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A Set of Stabilizing PID Controllers for Multi input-Multi output Systems

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Abstract

This paper is directed at the problem of designing a set of stabilizing proportional-integral-derivative (PID) controllers for each decoupled subsystem of a multi-input multi-output system based on PID stabilization theorem [1]. It is well known that state interconnection terms of i^{th} subsystem are arising out of N-interconnected subsystem dynamics can be treated as perturbation acting on that subsystem. An LMI optimization problem is formulated to ensure the stability of the composite system while the designed decentralized controllers are employed. A genetic algorithm based search technique is adopted to select an optimal PID controller gains from a designed search space of stabilizing controllers in order to have an optimum value of performance index. A linearized model of multi-machine infinite-bus system is considered for simulation to show the effectiveness of the design procedure.

Keywords: *Hermite-Biehler theorem, Stabilization, PID controller, Interval matrix, Linear matrix inequality (LMI), Lyapunov function*

1. Introduction

Proportional-Integral-Derivative (PID) controllers offer the simplest and most efficient solution to many real control problems for several decades and have found extensive industrial applications. A major obstacle of designing the best controller has been the difficulty in characterizing the entire set of stabilizing controllers. Very few papers are published [1, 2, 3, 15] where a class of stabilizing controllers is designed. An effective solution to this problem was obtained in [1]. This is accomplished by generalizing a classical stability result developed in the last century, the Hermite Biehler theorem. The characterization of all feedback gain values is useful for carrying out optimal designs with respect to various performance indices. In recent years, the time-delay systems have attracted recurring interests from research community. The problem of controlling time-delay systems exists frequently in many engineering fields where time-delays occur due to transportation, communication, process engineering, computation time and more recently network control systems. Time delays are often results in poor performance and can lead to instability [2]. Much of the research work has been focused on stability, controller synthesis and stabilization of time delay systems using so-called Lyapunov-Krasovskii functional and linear matrix inequality (LMI) approach [4]. LMI approach does provide an efficient tool for

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handling time-delay systems, especially with uncertain and immeasurable dead time and it establishes mostly only sufficient condition. A set of stabilizing PID controller design methodology for an arbitrary LTI system with time-delay is described in [2]. A new method for designing PID controller based on stability boundary locus in the (K_p , K_i) plane for fixed value of K_d or in the (K_p , K_d) plane for fixed value of K_i is proposed in [15]. However for a fixed K_p value it is not possible to obtain the (K_i , K_d) by the method in [15] . In this paper, we tried to obtain a class of PID controllers based on [1] for each subsystem of MIMO linear system. The significant results of Siljak et al. [5] demonstrate how the LMIs formulation can be useful to quadratically stabilize linear/nonlinear interconnected system via centralized /decentralized linear constant feedback laws. Motivated by the work of Siljak et al. [5], we have made an attempt based on LMI approach to prove how the designed set of controllers for each subsystem stabilizes the interconnected MIMO systems.

This paper is organized as follows. Section 2 provides a brief review of a set of PID controller based on Hermite Biehler theorem [1]. In section 3 a multi-input multi-output system is considered as a collection of linear subsystems with interconnection terms and describes how a set of decentralized PID controller for each decoupled system can be designed effectively using generalized form of Hermite-Biehler theorem and subsequently, stability analysis of the composite system (considering interaction terms) based on LMI technique is presented in the same section. Section 4 provides the simulation results of a linearized model of multi-machine infinite-bus system to illustrate the effectiveness of the proposed design technique. Concluding remarks are given in Section 5.

2. A Brief Description of All Stabilizing PID Controllers [1]

Consider the feedback control system shown in Figure 1. Here $r_d(t)$ is the reference signal, *y(t)* is the output, $G(s) = N(s)/D(s)$ is the plant to be controlled, $N(s)$ and $D(s)$ are coprime polynomials, and $C(s)$ is the PID controller to be designed which is described by

$$C(s) = K_p + \frac{K_i}{s} + sK_d \qquad (1)$$

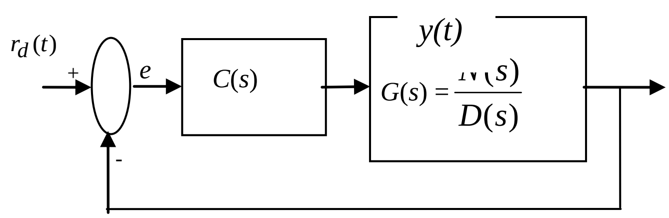

Figure 1. Feedback control system

The closed-loop characteristic polynomial is

$$\delta(s, K_p, K_i, K_d) = sD(s) + (K_i + s^2 K_d)N(s) + K_p sN(s) \qquad (2)$$

