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Model Reduction of Bilinear System using Genetic Algorithm

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Abstract

In designing control system, it is important to minimize the order of controller since the high order controller is impractical. In this paper we propose a method to obtain the minimum order of bilinear system by using the genetic algorithm. The process of optimization is to minimize the H_2-norm from the transfer function from input to output which implies suppressing the magnitude peaks of the frequency response. This problem is solved by using the genetic algorithm. To verify the proposed method, the simulation and numerical calculations is carried out on a nonlinear RC circuit and the results show that proposed method can able to approximate a high order bilinear model from 200th order to the 1st order better than the existing method.

Keywords: *Bilinear system, model order reduction, genetic algorithm*

1. Introduction

Many problems in sciences and engineering can be modeled in bilinear systems such as electrical networks, mechanical links, surface vehicle, heat transfer, nuclear fission, airplanes, fluid flow, socioeconomics, chemistry, demography, immunology, agriculture, respiratory chemostat, cardiovascular regulator, hormone regulation, urban processes, kidney water balance, combat model, predator-prey models and so on [18]. In several applications, it is posible that the bilinear model has state in high-order. In addition, designing control system using the H_∞-control theory tends to the high-order controller. Meanwhile, the high-order controller is impractical in implementation since it can cause the numerical problems and uncertainties. Hence, it is important to obtain a method for reducing the order of the bilinear system.

There are two ways to reduce order of a model, *i.e.*, firstly, we are first to reduce the order of the bilinear system and then the control system is designed. The secondly, we are first design the control system for the original bilinear system and then the order of controller is reduced. In this paper, we propose a method to obtain the low order of bilinear system based on algorithm genetic.

Designing control for the bilinear system have been proposed by some authors [6, 11, 12], and model reduction method to obtain the reduced bilinear system have been published in literatures, such as, the balanced truncation [1, 2, 5, 7, 13], moment matching through Krylov subspaces [3, 4, 8, 9, 15, 16]. The other methods can be found in [13]. The generalization of the balanced singular perturbation approximation for bilinear systems as the extension work of Liu [16] for the linear system has been published in [18].

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In this paper we propose a method to obtain the minimum order of bilinear system by using the genetic algorithm. The process of optimization is to minimize the H_2-norm from the transfer function from input to output which implies suppressing the magnitude peaks of the frequency response. This problem is solved by using the genetic algorithm.

This paper is organized as follows. We begin with the introduction in section1, the balanced bilinear system is introduced in Section 2. Meanwhile, in Section 3 we present the balanced singular perturbation. The genetic algorithm as the tools for the optimization is presented in Section 4. The numerical simulations and concusions are given in Section 5 and 6, recpectively.

2. Balanced Bilinear System

Consider a bilinear control systems

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} N_i x(t) u_i(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

with the initial condition $x(0)=0, x(t)\in \mathrm{R}^n$ is the state variable, and n is the dimention of the state space, $u(t)\in \mathrm{R}^m$ is the control input and $y(t)\in \mathrm{R}^p$ is the output of system, u_i denote i^{th} component of $u(t)$, $A\in \mathrm{R}^{n\times n}$, $B\in \mathrm{R}^{n\times m}$, $C\in \mathrm{R}^{n\times p}$, $N_i \in \mathrm{R}^{n\times n}$, $i=1,2,\ldots,m$ are constant matrices.

Assume that the pair (A,B) is controllable (locally controllable) and the pair (A,C) is observable (locally observable). Let the system is asymptotically stable for all $u\in L^2[0,\infty)$ if A is stable. By these properties, we can consider the energy functional is as follows

$$E_c(x_0) = \min_{\substack{u\in L^2(-\infty,0] \\ x(-\infty,x_0,u)=0}} \|u\|^2_{L^2(-\infty,0]}$$

$$E_o(x_0) = E_0^{\alpha}(x_0) = \max_{\substack{u\in L^2[0,\infty) \\ \|u\|_{L^2}\le\alpha}} \|y(.,x_0,u)\|^2_{L^2[0,\infty)},$$

where $\alpha>0$ is a fixed small parameter. There is some ambiguity concerning the definition of E_0. For the reasons of duality, Benner and Damm [7] prefer to consider y as the output of the following homogeneous system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{j=1}^{m} N_j x(t) u_j(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

