

## Aircraft Engine Fuel Flow Prediction Using Process Neural Network

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### **Abstract**

*Monitoring the aircraft engine fuel flow is critical to the flight safety and the aircraft maintenance economy. Aim at predicting the aircraft engine fuel flow accurately and quickly, an aircraft engine fuel flow prediction method based on the process neural network is proposed in this paper. The learning speed of the existing learning algorithms (e.g. BP learning algorithm) for process neural network is too slow for the practical application. A Levenberg-Marquardt learning algorithm based on the expansion of the orthogonal basis functions is developed to raise the adaptability of the process neural network to the real problems. Finally, the proposed prediction method with the corresponding learning algorithm is utilized to predict the fuel flow of some aircraft engine, the results indicate that the proposed prediction method seems to perform well and appears suitable for using as an aircraft engine health condition monitoring tool, and the comparative results also indicate that the Levenberg-Marquardt learning algorithm has a faster learning convergence speed and a higher prediction accuracy than the BP learning algorithm.*

**Keywords:** *Aircraft engine fuel flow; Process neural network; Time series prediction; Aircraft engine health condition monitoring; Orthogonal basis function*

### **1. Introduction**

Aircraft engine is the heart of the aircraft and has to operate at very high temperatures and under severe mechanical stresses. The performance of the aircraft engine deteriorates over time. The performance deterioration of the aircraft engine often reduces the fuel economy and the reliability of the engine. The health condition monitoring of the aircraft engine is a vital and urgent issue in the aircraft-in-service use and the flight safety. Health parameter such as fuel flow represents the engine component efficiencies and flow capacities [1, 2]. Monitoring the aircraft engine fuel flow can provide certain diagnostic and predictive information about the developing fault conditions, this is critical to the flight safety and the aircraft maintenance economy [3, 4].

The fuel flow of the aircraft engine is influenced by many complicated nonlinear time-varying factors. It is difficult or impossible to describe the variety of the fuel flow by a determinate mathematic model. Aim at solving this problem, the long and large amount of the collected data by the airlines can be shrunken into a time series model such as  $\{FF_m\}$ ,  $m=0,1,\dots$ , where  $m$  represents elapsed time. Using this model the tendency of the aircraft engine fuel flow can be predicted by some time series prediction methods. The traditional time series prediction methods like AR, MA, and ARMA are all linear mapping ones. There are only a few nonlinear models such as double linear model, yet they are limited in the degree, which makes them hard to find suitable expression for the varied process in the

complex systems [5]. In 1989, Hornik and Funahashi proved that multilayer feedforward neural networks can approximate any continuous function with any degree of accuracy [6, 7], respectively. It seems that the artificial neural networks have a great potential to the nonlinear time series prediction [8-10]. Formally, the time series prediction based on the artificial neural networks can be stated as: find a function  $G: R^N \rightarrow R$  such as to obtain an estimate of  $FF_{m+h}$  ( $h > 0$ ), from the  $N$  time steps back from time  $m$ , so that:  $FF_{m+h} = G(FF_m, FF_{m-1}, \dots, FF_{m-N+1})$ . Normally  $h$  will be one, so that  $G(\cdot)$  will be forecasting the next value of  $FF_m$ . It is obvious that the inputs of the artificial neural networks are all instantaneous discrete values, it is difficult to express how  $FF_{m-1}$  influences  $FF_m$  and how  $FF_{m-N+1}$  influences  $FF_{m-N}$ , and so on, that is to say, it is difficult to express the time accumulation which exists in the time series in fact. This is a major shortcoming of the time series prediction methods based on the artificial neural networks, which makes them have a low prediction accuracy [11].