The problem of stabilization using a PID controller is to determine the value of K_p, K_i and K_d for which the closed-loop characteristic polynomial $\delta(s, K_p, K_i, K_d)$ is Hurwitz. A new polynomial is constructed by multiplying $\delta(s, K_p, K_i, K_d)$ with $N^*(s)$ to separate controller parameters into two groups. The polynomial $N^*(s)$ is defined as $N^*(s)=N(-s)=N_e(s^2)-sN_o(s^2)$ where $N_o(s^2)$ and $N_e(s^2)$ are the odd and even parts of the polynomial $N(s)$. The real part of the resultant polynomial ($\delta(s, K_p, K_i, K_d) * N^*(s)$) is now a function of ($K_i, K_d$) where as the imaginary part is a function of K_p only (see (6)). This, in turn, will simplify the problem of determining stabilizing PID controller gains. Examining the signature expression of resulting polynomial one can write

$$l(\delta(s,K_p,K_i,K_d)N^*(s))-r(\delta(s,K_p,K_i,K_d)N^*(s))=l(\delta(s,K_p,K_i,K_d))-r(\delta(s,K_p,K_i,K_d))-(l(N(s))-r(N(s))). \tag{3}$$

where $l(\delta(s, K_p, K_i, K_d))$ and $r(\delta(s, K_p, K_i, K_d))$ indicate the number of roots of characteristic polynomial $\delta(s, K_p, K_i, K_d)$ in the left half and right half of complex plane respectively and similarly, $l(N(s))$ and $r(N(s))$ are the number of left half and right half poles in the complex plane. Now, the closed-loop characteristic polynomial $\delta(s, K_p, K_i, K_d)$, of degree n is Hurwitz if and only if $l(\delta(s, K_p, K_i, K_d)) = n$ and $r(\delta(s, K_p, K_i, K_d)) = 0$. Equation (3) can be restated for imaginary signature [1] as

$$\sigma_i(\delta(s,K_p,K_i,K_d)N^*(s))=n-(l(N(s))-r(N(s))). \tag{4}$$

We have to determine those values of K_p, K_i, K_d for which (4) holds and $\delta(s, K_p, K_i, K_d)\,N^*(s)$ has the following expression:

$$\begin{aligned}\delta(s,K_p,K_i,K_d)N^*(s)&=[s^2(N_e(s^2)D_o(s^2)-D_e(s^2)N_o(s^2))\\&+(K_i+K_ds^2)N_e(s^2)N_e(s^2)-s^2N_o(s^2)N_o(s^2))]\\&+s[D_e(s^2)N_e(s^2)-s^2D_o(s^2)N_o(s^2)+K_p(N_e(s^2)N_e(s^2)-s^2N_o(s^2)N_o(s^2))]\end{aligned} \tag{5}$$

where $D_o(s^2)$ and $D_e(s^2)$ are the odd and even parts of the polynomial $D(s)$. The new polynomial with $s = j\omega$ is given by

$$\delta(j\omega,K_p,K_i,K_d)N^*(j\omega)=p(\omega,K_i,K_d)+jq(\omega,K_p), \tag{6}$$

where

$$p(\omega,K_i,K_d)=p_1(\omega)+(K_i-K_d\omega^2)p_2(\omega),$$

$$q(\omega,K_p)=q_1(\omega)+K_pq_2(\omega),$$

$$p_1(\omega)=-\omega^2\,[D_o(-\omega^2)N_e(-\omega^2)-D_e(-\omega^2)N_o(-\omega^2)],$$
$$p_2(\omega)=[N_e(-\omega^2)N_e(-\omega^2)+\omega^2N_o(-\omega^2)N_o(-\omega^2)],$$

$$q_1(\omega)=\omega\,[D_e(-\omega^2)N_e(-\omega^2)+\omega^2D_o(-\omega^2)N_o(-\omega^2)],$$
$$q_2(\omega)=\omega\,[N_e(-\omega^2)N_e(-\omega^2)+\omega^2N_o(-\omega^2)N_o(-\omega^2)].$$

The new polynomial described by (6) is normalized in the following manner.

$$p_f(\omega,K_i,K_d)=\frac{p(\omega,K_i,K_d)}{(1+\omega^2)^{(m+n)/2}},\ q_f(\omega,K_p)=\frac{q(\omega,K_p)}{(1+\omega^2)^{(m+n)/2}}.$$

It can be seen that from equation (6), that K_i and K_d appear in the even power of ω where as K_p appears in the odd power of ω in the polynomial

$\delta(j\omega, K_p, K_i, K_d)N^*(j\omega)$. Furthermore for every fixed K_p, the zeros of q (ω, K_p) will not depend on K_i or K_d. The range of K_i and K_d for a fixed K_p can be obtained by solving a set of linear constraint equations that arises due to the two variables present in the expression p (ω, K_i, K_d) with the imaginary signature given by equation (4) need to be satisfied. For each fixed K_p a linear programming problem is to be solved to obtain K_i and K_d.

3. A Set of Stabilizing PID Controllers for a Linear MIMO System

Let us consider an interconnected system that arises out of 'N' interconnected subsystems and each subsystem is described by

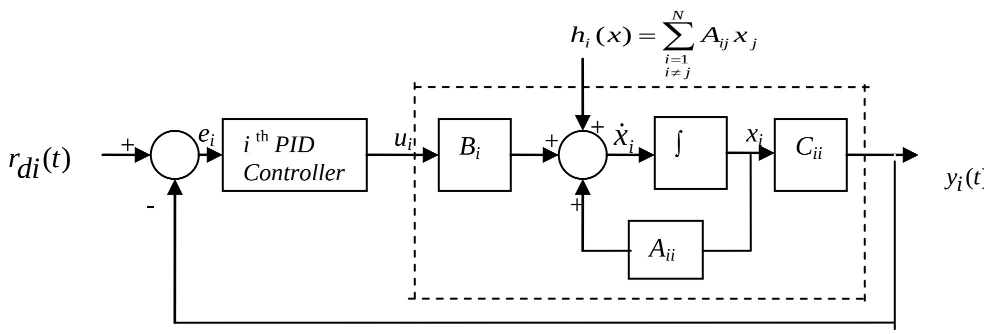

Figure 2. i^{th} subsystem with the controller for a MIMO linear system

$$\dot{x}_i(t) = A_{ii}x_i(t) + B_i u_i(t) + h_i(x(t)), \quad i = 1, 2, \ldots, N;$$
$$y_i(t) = C_{ni}x_i(t), \qquad (7)$$

