

# Design and Application of a NEW Seven-Dimensional Hyperchaotic System

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## **Abstract**

*Because the chaotic characteristics of the hyperchaotic system is more complex, so design a higher dimensional hyperchaotic system has become a new orientation of the chaos theory research. This article construct a seven-dimensional third-order hyperchaotic system, this syetem is proved to be hyperchaotic through the MATLAB simulation and the Lyapunov exponential calculation. This article also design the realizing hardware circuit and do its simulation with Multisim. The MATLAB simulation results are consistent with the Multisim simulation results for the designed system and have the same chaotic attractor, which shows the realizability of the system. The chaotic signal source is used for the image encryption and decryption in order to ensure the confidential transmission of images.*

**Keywords:** *seven-dimensional three-order hyperchaotic, attractor, image encryption*

## **1. Introduction**

In recent years, the chaotic system has been used more and more widely in secure communication with the further research on the chaos theory. Compared to other methods, the encryption method has such advantages as good security, strong dynamic storage capability and low power, which significantly improves the safety and reliability of information transmission. When the common chaotic system signal is used as the encrypted signal, the message can be deciphered and attacked easily. However, the chaotic characteristics of the hyperchaotic system are more complex Thus, to use the hyperchaotic system signal as the encrypted signal has more extensive application prospect [1-4]. In [5-7], the four-dimensional system, five-dimensional system, six-dimensional system and their realizing circuits are given, which lays a foundation of the construction of the higher dimensional hyperchaotic system<sup>[9]</sup>. when we take an ordinary chaotic systems signal as an encrypted signals , the information can easily be deciphered and attacked , and because the chaotic characteristic of hyperchaotic system is more complex, so hyperchaotic system as information encrypted signal has a broader application prospect.

In this paper, a seven-dimensional third-order hyperchaotic system is proposed [8]. Comparing with seven-dimensional chaotic system and its realizing circuit that is given in, not only the system have hyperchaotic characteristic, but also do its the nonlinear circuit part can be achieved with two multipliers merely. Meanwhile, the realizing circuit of seven-dimensional third-order hyperchaotic system is designed. The MATLAB simulation results of the system are consistent with the Multisim simulation results of the circuit and present the same chaotic attractors.The designed system is finally used for the image encryption in order to provide a new chaotic for the practical application of secure communication and image encryption based on the chaotic system.

## 2. Seven-dimensional Third-order Hyperchaotic System

The equation of seven-dimensional third-order hyperchaotic system is as follows:

$$\begin{cases} \dot{x}_1 = -ax_1 + ax_5 - bx_5x_6x_7 \\ \dot{x}_2 = -cx_2 - dx_6 + x_1x_6x_7 \\ \dot{x}_3 = -ax_3 + ax_5 - gx_1x_2x_7 \\ \dot{x}_4 = -ax_4 + ex_1 + x_1x_2x_3 \\ \dot{x}_5 = -ax_5 + ex_7 - x_2x_3x_4 \\ \dot{x}_6 = -ex_6 + ex_5 + x_3x_4x_5 \\ \dot{x}_7 = -bx_7 + fx_2 - hx_4x_5x_6 \end{cases} \quad (1)$$

where  $a = 15$ ,  $b = 5$ ,  $c = 0.5$ ,  $d = 25$ ,  $e = 10$ ,  $f = 4$ ,  $g = 0.1$  and  $h = 1.5$ . The system has seven Lyapunov exponents, where  $LE1 = 0.469044$ ,  $LE2 = 0.427538$ ,  $LE3 = -14.9674$ ,  $LE4 = -14.9907$ ,  $LE5 = -15.0002$ ,  $LE6 = -15.719$  and  $LE7 = -15.7193$ , as shown in Figure 1. System (1) has two positive Lyapunov exponents, which shows that the system is not only chaotic but also hyperchaotic.