From the point view of functional analysis,  $(FF_m, FF_{m-1}, \dots, FF_{m-N+1})$  can be used to generate a continuous function  $FF_m(t)$  by some mathematic method to express the time accumulation in the time series, where  $t \in [m-N+1, m]$ . Then, the time series prediction problem can be stated as: find a functional  $F: C([m-N+1, m]) \rightarrow R$  such as to obtain an estimate of  $FF_{m+1}$  on  $[m-N+1, m]$ , so that:  $FF_{m+1} = F(FF_m(t))$ . Thus, the time series prediction problem can be transformed from a function approximation problem into a functional approximation problem. It has been proved that multilayer feedforward process neural networks can approximate any continuous functional with any degree of accuracy [12,13]. An aircraft engine fuel flow prediction method based on the process neural network is proposed in this paper according to the approximation capability of the process neural network. The learning speed of the existing learning algorithms (e.g., BP learning algorithm) for process neural network is too slow for the practical application. A Levenberg-Marquardt (LM) learning algorithm based on the expansion of the orthogonal basis functions is developed in this study to raise the adaptability of the process neural network to the real problems. Finally, the proposed prediction method is utilized to predict the fuel flow of some aircraft engine.

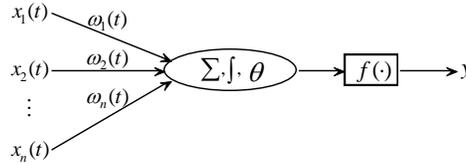
The rest of this paper is organized as follows: In Section 2, a time series prediction model based on the process neural network is proposed. In Section 3, a LM learning algorithm is developed for the proposed time series prediction model. To simplify the computation complexity of the learning algorithm, a set of appropriate orthogonal functions are selected as basis functions to expand the input functions and the connection weight functions of the time series prediction model at the same time. In Section 4, the proposed prediction method is utilized to predict the fuel flow of some aircraft engine, and the comparative results highlight the property of the proposed prediction method and the LM learning algorithm. Conclusions are given in Section 5.

## 2. Time Series Prediction Model Based on Process Neural Network

### 2.1. Process Neuron Model

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The process neuron model is composed of three sections, and the inputs and the connection weights of the process neuron are continuous time-varying functions. An aggregation operator on time is added to the process neuron, which provides the process neuron with the capability of handing simultaneously two items of dimension information of time and space. The architecture of the process neuron is depicted in Figure 1.



**Figure 1. The process neuron model**

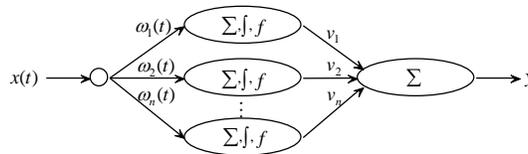
The output of the process neuron model can be expressed as:

$$y = f\left(\sum_{i=1}^n \int_0^T \omega_i(t)x_i(t)dt - \theta\right) \quad (1)$$

Where  $x_i(t) \in C[0, T]$  is the  $i$ -th input function,  $C[0, T]$  denotes the space of continuous functions on  $[0, T]$ ,  $\omega_i(t)$  is the  $i$ -th weight function,  $\theta$  is the threshold, and  $f(\cdot)$  is the activation function, it is usually a nonlinear function.

## 2.2. Time Series Prediction Model Based on Process Neural Network

A time series prediction model based on a 3-layer process neural network is proposed according to the practical demand of the aircraft engine fuel flow prediction. The first layer of the proposed time series prediction model is the input layer, which has only one unit. The second layer is the hidden layer, which is composed of  $n$  process neurons. The last layer is the output layer, which also has only one unit. The architecture of the proposed time series prediction model is depicted in Figure 2.



**Figure 2. Time series prediction model based on process neural network**

Let  $x(t) = FF_m(t)$ ,  $m = T$  and  $N = T + 1$ , then the estimate of  $FF_{m+1}$  can be expressed by  $y$  as:

$$y = \sum_{i=1}^n v_i f\left(\int_0^T \omega_i(t)FF_m(t)dt - \theta_i\right) - \theta \quad (2)$$

Where  $\omega_i(t)$  ( $i = 1, 2, \dots, n$ ) is the connection weight function between the  $i$ -th process neuron in the hidden layer and the input unit,  $v_i$  is the connection weight between the  $i$ -th process neuron in the hidden layer and the output unit.  $\theta_i$  is the threshold of the  $i$ -th process neuron in the hidden layer.  $\theta$  is the threshold of the output unit.  $f(\cdot)$  is the activation

function of the hidden layer, it is usually possible to express the derivative of the activation function in terms of itself, for example, the logistic function  $f(z) = \frac{1}{1 + e^{-z}}$ ,  $f'(z) = f(z) \cdot (1 - f(z))$ .