As a remark for the existing energy functions, Benner and Damm [7] define new output energy function. The definition is given by

Definition 2.1 [7]: Consider homogeneous system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{m} N_i x u_i$$
$$y = Cx(t)$$

where $y(t) \in \mathrm{R}^p$ and antistable, locally controllable dual system

$$\dot{\xi}(\mathrm{t}) = -\mathrm{A}^{\mathrm{T}}\xi(\mathrm{t}) - \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{N}_{\mathrm{i}}^{\mathrm{T}} \xi \mathrm{u}_i(t) + Cu(t)$$

For small $\mathrm{x}_0 \in \mathrm{R}^{\mathrm{n}}$ let $\mathrm{u} = \mathrm{u}_{\mathrm{x}_0}$ denote the control of minimal L^2-norm, so that $\lim_{t\to\infty} \xi(\mathrm{t}, \mathrm{x}_0, \mathrm{u}) = 0$. By this input consider the output $\mathrm{y}(., \mathrm{x}_0, \mathrm{u}_{\mathrm{x}_0})$ of (equation homogen) and define the output energy

$$\mathrm{E}_0(\mathrm{x}_0) = \left\| \mathrm{y}(., \mathrm{x}_0, \mathrm{u}_{\mathrm{x}_0}) \right\|^2_{\mathrm{L}^2[0,\infty)}.$$

Note that in contrast to maximizing over a class of bounded inputs, Benner and Damm choose a special input u_{x_0} for each x_0. In the context of locally stable bilinear systems, we can always rescale the input variable u so that the algebraic Gramians exist.

Thus, the bilinear system (1) can be replace by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} (\gamma N_i) x(t) \frac{u_i(t)}{\gamma} + (\gamma B) \frac{u(t)}{\gamma} \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

The balanced realization for the bilinear system has been developed by Al-Bayat [1]. Controllability and observability are built based on Volterra-Wiener representation

$$\begin{aligned} y(t) = {} & \int_0^t C \exp\left[A(t-\tau_1)\right](\gamma B) \frac{u(\tau_1)}{\gamma} d\tau_1 + \sum_{j=2}^{\infty} \int_0^t \int_0^{\tau_1} \cdots \int_0^{\tau_{j-1}} \sum_{i_1, i_2, \ldots, i_{id}=1}^{m} \exp\left[A(t-\tau_1)\right] \times \\ & (\gamma N_{i_1}) \exp\left[A(\tau_1 - \tau_2)\right] (\gamma N_{i_2}) \ldots \times (\gamma N_{i_d}) \exp\left[A(\tau_{j-1} - \tau_j)\right] (\gamma B) \frac{u_{i_1}(\tau_1)}{\gamma} \times \ldots \\ & \times \frac{u_{i_m}(\tau_j)}{\gamma} d\tau_1 \ldots d\tau_j \end{aligned} \tag{4}$$

Then the equations for algebraic Gramians are

$$\begin{aligned} AW_c + W_c A^T + \gamma^2 \sum_{i=1}^{m} N_i W_c N_i^T + BB^T &= 0 \\ A^T W_c + W_c A + \gamma^2 \sum_{i=1}^{m} N_i W_o N_i^T + C^T C &= 0 \end{aligned} \tag{5}$$

Choosing $\gamma > 0$ small enough for guarantee the existence of algebraic Gramians at the price of possibly decreasing the region where the energy estimates hold.

In order to obtain the controllability Gramian W_c , Al-Baiyat, et. al [1] rewrite (controllability Gramian) as Kronecker product linear matrix equation as

$$Gd = f \ ,$$

where

$$G = (A \otimes I + I \otimes A + [\gamma^2 \{N_1 \otimes N_1 + N_2 \otimes N_2 + \ldots + N_m \otimes N_m\}])$$

and

$$d = vec\left(W_c\right) = (d_{11}, d_{21}, \ldots, d_{n1}, d_{12,} d_{22}, \ldots, d_{n2}, \ldots, d_{1n}, \ldots, d_{nn})^T$$

$$f = vec\left(-BB^T\right).$$

The equation for the observability Gramian is treated similiarly.