where x_i is the state vector of the i^{th} subsystem, u_i and y_i are the input and output vectors, C_{ni}, is the output matrix of the i^{th} subsystem, and $h_i(x(t))$ is the interconnection term of the i^{th} subsystem and it is expressed as,

$$h_i(x(t)) = \sum_{\substack{i=1 \\ i\neq j}}^{N} A_{ij} x_j(t).$$

where the function $h_i(x(t))$ depends on the states of the system.

It is assumed that the pair (A_{ii}, B_i) is stabilizable and the i^{th} interconnection term can be bounded by a quadratic inequality

$$h_i^T(x(t))h_i(x(t)) \le \alpha_i^2 x(t)^T H_i^T H_i x(t), \text{ for } \quad i = 1, 2, \ldots, N, \qquad (8)$$

where $\alpha_i > 0$ are interconnection parameters or bounding parameters and H_i are $n_i \times n$ (n_i is the order of the subsystem and $n = \sum_{i=1}^{N} n_i$) constant matrices. The constraints can be interpreted as

$$\|h_i(x(t))\| \le \alpha_i \|H_i x\| \tag{9}$$

where $\|\cdot\|$ is the Euclidean norm. If we define the constant matrix H_i as a block matrix

$$H_i = [H_{i1}, H_{i2}, \ldots, H_{iN}] \tag{10}$$

with each block H_{ij} compatible dimension with the subsystem state vectors x_i, we can rewrite (9) as with $\xi_{ij} = \|H_{ij}\|$.

$$\|h_i(x(t))\| \le \alpha_i \left\| \sum_{j=1}^{N} H_{ij} x_j \right\| \le \alpha_i \sum_{j=1}^{N} \|H_{ij}\| \, \|x_j\| \tag{11}$$

and arrive at the inequality $$\|h_i(x(t))\| \le \alpha_i \sum_{j=1}^{N} \xi_{ij} \|x_j\| \tag{12}$$

which is the standard interconnection constraint [6]

The transfer function of the i^{th}-decoupled system (without interaction terms) is described by

$$C_{ni}(sI - A_{ii})^{-1} B_i \qquad \text{for} \qquad i = 1, 2, \ldots, N.$$

and the set of stabilizing PID controllers for each decoupled system was designed using the method briefly described in Section 2. Fig.2 shows the i^{th} subsystem with the PID controller. The input to the i^{th} subsystem is

$$u_i(t) = K_{pi} e_i(t) + K_{di} \dot{e}_i(t) + K_{ii} \int_0^t e_i(t) dt. \tag{13}$$

Assuming a zero reference input,

$$u_i(t) = K_{pi}(-y_i(t)) + K_{di}(-\dot{y}_i(t)) + K_{ii} \int_0^t (-y_i(t)) dt, \quad i = 1, 2, \ldots, N. \tag{14}$$

Let $$x_{ia}(t) = -K_{ii} \int_0^t y_i(t) dt. \tag{15}$$

So, $$\dot{x}_{ia}(t) = -K_{ii} y_i(t) = -K_{ii} C_{ni} x_i(t), \tag{16}$$

Hence, equation (14) can be written as

$$u_i(t) = K_{pi}(-y_i(t)) + K_{di}(-\dot{y}_i(t)) + x_{ia}(t), \qquad i = 1, 2, \ldots, N. \tag{17}$$

To study the stability of the interconnected system, we substitute for u_i (equation (17)) in equation (7) to get

$$E_i \dot{x}_i(t) = A_{in} x_i(t) + B_i x_{ia}(t) + h_i(x(t)),\ i = 1, 2, \ldots, N. \tag{18}$$

where $E_i = I + B_i K_{di} C_{ni}$, $A_{in} = A_{ii} - B_i K_{pi} C_{ni}$, and $h_i(x)$ given by equation(8). Equations (18) and (16) are augmented as

$$E_{in} \dot{x}_{in}(t) = A_{inw} x_{in}(t) + h_{in}(x(t)), \tag{19}$$

where $E_{in} = \begin{bmatrix} E_i & 0 \\ 0 & 1 \end{bmatrix}$, $A_{inw} = \begin{bmatrix} A_{in} & B_i \\ -K_{ii} C_{ni} & 0 \end{bmatrix}$, $h_{in}(x) = \begin{bmatrix} h_i(x) \\ 0 \end{bmatrix}$ and $x_{in} = \begin{bmatrix} x_i \\ x_{ia} \end{bmatrix}$.