### 3. LM Learning Algorithm Based on Orthogonal Basis Functions

Employing the aircraft engine fuel flow time series model  $\{FF_m\}$ , we can construct a sample set such as  $\{FF_q(t); d_q\}$  ( $q=1,2,\dots,Q$ ) to train the time series prediction model based on the process neural network, where  $d_q = FF_{q+1}$ . Suppose that  $y_q$  is the corresponding estimate of  $FF_{q+1}$  by the time series prediction model. Then, the sum squared error (SSE) of the time series prediction model can be defined as:

$$E = \sum_{q=1}^Q (d_q - y_q)^2 = \sum_{q=1}^Q [d_q - (\sum_{i=1}^n v_i f(\int_0^T \omega_i(t) FF_q(t) dt - \theta_i) - \theta)]^2 \quad (3)$$

To train the proposed time series prediction model,  $\omega_i(t)$ ,  $v_i$ ,  $\theta_i$  and  $\theta$  should be adjusted to minimize the SSE according to Equation (3).

Applying the gradient descent on this estimate of the SSE,  $\Delta\omega_i(t)$ ,  $\Delta v_i$ ,  $\Delta\theta_i$  and  $\Delta\theta$  can be computed by:

$$\begin{cases} \Delta\omega_i(t) = -\frac{\partial E}{\partial y_q} \frac{\partial y_q}{\partial \omega_i(t)} \\ \Delta v_i = -\frac{\partial E}{\partial y_q} \frac{\partial y_q}{\partial v_i} \\ \Delta\theta_i = -\frac{\partial E}{\partial y_q} \frac{\partial y_q}{\partial \theta_i} \\ \Delta\theta = -\frac{\partial E}{\partial y_q} \frac{\partial y_q}{\partial \theta} \end{cases} \quad (4)$$

Equation (4) is very difficult to be implemented by the computer directly. In order to raise the efficiency of the computation and the adaptability to the practical problems resolving of the proposed time series prediction model based on the process neural network, a set of appropriate orthogonal basis functions are introduced into the input space of the process neural network.

Suppose that  $FF_q(t) \in C[0, T]$ , by Weierstrass approximation theorem [13], if any  $\varepsilon > 0$  is given, then there exists a polynomial  $P(t)$  on  $[0, T]$  such that  $|FF_q(t) - P(t)| < \varepsilon$  for all  $t \in [0, T]$ . In short, any  $FF_q(t) \in C[0, T]$  can be uniformly approximated on  $[0, T]$  by polynomials  $P_k(t)$  ( $k = 1, 2, \dots$ ) to any degree of accuracy. Therefore,  $FF_q(t)$  can be written in the form:  $FF_q(t) = \sum_{k=1}^K c_{qk} P_k(t)$ , where the coefficient  $c_{qk} \in R$  is easy to find. Suppose

that the set of  $P_k(t)$  is linear independent. There is a relationship between orthogonality and independence. It is possible to convert a set of independent functions  $P_k(t)$  into a set of orthogonal functions  $b_k(t)$  that spans the same space. The standard procedure to accomplish this conversion is called Gram-Schmidt orthogonalization [12]. Therefore, the input function  $FF_q(t)$  and the connection weight function  $\omega_i(t)$  can be expanded as:  $FF_q(t) = \sum_{k=1}^K a_{qk} b_k(t)$ ,