The balanced realization is obtained by choosing a matrix T so that

$$T^{-1} W_c T^{-T} = \Sigma = T^T W_o T \ .$$

Assume that both W_c and W_o are positive definite, the Cholesky factors of both Gramians are

$$W_c = L_p L_p^T, W_o = L_o L_o^T$$

The singular value decomposition of the product of the Cholesky factors can be written as follows

$$L_p^T L_o = U \Sigma V^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$

And the matrix of the balancing transformation is

$$T = L_p V \Sigma^{-1/2}, \ T^{-1} = \Sigma^{-1/2} \mathrm{U}^{\mathrm{T}} L_o^T$$

Thus, the balanced bilinear system is as folows

$$\begin{aligned} \dot{x}_b(t) &= A_b x(t) + \sum_{i=1}^{m} N_{bi} x_b(t) u_i(t) + B_b u(t) \\ y(t) &= C_b x_b(t) \end{aligned} \qquad (6)$$

where $A_b = T^{-1}AT, B_b = T^{-1}B, N_{bi} = T^{-1}N_iT, C_b = CT$ and the Gramians new coordinates are

$$\overline{W_c} = \overline{W_o} = diag\left(\sigma_1, \ldots, \sigma_r, \sigma_{r+1}, \cdots, \sigma_n\right), \qquad \sigma_1 \geq \ldots \geq \sigma_n \, .$$

The state space form of the balanced bilinear system based on its Hankel singular value is as follows:

$$\begin{bmatrix} \dot{x}_{b_1}(t) \\ \dot{x}_{b_2}(t) \end{bmatrix} = \begin{bmatrix} A_{b_{11}} & A_{b_{12}} \\ A_{b_{21}} & A_{b_{22}} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} N_{b_{11_i}} & N_{b_{12_i}} \\ N_{b_{21_i}} & N_{b_{22_i}} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} u_i(t) + \begin{bmatrix} B_{b_1} \\ B_{b_2} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} C_{b_1} & C_{b_2} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} \tag{7}$$

The reduced order bilinear system can be obtained by neglecting state with small Hankel singular value. Thus we obtain state space with reduced order bilinear system as

$$\dot{x}_{b_1}(t) = A_{b_{11}} x_{b_1}(t) + \sum_{i=1}^{m} N_{b_{11_i}} x_{b_1}(t) u_i(t) + B_{b_1} u(t)$$
$$y(t) = C_{b_1} x_{b_1}(t)$$

3. Singular Perturbation Approach for Balanced Bilinear System

The singular perturbation approach has been applied by Liu [16] to reduce the linear system. In this paper we will use it for the bilinear system. To carry out this method, first we divide the state into the slow and the fast modes. Let $x_{b1}(t)$ be the slow mode and $x_{b2}(t)$ be the fast mode as in Eq. 7 and partition the matrix A_b, B_b, N_{bi}, C_b conform to the size of Σ such that the balanced bilinear system can be written as follows

$$\begin{bmatrix} \dot{x}_{b_1}(t) \\ \dot{x}_{b_2}(t) \end{bmatrix} = \begin{bmatrix} A_{b_{11}} & A_{b_{12}} \\ A_{b_{21}} & A_{b_{22}} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} N_{b_{11_i}} & N_{b_{12_i}} \\ N_{b_{21_i}} & N_{b_{22_i}} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} u_i + \begin{bmatrix} B_{b_1} \\ B_{b_2} \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} C_{b_1} & C_{b_2} \end{bmatrix} \begin{bmatrix} x_{b_1}(t) \\ x_{b_2}(t) \end{bmatrix} \tag{7}$$