We augment the descriptor system (19) with the system $\dot{x}_{\{in\}}(t) = \dot{x}_{\{in\}}$ to get the following equation and the corresponding for the augmented vector $z(t)$ is described below:

$$F\dot{z}(t) = \bar{A} z(t) + \bar{h} x((t)), \tag{20}$$

where $F = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{A} = \begin{bmatrix} 0 & I \\ A_{new} & -E_{new} \end{bmatrix}$, and $\bar{h}(x) = \begin{bmatrix} 0 \\ h_{new}(x) \end{bmatrix}$,

$z = [x_{1n}{}^T\ x_{2n}{}^T \cdots x_{Nn}{}^T\ \dot{x}_{1n}^T\ \dot{x}_{2n}^T \cdots \dot{x}_{Nn}^T]^T = [\bar{x}^T(t)\ \dot{\bar{x}}^T(t)]$ A_{new} = diag $\{A_{1nw}, A_{2nw}, \ldots, A_{Nnw}\}$ and E_{new} = diag $\{E_{1n}, E_{2n}, \ldots, E_{Nn}\}$ are constant matrices of appropriate dimensions. In the compact notation the interconnection function $h_{new} = (h_{1n}{}^T, h_{2n}{}^T, \ldots\ldots h_{Nn}{}^T)^T$ which is constrained as

$$h_{new}{}^T(x(t)) h_{new}(x(t)) \le x(t)^T \left(\sum_{i=1}^{N} \alpha_i^2 H_{in}{}^T H_{in} \right) x(t),$$
$$\le \bar{x}(t)^T \left(\sum_{i=1}^{N} \alpha_i^2 H_{in}{}^T H_{in} \right) \bar{x}(t), \tag{21}$$

where H_{in} are constant matrices of appropriate dimensions. It may be noted the system state interaction terms are now bounded by the quadratic inequality constraints (21) and the ranges of stabilizing PID controller parameter for each decoupled sub-system are designed based on the technique described in [1] and the controller parameters do no have any dependence on the H_{in} matrices. The closed loop system matrices A_{new} and E_{new} are of interval form and the stability analysis of the composite interval system (20) is then investigated in LMI framework.

Lemma: For matrices Q, S, R the following three statements are equivalent:

- $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$
- $Q > 0$ and $R - S^T Q^{-1} S > 0$

- $R > 0$ and $Q - S R^{-1} S^T > 0$

This is called Schur complement [7]. Using this relationship, some nonlinear matrix inequalities can be converted to an LMI.

Theorem 3.1: The interconnected system (7) is robustly stabilizable with degree α_i by the control law (14) if for matrices P_1, P_2, P_3 of compatible dimensions, and $\gamma_i > 0$, and there exists a feasible solution for the following LMI problem for all the corner matrices of A_{new} and E_{new} with

Minimize $\sum_{i=1}^{N} \gamma_i$, subject to $P_1 > 0$, and

$$\begin{bmatrix} {A_{new}^{r_1}}^T P_2 + P_2^T A_{new}^{r_1} & {A_{new}^{r_1}}^T P_3 + P_1 - P_2^T E_{new}^{r_2} & P_2^T & H_{1n}^T \cdots H_{Nn}^T \\ P_3^T A_{new}^{r_1} + P_1 - {E_{new}^{r_2}}^T P_2 & -{E_{new}^{r_2}}^T P_3 - P_3^T E_{new}^{r_2} & P_3^T & 0 \quad \cdots \quad 0 \\ P_2 & P_3 & -I & 0 \quad \cdots \quad 0 \\ H_{1n} & 0 & 0 & -\gamma_1 I \cdots \quad 0 \\ \vdots & \vdots & \vdots & \vdots \quad \ddots \quad \vdots \\ H_{Nn} & 0 & 0 & 0 \quad \cdots -\gamma_N I \end{bmatrix} < 0, \tag{22}$$

where $r_1, r_2 = 1, 2, \ldots, 2^{k^2}$, $k = n + N$ is the size of the matrices A_{new} and E_{new}, and $\gamma_i = 1/\alpha_i^{\ 2}$.

Proof:

Let us choose a Lyapunov function candidate [8] for the descriptor system (20) as

$$V(t) = z^T(t) F P z(t) \tag{23}$$

where $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$ is nonsingular with $P_1 = P_1^T > 0$, and $FP = (FP)^T$ due to special structures of F and P. We compute

$$\dot{V} = z(t)^T (\overline{A}^T P + P^T \overline{A}) z(t) + \overline{h}(x(t))^T P z(t) + z(t)^T P^T \overline{h}(x(t))$$

In order that the descriptor system (20) to be stable, it is required that

$$\begin{aligned} & P_1 > 0, \\ & z(t)^T (\overline{A}^T P + P^T \overline{A}) z(t) + \overline{h}^T (x(t)) P z(t) + z(t) + z(t)^T P^T \overline{h}(x(t)) < 0 \end{aligned} \tag{24}$$

Inequality (24) is rewritten in expanded form

$$P_1 > 0,$$

$$\begin{aligned} & \overline{x}^T (A_{new}^T P_2 + P_2^T A_{new}) \overline{x} + \dot{\overline{x}}^T (-{E_{new}}^T P_3 - P_3^T E_{new}) \dot{\overline{x}} \\ & + \overline{x}^T (A_{new}^T P_3 + P_1 - P_2^T E_{new}) \dot{\overline{x}} + \dot{\overline{x}}^T (P_1 - {E_{new}}^T P_2 \end{aligned}$$

$$+P_3^T A_{new})\bar{x} + h_{new}(x)^T P_2 \bar{x} + h_{new}(x)^T P_3 \dot{\bar{x}} + \bar{x}^T P_2{}^T h_{new}(x) + \dot{\bar{x}}^T P_3{}^T h_{new}(x) < 0 \tag{25}$$

or $P_1 > 0,$

$$\begin{bmatrix} \bar{x}(t) \\ \dot{\bar{x}}(t) \\ h_{new}(x) \end{bmatrix}^T \begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T \\ P_3^T A_{new} + P_1 - E_{new}{}^T P_2 & -E_{new}{}^T P_3 - P_3^T E_{new} & P_3^T \\ P_2 & P_3 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \dot{\bar{x}}(t) \\ h_{new}(x) \end{bmatrix} < 0 \tag{26}$$