$\omega_i(t) = \sum_{k=1}^K \omega_{ik} b_k(t)$ ,  $a_{qk}, \omega_{ik} \in R$ , at the same time, respectively. According to the property

of the orthogonal basis functions,  $\int_0^T b_k(t) b_l(t) dt = \begin{cases} 1 & l = k \\ 0 & l \neq k \end{cases}$ . Thus, Equation (2) can be

substituted by:

$$y = \sum_{i=1}^n v_i f\left(\sum_{k=1}^K \omega_{ik} a_{qk} - \theta_i\right) - \theta \quad (5)$$

This allows us to conveniently write out an expression for the SSE of the proposed time series prediction model as:

$$E = \sum_{q=1}^Q [d_q - \left(\sum_{i=1}^n v_i f\left(\sum_{k=1}^K \omega_{ik} a_{qk} - \theta_i\right) - \theta\right)]^2 \quad (6)$$

The learning algorithm will adjust  $\omega_{ik}$ ,  $v_i$ ,  $\theta_i$  and  $\theta$  to minimize the SSE of the proposed time series prediction model based on the process neural network according to Equation (6).

For the convenience of analysis, let  $R^T = [e_1, e_2, \dots, e_Q]$ , where  $e_q = d_q - y_q$  and  $R^T$  denotes the transpose of  $R$ , let  $W^T = [w_1, w_2, \dots, w_L] = [\omega_{11}, \dots, \omega_{nK}, \theta_1, \dots, \theta_n, v_1, \dots, v_n, \theta]$ , where  $L = n \times K + n + n + 1 = n \times (K + 2) + 1$  and  $W^T$  denotes the transpose of  $W$ .

According to the LM learning algorithm [11-13], the update rule for  $W$  at each iteration can be defined as:

$$W(s+1) = W(s) - [J^T(W(s)) \cdot J(W(s)) + \mu(s) \cdot I]^{-1} \cdot J^T(W(s)) \cdot R(W(s)) \quad (7)$$

Where  $s$  is the iteration;  $I$  is the identity matrix;  $\mu$  is the learning rate, the LM learning algorithm begins with  $\mu$  set to some small value. If an iteration does not yield a smaller value for  $E$ , then the iteration is repeated with  $\mu$  multiplied by some factor  $\lambda > 1$ . Eventually  $E$  should decrease, since we would be taking a small step in the direction of steepest descent. If an iteration does produce a smaller value for  $E$ , then  $\mu$  is divided by  $\lambda$  for the next iteration. This guarantees that the LM learning algorithm will always be able to reduce  $E$  at each iteration;  $J(W)_{Q \times L}$  is a Jacobian matrix about  $W$ :

$$J(W) = \begin{bmatrix} \frac{\partial e_1}{\omega_{11}}, \dots, \frac{\partial e_1}{\omega_{n1}}, \dots, \frac{\partial e_1}{\omega_{nK}}, \frac{\partial e_1}{\theta_1}, \dots, \frac{\partial e_1}{\theta_n}, \frac{\partial e_1}{v_1}, \dots, \frac{\partial e_1}{v_n}, \frac{\partial e_1}{\theta} \\ \frac{\partial e_2}{\omega_{11}}, \dots, \frac{\partial e_2}{\omega_{n1}}, \dots, \frac{\partial e_2}{\omega_{nK}}, \frac{\partial e_2}{\theta_1}, \dots, \frac{\partial e_2}{\theta_n}, \frac{\partial e_2}{v_1}, \dots, \frac{\partial e_2}{v_n}, \frac{\partial e_2}{\theta} \\ \vdots \\ \frac{\partial e_Q}{\omega_{11}}, \dots, \frac{\partial e_Q}{\omega_{n1}}, \dots, \frac{\partial e_Q}{\omega_{nK}}, \frac{\partial e_Q}{\theta_1}, \dots, \frac{\partial e_Q}{\theta_n}, \frac{\partial e_Q}{v_1}, \dots, \frac{\partial e_Q}{v_n}, \frac{\partial e_Q}{\theta} \end{bmatrix} \quad (8)$$