Now let $z(t) = \Gamma^{-1}(\varepsilon) x_b(t)$, where $T(\varepsilon) = \begin{bmatrix} I & 0 \\ 0 & \varepsilon^{-1/2} I \end{bmatrix}$, so that the balanced bilinear system can be written into the singularly perturbed system

$$\dot{z}_1(t) = A_{b_{11}} z_1(t) + \frac{1}{\sqrt{\varepsilon}} A_{b_{12}} z_2(t) + \sum_{i=1}^{m} \left(N_{b_{11i}} z_1(t) + \frac{1}{\sqrt{\varepsilon}} N_{b_{12i}} z_2(t) \right) u_i(t) + B_{b1} u(t)$$
$$\sqrt{\varepsilon} \dot{z}_2(t) = A_{b_{21}} z_1(t) + \frac{1}{\sqrt{\varepsilon}} A_{b_{22}} z_2(t) + \sum_{i=1}^{m} \left(N_{b_{21i}} z_1(t) + \frac{1}{\sqrt{\varepsilon}} N_{b_{22i}} z_2(t) \right) u_i(t) + B_{b2} u(t)$$
$$y(t) = C_{b1} z_1(t) + \frac{1}{\sqrt{\varepsilon}} C_{b2} z_2(t) \tag{8}$$

To vanish small singular values, introduce the scaled variable $z_2 \to \sqrt{\varepsilon} z_2$, the equations (8) can be written in the following form

$$\dot{z}_1(t) = A_{b_{11}} z_1(t) + A_{b_{12}} z_2(t) + \sum_{i=1}^{m} \left(N_{b_{11i}} z_1(t) + N_{b_{12i}} z_2(t) \right) u_i(t) + B_{b1} u(t)$$
$$\varepsilon \dot{z}_2(t) = A_{b_{21}} z_1(t) + A_{b_{22}} z_2(t) + \sum_{i=1}^{m} \left(N_{b_{21i}} z_1(t) + N_{b_{22i}} z_2(t) \right) u_i(t) + B_{b2} u(t)$$
$$y(t) = C_{b1} z_1(t) + C_{b2} z_2(t) \tag{9}$$

By assumption the controls u are bounded continuous function with finite energy, implies that the control decays asymptotically as $t \to \infty$ which guarantees that the fast dynamics relax to a stationary distribution. And also by assumption that the controls u are from the class of relatively slow controls implies that the controls act on an intermediate time scale between the slow and the fast modes.

Let $z_1 = w$ and ignore the output equation, the fast dynamics are obtained as the solution of

$$\dot{z}_2(t) = A_{b_{21}} w(t) + A_{b_{22}} z_2(t) + \sum_{i=1}^{m} \left(N_{b_{21i}} w(t) + N_{b_{22i}} z_2(t) \right) u_i(t) + B_{b2} u(t)$$

If we let $\varphi_w(z_2)$ denotes solutions, we can write $\lim_{t \to \infty} \varphi_w(z_2) = -A_{b_{22}}^{-1} A_{b_{21}} w(t)$, or $z_2(t) = -A_{b_{22}}^{-1} A_{b_{21}} z_1(t)$ **.** Now we obtain the reduced-order bilinear system based on the singular perturbation as the following equation:

$$\begin{aligned} \dot{z}_1(t) &= \left(A_{b_{11}} - A_{b_{12}} A_{b_{22}}^{-1} A_{b_{21}} \right) z_1(t) + \sum_{i=1}^{m} \left(N_{b_{11i}} - N_{b_{12i}} A_{b_{22}}^{-1} A_{b_{21}} \right) z_1(t) u_i(t) + B_{b1} u(t) \\ y(t) &= \left(C_{b1} - C_{b2} A_{b_{22}}^{-1} A_{b_{21}} \right) z_1(t) \end{aligned} \tag{10}$$

4. Genetic Algorithm

A search algorithm that has been extensively studied and applied to optimization, design, machine learning and other varied types of computational intelligence applications, is the Genetic Algorithm (GA) [10]. The Genetic Algorithms are regarded as one of the optimization technique based on an evolution process of living things. In this algorithm, the parameters to be optimized are coded as a row of gene into a chromosome, and genetic operations such as selection, crossover and mutation are performed. When the length of chromosome (length of genes) and the fitness function are given by a design engineer, the chromosome including design parameters is evolved in the process of genetic operations. One of effectiveness of applying the GA in optimization problem is the capability of searching for global optimization parameters. Applications of the GA in various fields of Engineering is rapidly growing with contributions in both the theoretical aspects of the working of the GA with enhancements, as well as typical applications, where traditional optimization and design techniques are proving inefficient with increasing complexity of the systems.