The constraint (21) is equivalent to the quadratic inequality

$$\begin{bmatrix} \bar{x}(t)^T & h_{new}{}^T(x(t)) \end{bmatrix} \begin{bmatrix} -\sum_{i=1}^{N} \alpha_i^2 H_{in}^T H_{in} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ h_{new}(x(t)) \end{bmatrix} \le 0.$$

$$\tau > 0. \tag{27}$$

By using S-procedure [9], it is possible to combine quadratic inequalities (26) and (27) into one single linear matrix inequality (LMI) form such that

$$\begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} + \tau \sum_{i=1}^{N} \alpha_i^2 H_{in}^T H_{in} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T \\ P_3^T A_{new} + P_1 - E_{new}{}^T P_2 & -E_{new}{}^T P_3 - P_3^T E_{new} & P_3^T \\ p_2 & P_3 & -\tau I \end{bmatrix} < 0 \tag{28}$$

where $P_1 > 0$ and a number $\tau > 0$. By repeatedly applying the Schur complement formula on equation (28) with $\tau = 1$, the equation (28) can be rewritten as

$$\begin{bmatrix} A_{new}^T P_2 + P_2^T A_{new} & A_{new}^T P_3 + P_1 - P_2^T E_{new} & P_2^T & H_{1n}^T & \cdots & H_{Nn}^T \\ P_3^T A_{new} + P_1 - E_{new}{}^T P_2 & -E_{new}{}^T P_3 - P_3^T E_{new} & P_3^T & 0 & \cdots & 0 \\ P_2 & P_3 & -I & 0 & \cdots & 0 \\ H_{1n} & 0 & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{Nn} & 0 & 0 & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \tag{29}$$

where $\gamma_i = \dfrac{1}{\alpha_i^2}$

The matrices A_{new} and E_{new} of equation (20) are interval matrices. As discussed in [10-12], a sufficient condition for the stability robustness of interval matrices, *i.e.*, matrices having the elements varying within given bounds, requires that the right hand side of the Lyapunov equation be negative definite when evaluated at the so-called corner matrices. The corner matrices of a ($n \times n$) *for an* interval matrix A is defined as $A^r = (a_{ij}{}^r),\ r = 1,2,....2^{n^2}$, with $a_{ij}{}^r = al_{ij}\ or\ au_{ij}$, $i, j = 1,2,....n$. where, $a_{ij}{}^r = al_{ij}\ and\ au_{ij}$ are minimum and maximum respectively of ij^{th} element of interval matrix. Hence, the inequality condition (29) should be satisfied for all the corner matrices of A_{new} and E_{new} for system (20) to be asymptotically stable. The matrix is H_{in} formed with the knowledge of $h_i(x(t))$ such that constraint (27) is satisfied and the bounding parameter α_i is to be maximized. Hence equation (28) can be reformulated as an LMI optimization problem as stated in equation (22). In other words, system (7) is robustly stabilized by the set of decoupled stabilizing PID controllers provided the LMI problem (22) has a feasible solution for all corner matrices. This completes the proof.

Remarks: If the feasibility condition (22) fails, the designed range of controller parameters obtained for each decoupled subsystem may not ensure the stability of the composite system. However, at the initial stage, one may proceed to meet the LMI condition by introducing a local output feedback to those subsystems that are strongly affected by the state interaction terms arising from the other subsystems. To make the outcomes of problem (22) practical, one must limit the lower bound of bounding parameters α_i by $\bar{\alpha}_i$ and thus the stability analysis of the closed loop system can now be formulated by augmenting an additional LMI condition (*i.e.*, equivalent to $\alpha_i > \bar{\alpha}_i$, for $i = 1,2,...N$) to (22) and then solve the optimization problem.

4. Simulation Results

A multi-machine infinite-bus system is considered for verifying the results given in Section 3. The composite system is arising out of N interconnected subsystems and matrices associated with each subsystem are given below. The following numerical values are taken from [13]

$$A_{11} = \begin{bmatrix} -0.922 & 1 & -0.266 & -0.009 \\ -2.75 & -2.78 & -1.36 & -0.37 \\ 0 & 0 & 0 & 1 \\ -4.95 & 0 & -55.5 & -0.39 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.024 & 0 & -0.087 & -0.002 \\ -0.158 & 0 & 1.11 & -0.011 \\ 0 & 0 & 0 & 0 \\ 0.222 & 0 & 8.17 & 0.004 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 0.021 & 0 & 0.121 & 0.003 \\ -1.1 & 0 & -1.62 & -0.015 \\ 0 & 0 & 0 & 0 \\ -2.43 & 0 & 1.37 & -0.034 \end{bmatrix}, A_{22} = \begin{bmatrix} -0.21 & 1 & -1.6 & -0.005 \\ -1.9 & -1.8 & 9.3 & -0.12 \\ 0 & 0 & 0 & 1 \\ -3.1 & 0 & -56 & 0.032 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 36.1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 78.9 \\ 0 \\ 0 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \qquad (30)$$

$x_i = [\Delta\chi_{fi} \quad \Delta\delta_i \quad \Delta\omega_i \quad \Delta E_{qi}]^T$ is the state vector, χ_f = flux linkages, δ = rotor angle, ω = angular velocity, E_q = field voltage.