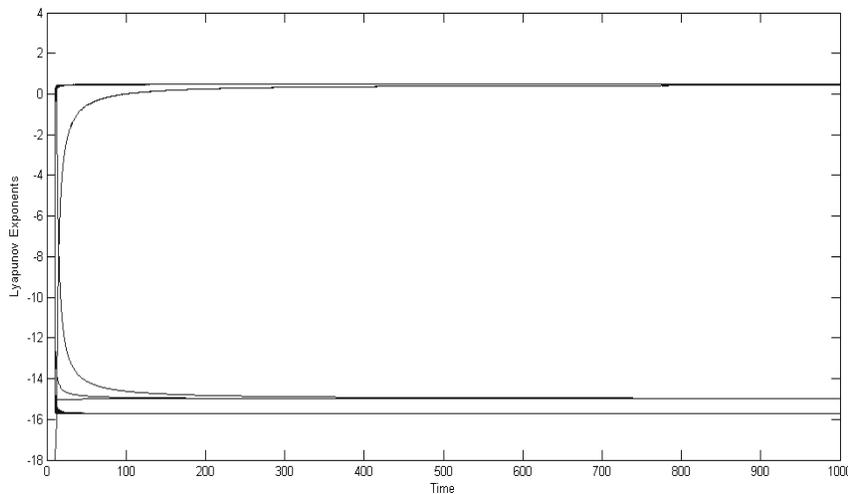
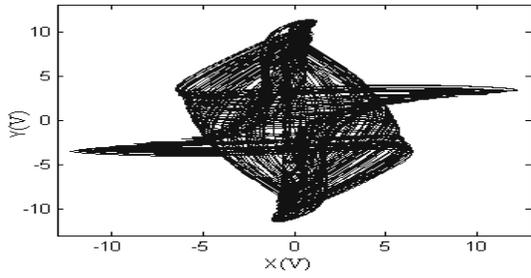


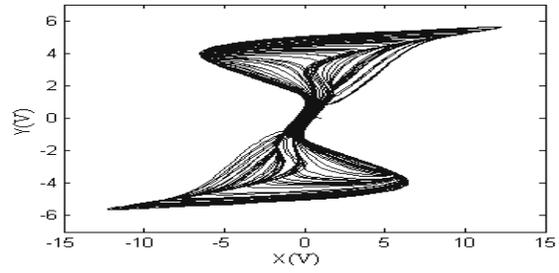
Figure 1. Lyapunov exponent

## 3. MATLAB Simulation and Chaotic Attractors

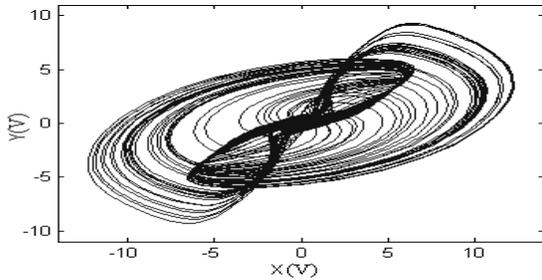
The simulation research on System (1) is carried out with MATLAB. The system has all the two-dimensional and three-dimensional attractors. Some chaotic attractors are shown in Figure 2.