In this paper, the genetic algorithm is used to obtain the reduced order bilinear system. The problem can be stated as follows:

Let the system bilinear with n-order

$$\begin{aligned} \dot{x}(t) &= Ax(t) + N_1 x(t) u_1(t) + N_2 x(t) u_2(t) + B \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \\ y(t) &= Cx(t) \end{aligned} \tag{11}$$

Find the r-order bilinear system ($r < n$)

$$\begin{aligned} \dot{x}_r(t) &= A_r x_r(t) + N_{1r} x_r(t) u_1(t) + N_{2r} x_r(t) u_2(t) + B_r \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \\ y_r(t) &= C_r x_r(t) \end{aligned} \tag{12}$$

which minimize $\left\| y - y_r \right\|_2$.

Generally, the procedures optimization by using the GA is as follows:

1. Initialize random population.
2. Create valid function using grammar.
3. Evaluate fitness value of the chromosome.
4. If the improvement of the fitness tends to zero, stop the procedures. Otherwise proceed to the next step.
5. Generate new population using the genetic operations, go to step2.

6. Numerical Simulations

Consider a MIMO nonlinear system of RC circuit with nonlinear resistor and a independent current source as describe in Lin et. al [16].

$$\dot{x}(t) = Ax(t) + N_1 x(t) u_1(t) + N_2 x(t) u_2(t) + Bu(t)$$
$$y(t) = C^T x(t)$$

where

$$A = \begin{pmatrix} -5 & 2 & & & & \\ 2 & -5 & 2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & 2 & -5 & 2 & \\ & & & 2 & -5 & 2 \\ & & & & 2 & -5 \end{pmatrix} \in \mathbb{R}^{200\times200},\; N_1 = \begin{pmatrix} 0 & -3 & & & & \\ 3 & 0 & -3 & & & \\ & \ddots & \ddots & \ddots & & \\ & & 3 & 0 & -3 & \\ & & & 3 & 0 & -3 \\ & & & & 3 & 0 \end{pmatrix} \in \mathbb{R}^{200\times200}$$

$$N_2 = N_1 + I$$

$$B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{pmatrix} \in \mathbb{R}^{200\times2},\; C = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix} \in \mathbb{R}^{3\times200}.$$

Let we want to reduce the bilinear system into the first order bilinear system:

$$\dot{x}(t) = ax(t) + n_1 x(t) u_1(t) + n_2 x(t) u_2(t) + \begin{pmatrix} b_1 & b_2 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$
$$y(t) = cx(t)$$

By using the genetic algorithm, we would like to find 6 variables: $(a, n_1, n_2, b_1, b_2, c)$.

The same procedure is also applied if we want to reduce into the second order bilinear system which has the equation:

$$\dot{x}(t)=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}x(t)+\begin{pmatrix} n_{111} & n_{112} \\ n_{121} & n_{122} \end{pmatrix}x(t)u_1(t)+\begin{pmatrix} n_{211} & n_{212} \\ n_{221} & n_{222} \end{pmatrix}x(t)u_2(t)+\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$
$$y(t)=\begin{pmatrix} c_1 & c_2 \end{pmatrix}x(t)\quad .$$

We have to find 18 parameters

$$\begin{pmatrix} a_{11} & a_{12} & a_{21} & a_{22} & n_{111} & n_{112} & n_{121} & n_{122} & n_{211} & n_{212} & n_{221} & n_{222} & b_{11} & b_{12} & b_{21} & b_{22} & c_1 & c_2 \end{pmatrix}$$

In the first step, we reduce the original bilinear system into the 3rd order bilinear system. The performance of the 3rd bilinear system obtained by the genetic algorithm is compared to that of the singular perturbation approach (SPA) and the balanced truncation (BT) as shown in Figure 1. From this result, we can conclude that the genetic algorithm is better than the others.

The peformance of the SPA is same as that of the balanced truncation.