4.1 A Set of Stabilizing PID Controllers

The transfer function of each subsystem $C_i(sI - A_{ii})^{-1}B_i$; i =1, 2 is obtained using the numerical values of the matrices given in (30) and a set of stabilizing PID controllers was designed by following the procedure given in Section 2 and the corresponding stabilizing PID controllers for subsystem 1 and subsystem 2 obtained are shown in Figure 3 and Figure 4 respectively. The transfer function of each subsystem $C_i(sI - A_{ii})^{-1}B_i$; i =1, 2 is obtained using the numerical values of the matrices given in (30) and a set of stabilizing PID controllers was designed by following the procedure given in Section 2 and the corresponding stabilizing PID controllers for subsystem 1 and subsystem 2 obtained are shown in Figure 3 and Figure 4 respectively.

From the planes of stabilizing region a common largest rectangular region formed with the ranges of K_i and K_d values inside the shaded planes a mid the different range of K_p values is shown below.

$$K_{p1} \in [-1.5 \quad -0.6], \quad K_{i1} \in [-4.01 \ -0.23], \quad K_{d1} \in [0.09 \quad 0.39],$$
$$K_{p2} \in [-0.8 \quad 0.09],$$
$$K_{i2} \in [-1 \quad -0.1], \qquad K_{d2} \in [0.04 \quad 0.22]. \tag{31}$$

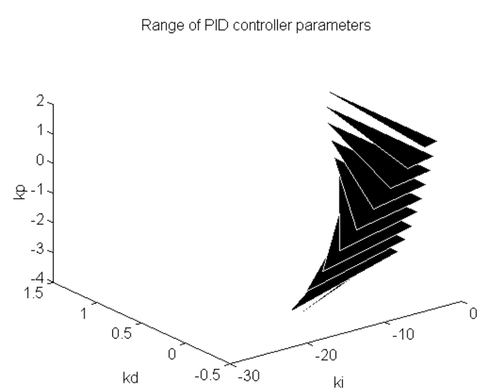

Figure 3. The stabilizing set of K_p, K_i, K_d values for subsystem 1

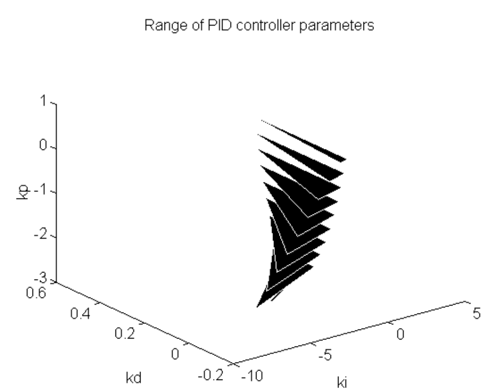

Figure 4. The stabilizing set of K_p, K_i, K_d values for subsystem 2

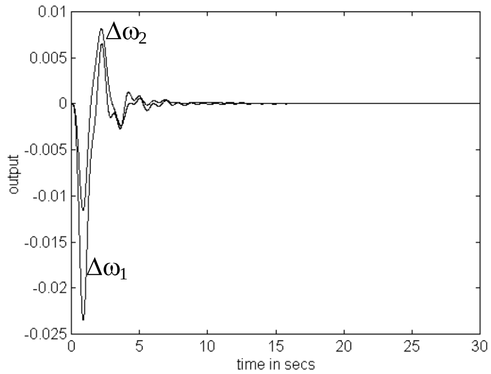

Figure 5. Output responses for 0.05 p.u. step change in mechanical torque (optimal PID controller gains K_{p1}^*= –1.15, K_{i1}^*= –3.1451, K_{d1}^*= 0.1014, K_{p2}^*= –0.1032, K_{i2}^*= –0.126, K_{d2}^*= 0.2093)

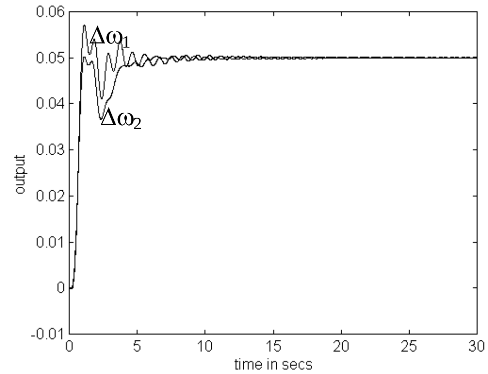

Figure 6. Output responses for 0.05 p.u. step change in electrical voltage (optimal PID controller gains K_{p1}^*= –1.15, K_{i1}^*= –2.277, K_{d1}^*= 0.1293, K_{p2}^*= –0.7656, K_{i2}^*= –0.9771, K_{d2}^*= 0.0422)

