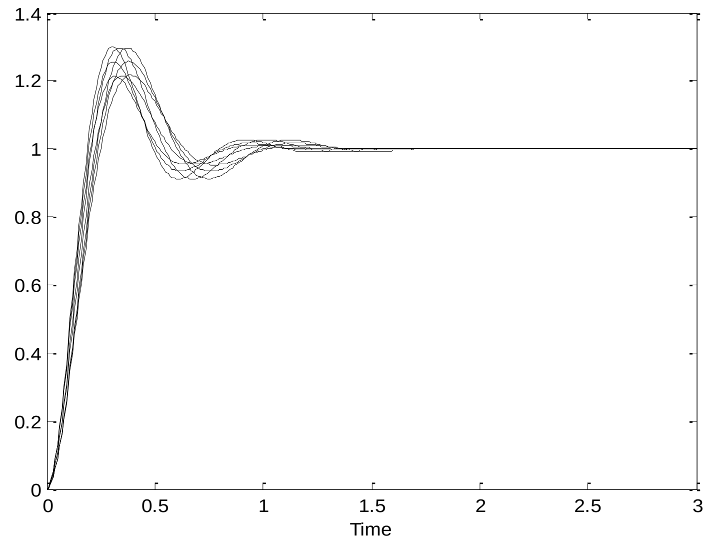

Figure 2. Safe functioning domain of a second order system

4.2.1. Third order system

Let consider a third order system given by transfer function:

$$H_3(p) = \frac{1}{p^3 + (5x_1 - x_2)p^2 + (x_1 x_2 + 2)p + x_1 x_2} \qquad (36)$$

Where x_1 and x_2 define the system parameters

Let $x_1 = 1$ and $x_2 = 2$.

The tolerance interval which guarantee a safe functioning are:

$$\Delta x_1 = [x_1 - 10\% \; ; \; x_1 + 10\%]$$

and $$\Delta x_2 = [x_2 - 10\% \; ; \; x_2 + 10\%]$$

The different cases of both parameter variations are given by Table 2, which the functioning domain is given by Figure 3.

Table 2. The different cases of the parameter x1 and x2 variations

Case n°	Variation of x_1	Variation of x_2	Expression of H(p)
1	-10%	-10%	$\frac{1}{p^3 + 2.7p^2 + 3.62p + 1.62}$
2	-10%	0%	$\frac{1}{p^3 + 2.5p^2 + 3.8p + 1.8}$

3	-10%	10%	$\frac{1}{p^3+2.3p^2+3.98p+1.98}$
4	0%	-10%	$\frac{1}{p^3+3.2p^2+3.8p+1.8}$
5	0%	10%	$\frac{1}{p^3+2.8p^2+4.2p+2.2}$
6	10%	-10%	$\frac{1}{p^3+3.7p^2+3.98p+1.98}$
7	10%	0%	$\frac{1}{p^3+3.5p^2+4.2p+2.2}$
8	10%	10%	$\frac{1}{p^3+3.3p^2+4.42p+2.42}$

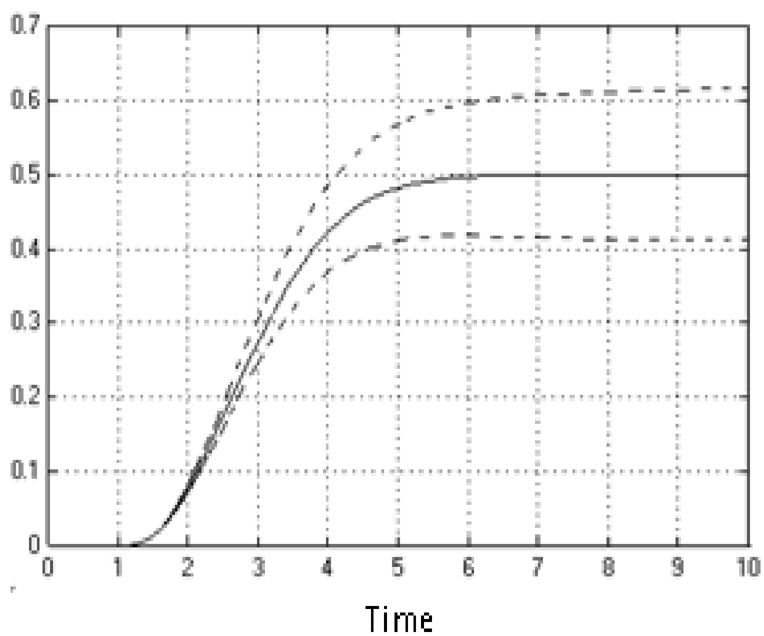

Figure 3. Domain of safe functioning of a third order system

4.3. Determination of the fault causes

We propose to determine the parameter which causes the fault by using the sensibility notion detailed previously.