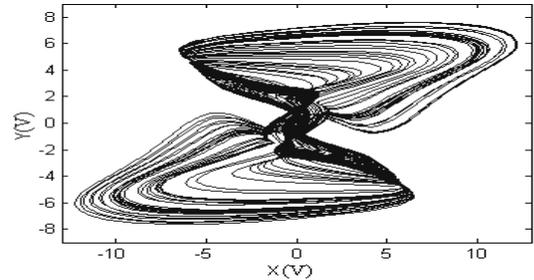
(a)  $x_1 - x_2$  flat attractor



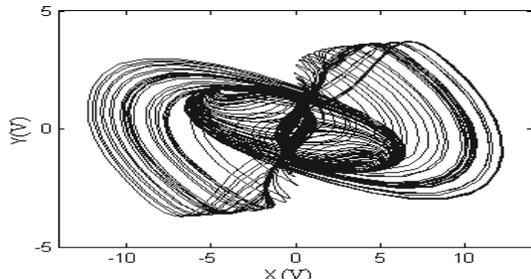
(b)  $x_1 - x_3$  flat attractor



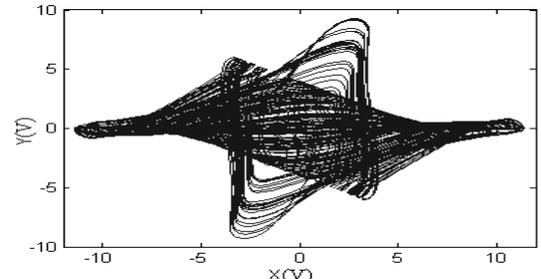
(c)  $x_1 - x_4$  flat attractor



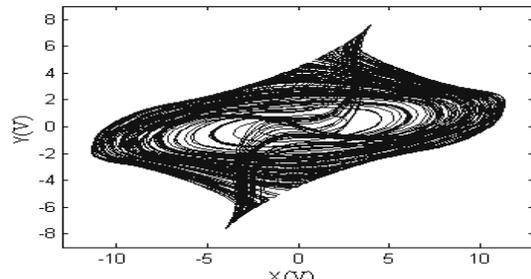
(d)  $x_1 - x_5$  flat attractor



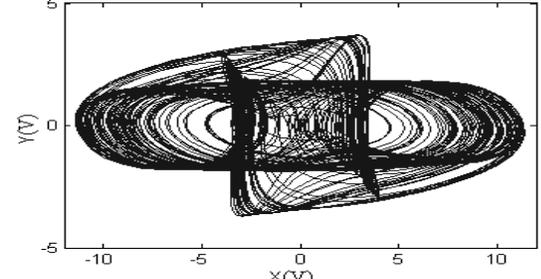
(e)  $x_1 - x_6$  flat attractor



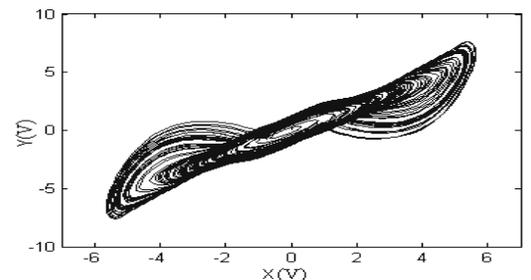
(f)  $x_2 - x_4$  flat attractor



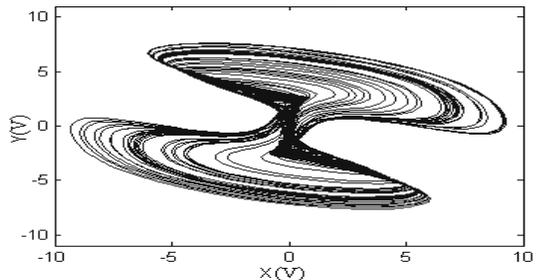
(g)  $x_2 - x_5$  flat attractor



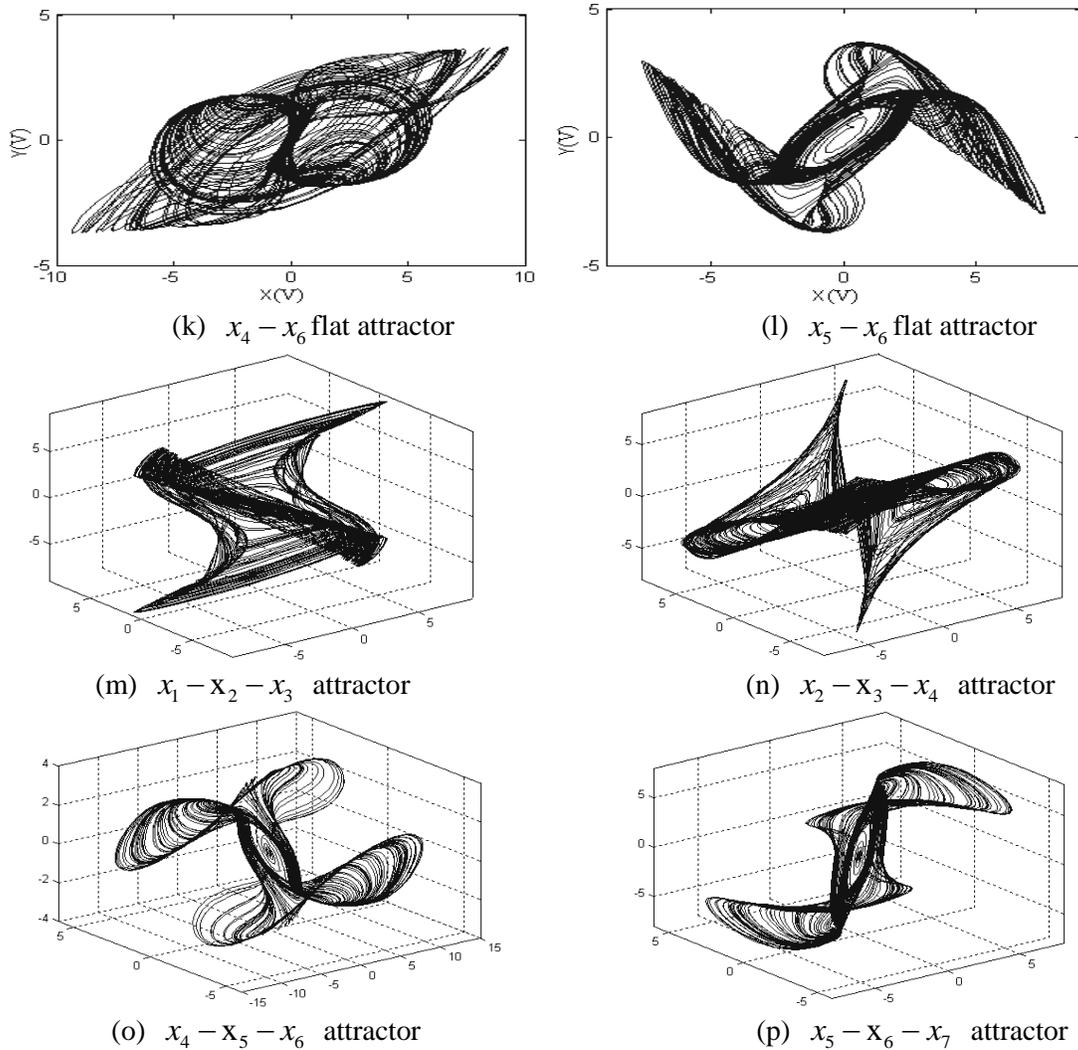
(h)  $x_2 - x_6$  flat attractor