From the class of stabilizing controllers given in equation (31) for each of the subsystems an optimal controller parameter $(K_{pi}^*, K_{ii}^*, K_{di}^*)$ based on genetic algorithm was obtained, by maximizing the fitness function J_f where J_f is defined as

$$J_f = \frac{1}{1+J}, \quad J = \int_0^t e_i^{\,2} dt \quad i = 1, 2 \tag{32}$$

The genetic operations applied were arithmetic crossover, uniform mutation and ranking selection. The population size of 50 was taken and GA was run for 25 generations. The optimal controller parameters are obtained by maximizing the fitness function (32) for 0.05 p.u. step change in mechanical torque in the first machine and assumed all initial states are zero. $K_{p1}^* = -1.1500, \quad K_{i1}^* = -3.1451, \quad K_{d1}^* = 0.1014,$

$$K_{p2}^* = -0.1032, \quad K_{i2}^* = -0.1260, \quad K_{d2}^* = 0.2093. \tag{33}$$

Simulation result of output responses of the interconnected system (30) using the optimal controller parameters (see equation (33)) is shown in the Fig. 5. The optimal controller gains are obtained by maximizing the fitness function (32) for 0.05 p.u. step change in electrical voltage in both machines and also with an initial condition of state 2 as 0.1.

$$K_{p1}^* = -1.1500, \quad K_{i1}^* = -2.2770, \quad K_{d1}^* = 0.1293,$$
$$K_{p2}^* = -0.7656, \quad K_{i2}^* = -0.9771, \quad K_{d2}^* = 0.0422. \tag{34}$$

The system (30) is simulated with the optimal gain parameters given in equation (34) and the response of outputs of the interconnected system is shown in Figure 6. It is observed from Figures 5 and 6 that the response of interconnected system is stable even though the set of stabilizing PID controller was designed based on the information of each decoupled subsystem dynamics.

4.2 Stability Analysis

The stability analysis of interval matrix, arising from a set of stabilizing controller of each subsystem, was studied by solving the LMI optimization problem (22) for all the corner matrices of A_{new} and E_{new}. For the designed ranges of controller gains (equation (31)) the A_{new} and E_{new} matrices are found out from equations (19) and (20) as

$$A_{new} = diag\{A_{1nw}, A_{2nw}\},$$
$$E_{new} = diag\{E_{1n}, E_{2n}\}, \quad (35)$$

where

$$A_{1nw} = \begin{bmatrix} -0.922 & 1 & -0.266 & -0.009 & 0 \\ -2.75 & -2.78 & [20.3 \;\; 52.8] & -0.37 & 36.1 \\ 0 & 0 & 0 & 1 & 0 \\ -4.95 & 0 & -55.5 & -0.39 & 0 \\ 0 & 0 & [0.23 \;\; 4.01] & 0 & 0 \end{bmatrix}, \; E_{1n} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & [3.25 \;\; 14.1] & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_{2nw} = \begin{bmatrix} -0.21 & 1 & -1.6 & -0.005 & 0 \\ -1.9 & -1.8 & [2.2 \;\; 72.42] & -0.012 & 78.9 \\ 0 & 0 & 0 & 1 & 0 \\ -3.1 & 0 & -56 & 0.032 & 0 \\ 0 & 0 & [0.1 \;\; 1] & 0 & 0 \end{bmatrix}, \; E_{2n} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & [3.16 \;\; 17.4] & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The interconnection functions are h_{1n} and h_{2n}, given by equation (19), from which H_{1n}, H_{2n} are obtained as

$$H_{1n} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.024 & 0 & -0.087 & -0.002 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.158 & 0 & 1.11 & -0.011 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.222 & 0 & 8.17 & 0.004 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_{2n} = \begin{bmatrix} 0.021 & 0 & 0.121 & 0.003 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1.1 & 0 & -1.62 & -0.015 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.43 & 0 & 1.37 & -0.034 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (36)$$

Two elements in each of A_{1nw}, A_{2nw} and E_{new} are of interval form. Thus four corner matrices are obtained for each of A_{1nw}, A_{2nw} and E_{new}. Table 1 shows the corner matrices of A_{1nw}, A_{2nw} and E_{new}. Thus sixteen corner matrices are possible for A_{new} with eight corner matrices of A_{1nw} and A_{2nw}. These corner matrices of A_{new} are given in

Table 2. These sixteen combinations of A_{new} and four corner matrices of E_{new} are considered, thus giving us sixty four combinations for which the optimization problem

Table 1. Corner matrices of *A_{1nw}*, *A_{2nw}* and *E_{new}*

$A^1_{1nw} = \{A_{1nw}\underline{(2,3)},\ A_{1nw}\underline{(5,3)}$	$A^1_{2nw} = \{A_{2nw}\underline{(2,3)},\ A_{2nw}\underline{(5,3)}$	$E^1_{new} = \{E_{1n}\underline{(2,3)},\ E_{2n}\underline{(2,3)}\}$
$A^2_{1nw} = \{A_{1nw}\underline{(2,3)},\ A_{1nw}\overline{(5,3)}$	$A^2_{2nw} = \{A_{2nw}\underline{(2,3)},\ A_{2nw}\overline{(5,3)}$	$E^2_{new} = \{E_{1n}\underline{(2,3)},\ E_{2n}\overline{(2,3)}\}$
$A^3_{1nw} = \{A_{1nw}\overline{(2,3)},\ A_{1nw}\underline{(5,3)}\}$	$A^3_{2nw} = \{A_{2nw}\overline{(2,3)},\ A_{2nw}\underline{(5,3)}\}$	$E^3_{new} = \{E_{1n}\overline{(2,3)},\ E_{2n}\underline{(2,3)}\}$
$A^4_{1nw} = \{A_{1nw}\overline{(2,3)},\ A_{1nw}\overline{(5,3)}\}$	$A^4_{2nw} = \{A_{2nw}\overline{(2,3)},\ A_{2nw}\overline{(5,3)}\}$	$E^4_{new} = \{E_{1n}\overline{(2,3)},\ E_{2n}\overline{(2,3)}\}$

where $A_{inw}\underline{(2,3)}$ and $A_{inw}\overline{(2,3)}$ denote the lower and upper limits of (2, 3) th element of matrix A_{inw}.