4.3.1. Second order system

The Laplace transform of a second order step response is:

$$S(p) = \frac{1}{\frac{1}{\omega_0^2}\ p^3\ +\ \frac{2\ \xi}{\omega_0}\ p^2\ +\ p} \qquad (37)$$

and its transfer function is:

$$H(p) = \frac{1}{\frac{1}{\omega_0^2}\ p^2\ +\ \frac{2\ \xi}{\omega_0}\ p\ +\ 1} \qquad (38)$$

Let's determine the sensibility matrix:

Consider the polynomial $Q(p)$ defined as:

$$Q(p) = \frac{1}{\omega_0^2}\ p^2 + \frac{2\ \xi}{\omega_0}\ p\ +\ 1 \qquad (39)$$

Since the sensibility considers essentially, the pole variation we consider the polynomial $P(p)$ which has the same roots as $Q(p)$ given by:

$$P(p)\ =\ p^2\ +\ 2\ \xi\ \omega_0\ p\ +\ \omega_0^2 \qquad (40)$$

Let

$\omega_0 = x_1 = 10\ \text{rad.s}^{-1}$ and $\xi = x_2 = 0.4$

Which yields:

$$P(p) = p^2 + a_1 p + a_0 = p^2 + 8p + 100 \qquad (41)$$

with $a_0 = x_1^2$ et $a_1 = 2x_1x_2$

The matrix F, the components of which are $f_{ij} = \frac{\partial a_i}{\partial x_j}$, is :

$$F = \begin{bmatrix} 20 & 0 \\ 0.8 & 20 \end{bmatrix}$$

The polynomial $P(p)$ can be written as:

$$P(p) = p^2 + b_1 p + b_2$$

The matrix D with $d_{ij} = \frac{\partial a_i}{\partial b_j}$ is:

$$D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The diagonal matrices B and X such as $B_{ii} = b_i$ and $X_{ii} = x_i$ are:

$$B = \begin{bmatrix} 8 & 0 \\ 0 & 100 \end{bmatrix} \qquad X = \begin{bmatrix} 10 & 0 \\ 0 & 0.4 \end{bmatrix}$$

and the sensibility matrix is defined as:

$$S = B^{-1}.D^{-1}.F.X = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

Which means $\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} . \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \end{bmatrix}$

Based on this result we can state the following:

- A 1% variation on ω_0 ($\frac{\delta\omega_0}{\omega_0} = 0.01$) implies:
  - variation of b_1 of 1% ($\frac{\delta b_1}{b_1} = 0.01$)
  - variation of b_2 of 2% ($\frac{\delta b_2}{b_2} = 0.02$).
- A 1% variation on ξ implies a 1% variation on b_1 but no variation on b_2.

Therefore since the tolerance intervals are:

$$\Delta\omega_0 = [\omega_0 - 10\% \ ; \ \omega_0 + 10\%]$$

and $\Delta\xi = [\xi - 10\% \ ; \ \xi + 10\%]$

and since the system is faulty (the output s(t) is outside the functioning domain) we can presume that:

- A value of b_1 outside the interval $[b_{01} - 10\% \ , \ b_{01} + 10\%]$ and a value b_2 outside the interval $[b_{02} - 20\% \ , \ b_{02} + 20\%]$ imply that the faulty is due to ω_0 .
- A value of b_1 outside the interval $[b_{01} - 10\% \ , \ b_{01} + 10\%]$ and et de $b_2 \neq b_{02}$ means that the faulty results from ξ.

Where b_{01} and b_{02} are system coefficient before the fault.

Example 1

Let the second order transfer function $H(p) = \frac{1}{0.009p^2 + 0.09p + 1}$ its step response doesn't belong to safe functioning domain indicted by 4.2.1. We propose to determine the cause of its fault.