(i)  $x_3 - x_5$  flat attractor



(j)  $x_4 - x_5$  flat attractor



**Figure 2. MATLAB simulation results**

System (1) has two-dimensional and three-dimensional attractors.

#### 4. Realizing Hardware Circuit and Chaotic Attractors

The realizing circuit is designed to verify the realizability of System (1), as shown in Figure 3. The component parameters in the circuit are one-to-one corresponded with the equation coefficients in System (1), where the multipliers can be implemented by the combination of AD633.

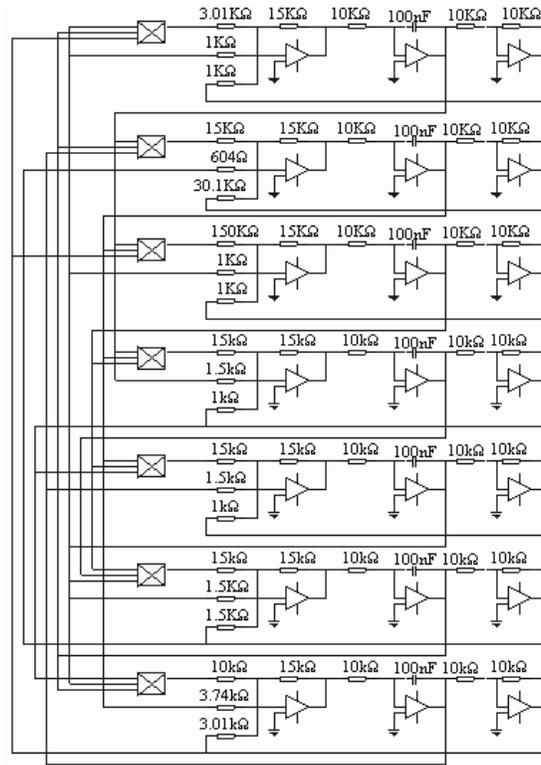
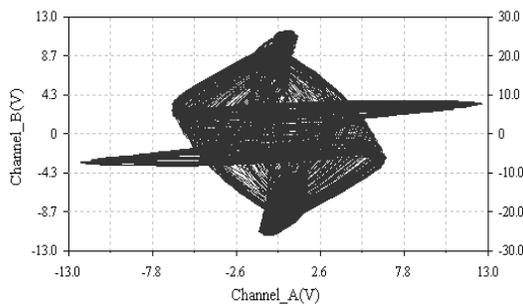
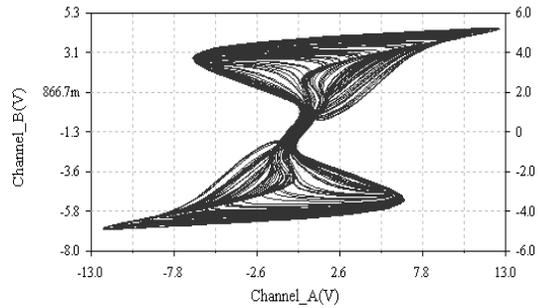