Let  $p_q = \sum_{k=1}^K \omega_{ik} a_{qk} - \theta_i$ , then the elements of  $J(W)$  can be computed by:

$$\begin{cases} \frac{\partial e_q}{\partial \omega_{ik}} = -\sum_{i=1}^n v_i f'(p_q) a_{qk} \\ \frac{\partial e_q}{\partial \theta_i} = \sum_{i=1}^n v_i f'(p_q) \\ \frac{\partial e_q}{\partial v_i} = -f(p_q) \\ \frac{\partial e_q}{\partial \theta} = 1 \end{cases} \quad (9)$$

Thus, the iterations of the LM learning algorithm based on the expansion of the orthogonal basis functions can be summarized as follows:

step1 Select appropriate orthogonal basis functions to expand the input functions and the corresponding weight functions of the time series prediction model.

step2 Initialize  $W$ ; set output error goal  $\varepsilon > 0$ ; initialize iteration  $s = 0$ , and set the max iteration number as  $M$ .

step 3 Utilize Equation (9) and Equation (8) to calculate the Jacobian matrix  $J(W)$ .

step 4 Update  $W$  according to Equation (7), if  $E(s+1)$  is greater than  $E(s)$ , then  $\mu$  multiply by  $\lambda > 1$ , repeat step4; otherwise, divide  $\mu$  by  $\lambda$ ,  $S = S + 1$ , go to Step5.

step5 If  $E < \varepsilon$  or  $s > M$ , go to step6; otherwise, go to step3.

step6 Output results; stop.

## 4. Application Test

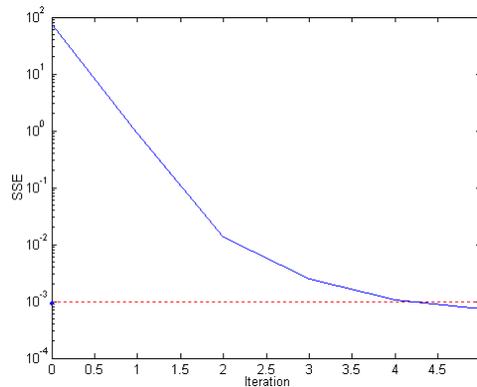
Aircraft engine is a complicated nonlinear system, which operates under high temperature and speed conditions. Operating such a modern technical system, calls for good maintenance to keep the system in an optimal operational condition. Condition-based maintenance is preferred over scheduled maintenance. The scheduled maintenance follows a set of schedule: after a specified usage period, the engines are disassembled and overhauled, irrespective of their health condition. Such scheduled maintenance is costly. Nowadays, condition-based maintenance is becoming more widely adopted, which advocates maintenance only when it is necessary and at the appropriate time instead of after a fixed number of operating hours or cycles. The condition-based maintenance method improved both the economics and the reliability of the operation [10].

For the operational health of the engine to be regularly monitored for gas path faults, health parameters such as shaft speed, pressures, temperatures, and fuel flow are required. Fuel flow

is an important health parameter of the aircraft engine, which represents the engine component efficiencies and flow capacities. By monitoring the fuel flow, maintenance crew can judge the performance deterioration of the aircraft engine. Predicting the tendency of the aircraft engine fuel flow can deduce the gas path faults of the aircraft engine in advance, this is critical to the flight safety and the air transport economics. However, the fuel flow is influenced by many complicated factors and varying continuously with time. It is difficult or impossible to predict the fuel flow by a determinate mathematic model. In this paper, the prediction of the fuel flow by the time series prediction model based on process neural network is presented.