The polynomial P(p) obtained from H(p) is:

$$P(p) = p^2 + 10\,p + 111.11\,b_2$$

The non-faulty system transfer function is:

$$H_0(p) = \frac{1}{0.01\,p^2 + 0.08\,p + 1}$$

Which mean $b_{01} = 8$ and $b_{02} = 100$

Therefore $b_1 = 10 = b_{01} + 25\%$

and $b_2 = b_{02} + 11.11\%$

We can conclude that the fault is due to ω_0.

4.3.2. Third order system

The transfer function of our third order system is:

$$S(p) = \frac{1}{p^4 + (5x_1 - x_2)\,p^3 + (x_1x_2 + 2)\,p^2 + x_1x_2\,p} \quad (42)$$

and the corresponding step response Laplace transform is :

$$H(p) = \frac{1}{p^3 + (5x_1 - x_2)\,p^2 + (x_1x_2 + 2)\,p + x_1x_2} \quad (43)$$

and the polynomial P(p) is:

$$P(p) = p^3 + (5x_1 - x_2)\,p^2 + (x_1x_2 + 2)\,p + x_1x_2 \quad (44)$$

with $x_1 = 1$ and $x_2 = 2$

The matrix F is: $F = \begin{bmatrix} 2 & 1 \\ 2 & 1 \\ 5 & -1 \end{bmatrix}$

$$P(p) = p^3 + 3p^2 + 4p + 2$$
$$= (p+1)\,(p^2 + 2p + 2)$$

$$\begin{bmatrix} da_0 \\ da_1 \\ da_2 \end{bmatrix} = \begin{bmatrix} 0 & b_3 & b_2 \\ b_3 & 1 & b_1 \\ 1 & 0 & 1 \end{bmatrix} . \begin{bmatrix} db_1 \\ db_2 \\ db_3 \end{bmatrix} = \Delta a = D.\Delta b$$

$b_1 = 2$, $b_2 = 2$ et $b_3 = 1$

Then $D = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$, $D^{-1} = -\begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 2 \\ -1 & 1 & -1 \end{bmatrix}$,

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}.$$

By taking $db_i = \delta b_i$ and $dx_j = \delta x_j$, we have:

$$\begin{bmatrix} \Delta B_1 \\ \Delta B_2 \\ \Delta B_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -4 & 3 \\ 5 & -2 \end{bmatrix} . \begin{bmatrix} \Delta X_1 \\ \\ \Delta X_2 \end{bmatrix} \tag{45}$$

Based on this result we can presume that:

- 1% variation on x_1 implies a -4% variation on b_1 and 5%.variation on b_3.
- 1% variation on x_2 leads to 3% variation on b_2 and 2%.variation on b_3.

Therefore, since the tolerance intervals are $\Delta x_1 = [x_1 - 10\% \; ; \; x_1 + 10\%]$ and $\Delta x_2 = [x_2 - 10\% \; ; \; x_2 + 10\%]$ and since the system is faulty, we can conclude that:

- A value of b_2 outside the interval $[b_{02} - 40\% \; , \; b_{02} + 40\%]$ and a value of b_3 outside the interval $[b_{03} - 50\% \; , \; b_{03} + 50\%]$ means that the fault results from x_1
- A value of b_2 outside the interval $[b_{02} - 30\% \; , \; b_{02} + 30\%]$ and a value of b_3 outside the interval $[b_{03} - 20\% \; , \; b_{03} + 20\%]$ implies that the fault results from means that the fault results from x_2.

where b_{02} and b_{03} are the system coefficients before the fault.

Example 2

The comparison of the step reponse of the transfer function $H(p) = \dfrac{1}{p^3 + 3.4p^2 + 4.1p + 1.82}$ with that of the domain given by figure 3 confirm the fault existence. We propose to determine the cause leading to this fault.

From the transfer H(p), the polynomial P(p) is:

$$\begin{aligned} P(p) &= (p + b_3)\,(p^2 + b_1\,p + b_2) \\ &= (p + 1.4)\,(p^2 + 2p + 1.3) \end{aligned}$$

Or the safe system is given by the transfer function:

$$H(p) = \frac{1}{p^3 + 3p^2 + 4p + 2}$$

Which means $b_{01} = 2$, $b_{02} = 2$ and $b_{03} = 1$ and which assume that $b_1 = b_{01}$, $b_2 = b_{02} + 40\%$ and $b_3 = b_{03} - 35\%$.