Figure 3. Realizing circuit of system (1)

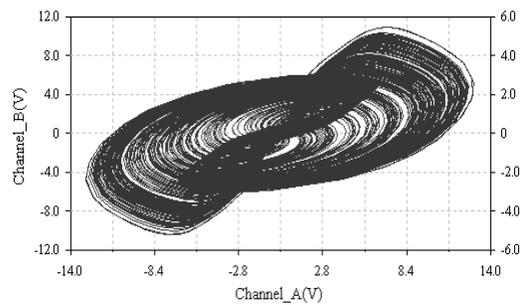
The MULTISIM simulation waveforms of the circuit are shown in Figure 4.



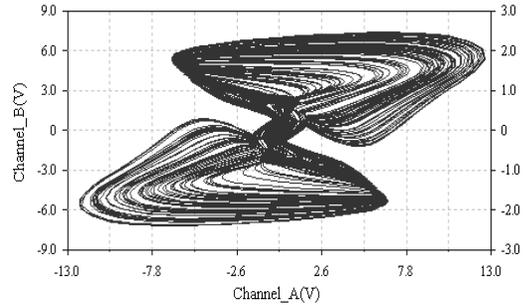
(a)  $x_1 - x_2$  flat attractor



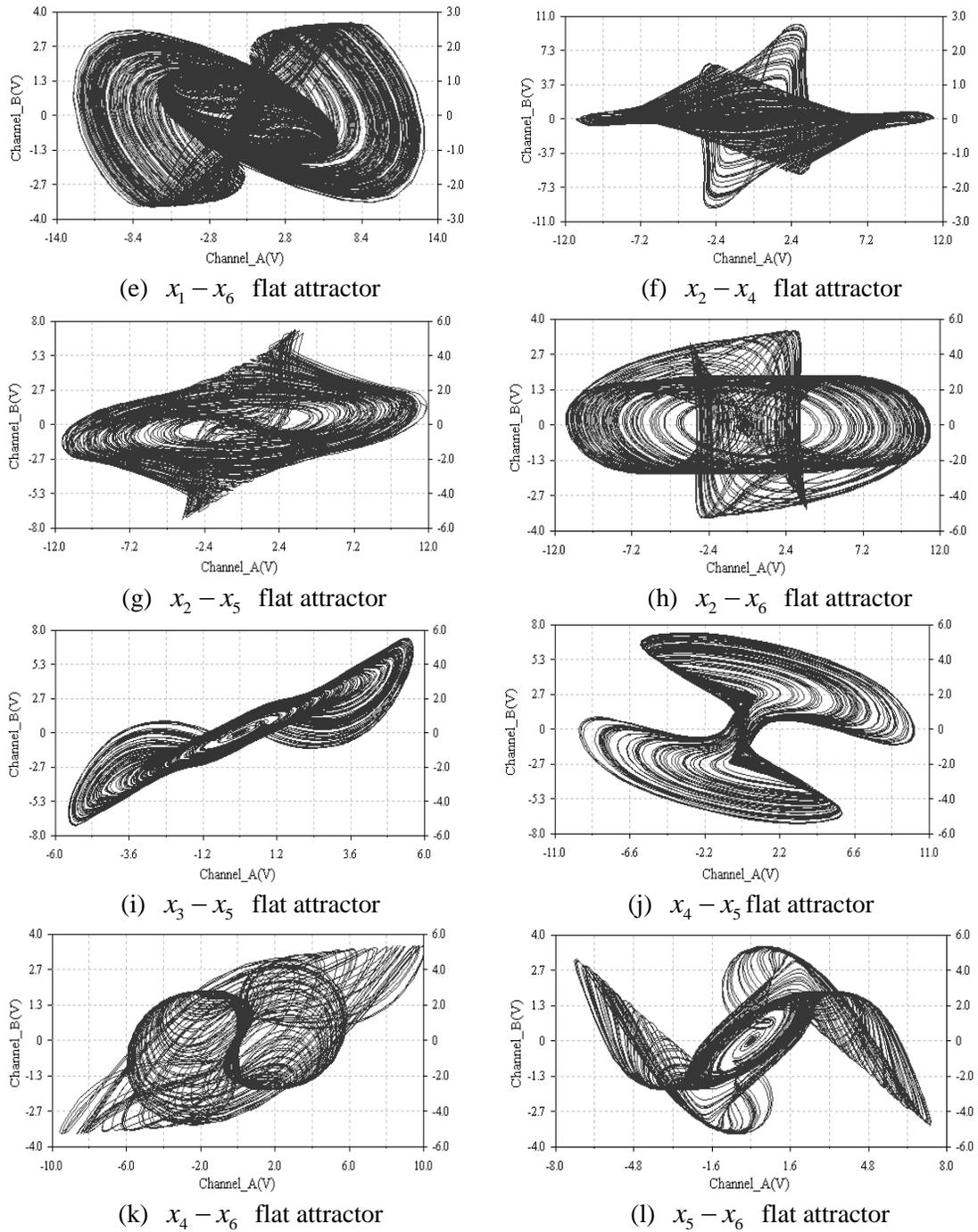
(b)  $x_1 - x_3$  flat attractor



(c)  $x_1 - x_4$  flat attractor



(d)  $x_1 - x_4$  flat attractor



**Figure 4. MULTISIM simulation results of circuit**

It can be seen from Figure 4 that the MULTISIM simulation results of the circuit are completely consistent with the MATLAB simulation results of System (1), which proves that the designed hyperchaotic system is not only existent but also realizable.

## 5. Image Encryption Based on the Seven-dimensional Third-order Hyperchaotic System

The image scrambling technology is used for image encryption in this paper.

The encryption algorithm can be briefly described as follows:

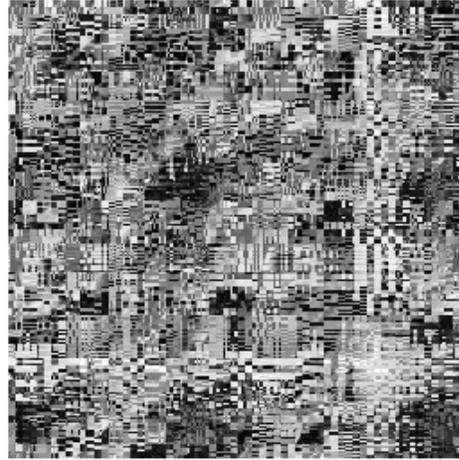
The real value sequence generated by the differential equations of the seven-dimensional third-order hyperchaotic system is extracted for the image encryption, and then the original image with  $256 \times 256$  format is scrambled and replaced according to row or column. As shown in Figures 5 and 6, firstly, the