Table 2. Corner matrices of *A_{new}*

A^1_{new}={ A^1_{1nw} A^1_{2nw} }	A^5_{new}={ A^1_{1nw} A^2_{2nw} }	A^9_{new}={ A^1_{1nw} A^3_{2nw} }	A^{13}_{new}={ A^1_{1nw} A^4_{2nw} }
A^2_{new}={ A^2_{1nw} A^1_{2nw} }	A^6_{new}={ A^2_{1nw} A^2_{2nw} }	A^{10}_{new}={ A^2_{1nw} A^3_{2nw} }	A^{14}_{new}={ A^2_{1nw} A^4_{2nw} }
A^3_{new}={ A^3_{1nw} A^1_{2nw} }	A^7_{new}={ A^3_{1nw} A^2_{2nw} }	A^{11}_{new}={ A^3_{1nw} A^3_{2nw} }	A^{15}_{new}={ A^3_{1nw} A^4_{2nw} }
A^4_{new}={ A^4_{1nw} A^1_{2nw} }	A^8_{new}={ A^4_{1nw} A^2_{2nw} }	A^{12}_{new}={ A^4_{1nw} A^3_{2nw} }	A^{16}_{new}={ A^4_{1nw} A^4_{2nw} }

Table 3. α_1, α_2 **values obtained solving LMI problem with PID controller**

	E^1_{new}		E^2_{new}		E^3_{new}		E^4_{new}	
	α_1	α_2	α_1	α_2	α_1	α_2	α_1	α_2
A^1_{new}	0.0217	0.1105	0.0287	0.1107	0.0217	0.080	0.0287	0.0800
A^2_{new}	0.0745	0.1110	0.0433	0.1107	0.0745	0.080	0.0433	0.0800
A^3_{new}	0.0217	0.0826	0.0287	0.0833	0.0217	0.0579	0.0287	0.0580
A^4_{new}	0.0744	0.0831	0.0434	0.0832	0.0744	0.0579	0.0434	0.0580
A^5_{new}	0.0167	0.0714	0.0240	0.1096	0.0167	0.0764	0.024	0.0796
A^6_{new}	0.0692	0.1110	0.0743	0.1109	0.0692	0.0800	0.0743	0.0800
A^7_{new}	0.0167	0.0780	0.0240	0.0827	0.0167	0.0590	0.024	0.0578
A^8_{new}	0.0692	0.0832	0.7430	0.0832	0.0691	0.0580	0.0743	0.0579
A^9_{new}	0.0218	0.0693	0.0287	0.0693	0.0217	0.0691	0.0287	0.0693
A^{10}_{new}	0.0744	0.0693	0.0433	0.0692	0.0745	0.0693	0.0433	0.0692
A^{11}_{new}	0.0218	0.0693	0.0287	0.0693	0.0218	0.0693	0.0287	0.0693
A^{12}_{new}	0.0743	0.0693	0.0434	0.0693	0.0745	0.0693	0.0434	0.0693
A^{13}_{new}	0.0167	0.0691	0.0240	0.0693	0.0167	0.0693	0.024	0.0693

A_{new}^{14}	0.0691	0.0693	0.7430	0.0693	0.0691	0.0693	0.0743	0.0693
A_{new}^{15}	0.0167	0.0692	0.0240	0.0692	0.0167	0.0692	0.024	0.0693
A_{new}^{16}	0.0691	0.0693	0.7430	0.0693	0.0691	0.0693	0.0742	0.0693

(22) is solved using LMI control toolbox [14] with H_{1n} and H_{2n} as given in equation (36). Table 3 shows the α_1, α_2 values obtained by solving the LMI problem (22) with the designed ranges given by (31). There exist feasible solutions for all the sixty-four combinations and hence it is concluded that the set of PID controllers designed for the linear decoupled subsystems stabilize the multi-machine infinite-bus system described by equation (30). The finite numerical values of $\alpha_i, i = 1,2$ can be interpreted as a measure for the decentralized robust stability of system with respect to interaction terms.. This method succeeds exactly may be due to the fact that a set of stabilizing controllers exist for each of the subsystems by method in [1].

5. Conclusions

A class of stabilizing PID controller for each decoupled system of a MIMO system is designed based on PID stabilization theorem. If there is a set of stabilizing controller for each decoupled system, then it is shown by LMI formulation how a set of designed linear controllers for each of the subsystem stabilizes the interconnected system. Simulation results are presented to illustrate the design procedure and also to show the effectiveness of the proposed set of stabilizing PID controller for a multi-machine infinite-bus system. The genetic algorithm based optimization technique is employed to get an optimal controller gains from the designed range of stabilizing controller parameters.

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