Referring to previous results we can affirm that the fault is due to x_2.

4.4. Graphical determination of fault causes

Assume that the response of the second order system is not completely in the domain as described by Figure 2. The purpose is to seek which of parameter ω_0 and ξ is the cause of the abnormal functioning of the system. We remind that the step response of an oscillating second order system is:

$$s(t)=1-e^{-\xi\omega_0 t}\cos\left(\omega_0.\sqrt{1-\xi^2}.t\right)-\frac{e^{-\xi\omega_0 t}.\xi.\sin\left(\omega_0.\sqrt{1-\xi^2}.t\right)}{\sqrt{1-\xi^2}}$$

The coordinates of two points of the above response provide the values of ω_0 and ξ. The parameter located outside the tolerance domains will be the fault cause. In Figure 4, form plots of the step response are drawn for different values of ω_0 and ξ. As shown in Table 3, only the green plot is not fault as confirmed by Figure 5 where the plots are superposed to the safe functioning domain.

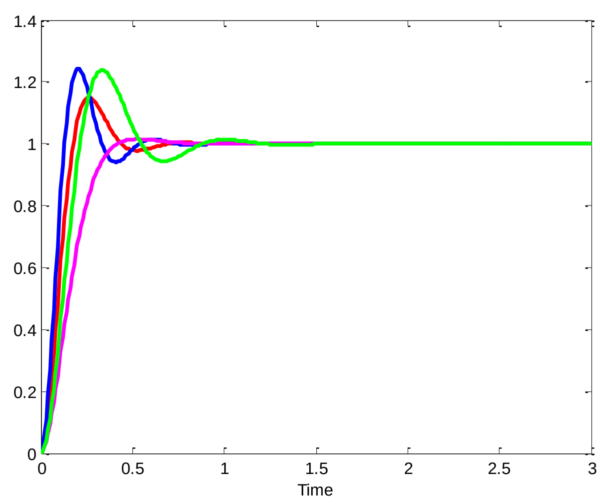

Figure 4. Step response of the second order system for different values of ω_0 and ξ

Table 2

Curve color	**Chosen coordinates**	**Values of ω_0 and ξ**	**Fault cause**
Green	(0.936 ;1.767)	$\omega_0 = 10.321$	no fault
	(1.218 ;1.30)	$\xi = 0.417$	

Blue	(0.927 ;1.39)	$\omega_0 = 16.899$	ω_0
	(1.236 ;1.23)	$\xi = 0.411$	
Red	(0.963 ;1.5)	$\omega_0 = 14.012$	ω_0 and ξ
	(1.145 ;1.27)	$\xi = 0.521$	
Violet	(0.8 ;1.25)	$\omega_0 = 9.921$	ξ
	(1.009 ;1.53)	$\xi = 0.797$	

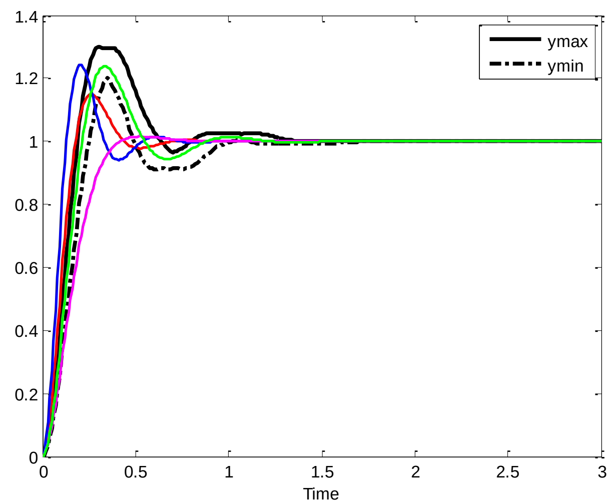

Figure 5. Superposition of step responses with the safe functioning domain

Remark

This graphical approach may suffer from some results uncertainty, due the risk of inaccuracy in determining the points coordinates. Moreover even if the results are convincing for the second and the third order, this assumption can't be assumed for higher order because of the equation complexity.

5. Conclusion

In this paper we have used the sensitivity notion for the fault detection and localization in the second order and the third order linear system given by their transfer functions. Although its sufficiency, this approach raises the deficiency of affecting both the system and the sensor. Therefore to insure that the fault cause result from system parameters we have guarantee the safe functioning of the sensor.

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