original image with  $256 \times 256$  format is divided into the image blocks with  $32 \times 32$  format, which results in  $8 \times 8 = 64$  small image blocks [10]; secondly, the 64 small image blocks is scrambled in the entire image with the  $8 \times 8$  magic squares or Hilbert transform; finally, the upper and lower elements and left and right elements of the entire row (or column) in the obtained image is exchanged with those of even rows and even columns, in order to cancel the position correlation between the adjacent rows (or columns) of the image, so that the encrypted image is obtained [4-6].

The intensity distributions of the gradation value of the original image and the encrypted image are shown in Figure 5.

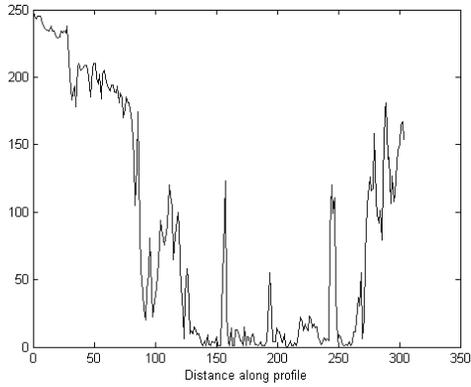
It can be seen from Figure 5 that the intensity distribution of the gradation values of the encrypted image is more uniform and random than that of the original image, which improves the image's anti-attack capability and ensures its safety performance of the information transmission. Furthermore, in order to improve the safety performance. The encryption algorithm of the original image can be iterated so as for a better security for information transmission of the image.