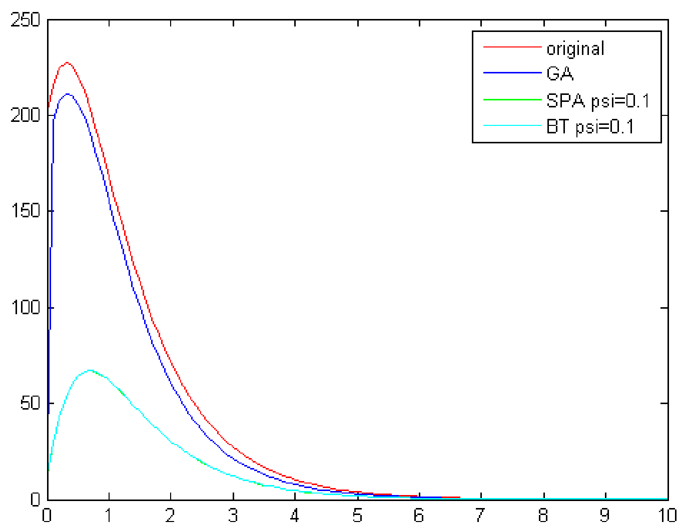

Figure 1. The performance of the 3rd order bilinear system

If the original system is reduced to the 2nd order, as shown in Figure 2, the performance of the genetic algorithm is better than the SPA and BT. The same result is obtained if the original system is reduced into the 1st order as shown in Figure 3. The values of $\|y-y_r\|_2$ for each method can be seen in Table 1. Form this results we see that the performance of genetic algorithm is better.

Table 1. The values of $\|y-y_r\|_2$ for each aproximation methods

Method	1st order	2nd order	3rd-order	5th-order	7th-order
Genetic Algorithm	202	204	211	323	3.22E+06
Balanced truncation	566	566	566	566	566
Singular Perturbation	566	566	566	566	566

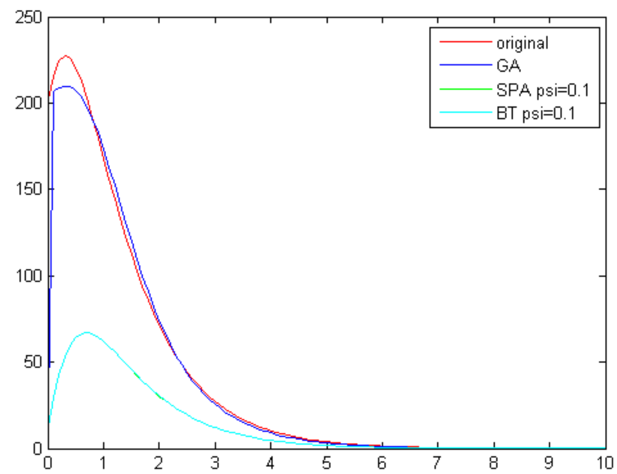

Figure 2. The performance of the 2nd order bilinear system

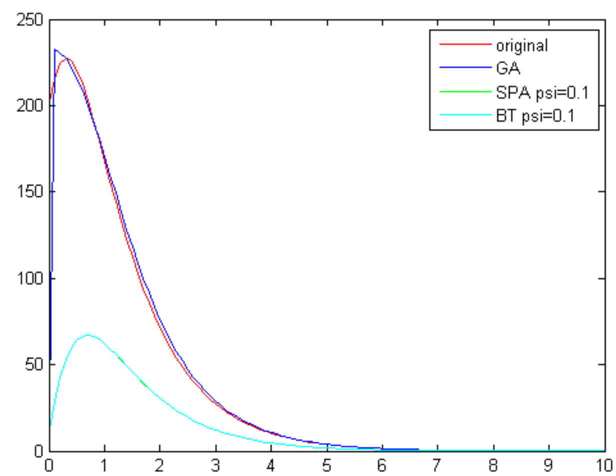

Figure 3. The performance of the 1st order bilinear system

7. Conclusions

In this paper the genetic algorithm is developed to reduce the order of the bilinear system. In this algorithm, the parameters to be optimized are coded as a row of gene into a chromosome, and genetic operations such as selection, crossover and mutation are performed. From the simulation results, we obtained that the performance of the genetic algorithm is better than the singular perturbation approach and the balanced truncation.

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