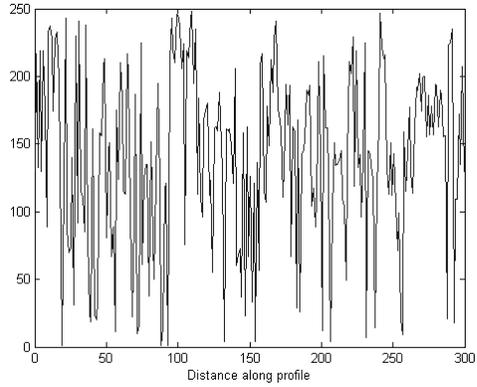
(a) the original image



(b) the encrypted image



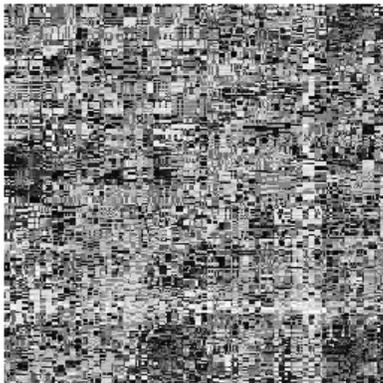
(c) the intensity distribution of the gradation value of the original value image



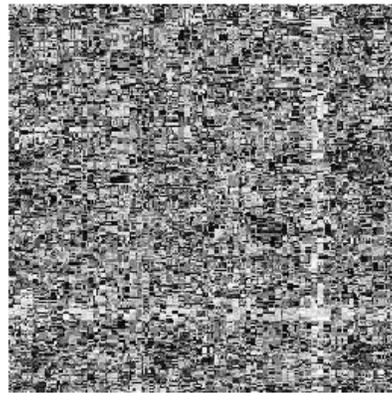
(d) the intensity distribution of the value of the encrypted image

**Figure 5. The intensity distributions of the gradation value of the original image and the encrypted image**

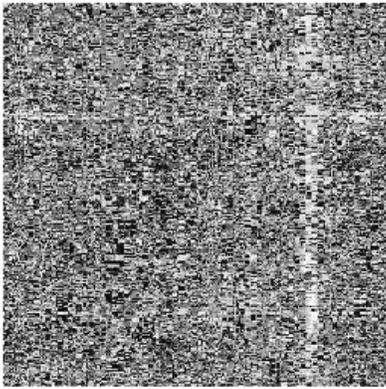
The output generated by the chaotic signal source multiplied randomly by a certain multiple is assigned and pretreated to generate a binary sequence, and then the generated binary sequence is mixed with the binary sequence of the original image to set a password by the manner of addition, subtraction and iteration with the image transformation algorithm, which is more conducive to confidentiality. The process of the image decryption is the inverse process of the image encryption.



(a) the encrypted image of 1 time iteration



(b) the encrypted image of 3 time iteration



(c) the error decrypted image



(d) the correct decrypted image

**Figure 6. Encryption and decryption simulation of the original image**

## 6. Conclusion

In this paper, a seven-dimensional third-order hyperchaotic system and its realizing hardware circuit is proposed, and the following conclusions are obtained through the analysis of Lyapunov exponent and the system simulation:

(1) The proposed seven-dimensional third-order hyperchaotic system has more complex hyperchaotic property. The MATLAB simulation results of the system are completely consistent with the MULTISIM simulation results of the corresponding realizing circuit, which proves that the designed hyperchaotic system can be realized.

(2) The seven-dimensional third-order hyperchaotic signal source applies to the image encryption and decryption so as for the efficacy of the secure transmission, which shows the practicability of the designed hyperchaotic system.

With the further deepened research on the chaos theory, to design and construct a new chaotic system with higher dimension and its corresponding hardware circuit will be a new orientation of the research on the chaos theory and application.

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