The data used in this paper was taken from some aircraft of Air China, and the sampling interval is about 18 flight cycles. We get a time series of the aircraft engine fuel flow with 80 discrete points such as  $\{FF_m\}_{m=1}^{80}$ .  $(FF_j, FF_{j+1}, \dots, FF_{j+9})$  is used to generate an input function  $IF_j$  by the nonlinear least-squares method, where  $j=1, 2, \dots, 70$ , and  $FF_{j+10}$  is used as the desired output of the proposed time series prediction model based on the process neural network corresponding to  $IF_j$ . Thus, we get 70 samples such as  $\{IF_j; FF_{j+10}\}_{j=1}^{70}$ . The samples  $\{IF_j; FF_{j+10}\}_{j=1}^{50}$  are selected to train the time series prediction model. The time series prediction model used in this application is composed of 3 layers. The input layer has one unit, the hidden layer is consisted of 10 process neurons, and the last layer is the output layer with one unit. The orthogonal Legendre basis functions are selected to expand the input functions and the corresponding connection weight functions of the process neural network. The error goal is set as 0.001, the initial learning rate is set as 0.01, and the max training iteration number is set as 3000.

The samples  $\{IF_j; FF_{j+10}\}_{j=1}^{50}$  are continuously entered to the process neural network for training with the LM learning algorithm. After 5 training iterations, the aircraft engine fuel flow time series prediction model based on the process neural network has converged, and the learning error curve is depicted in Figure 3.



**Figure 3. Learning error curve of the LM learning algorithm**

The samples  $\{IF_j; FF_{j+10}\}_{j=51}^{70}$  are selected to test the aircraft engine fuel flow time series prediction model. The test results as shown in Table 2 indicate that the proposed time series prediction model seems to perform well and appears suitable for using as a predictive maintenance tool.

**Table 2. Aircraft engine fuel flow prediction results**

No.	Actual value	LM		BP	
		Prediction	Relative error(%)	Prediction	Relative error(%)
51	2.0876	2.1000	0.59	2.1305	2.06
52	2.1330	2.1458	0.60	2.1603	1.28
53	2.1394	2.1923	2.47	2.1014	1.78
54	2.1354	2.1414	0.28	2.0819	2.50
55	2.1808	2.0927	4.04	2.2307	2.29
56	2.2128	2.2207	0.36	2.2340	0.96
57	2.1854	2.2082	1.05	2.1602	1.15
58	2.1968	2.0870	5.00	2.0588	6.28
59	2.2038	2.1614	1.93	2.1386	2.96
60	2.1556	2.1121	2.02	2.2909	6.28

In order to compare the performance of the LM learning algorithm with the BP learning algorithm, the aircraft engine fuel flow time series prediction model is trained by the BP learning algorithm under the completely same conditions. After 3000 training iterations, the aircraft engine fuel flow time series prediction model based on the process neural network has met the training condition, and the learning error curve is depicted in Figure 4. The prediction results of the aircraft engine fuel flow time series based on the BP learning algorithm can be found in Table 2.

## 5. Conclusions

The engine plays a significant role as the heart of an aircraft. The aircraft engine health monitoring is essential in terms of the flight safety and also for reduction of the maintenance cost. Fuel flow is one of the most important health parameters of the aircraft engine, which represents the engine component efficiencies and flow capacities. By monitoring the fuel flow, maintenance crew can judge the performance deterioration of the aircraft engine and can find the latent gas path faults in the aircraft engine in advance. But the fuel flow is influenced by many complicated factors and varying continuously with time. It is difficult for traditional methods to predict the tendency of the fuel flow accurately. By analyzing previous data collected by the airlines, the tendency of the fuel flow can be predicted. But the conventional prediction methods neglect the time accumulation existing in the fuel flow time series, this makes them have a low prediction accuracy. In order to resolve this problem, an aircraft engine fuel flow prediction model based on process neural network is proposed in this paper. The inputs and the corresponding connection weights of the process neural network are time-varying functions. With this advantage, process neural network can well predict nonlinear and complex time series with a higher accuracy. The learning speed of the existing learning algorithms for the process neural network is too slow for the practical application. In order to raise the efficiency and the adaptability of the process neural network to the real problems, a LM learning algorithm is developed for the process neural network in this study. To simplify the computation complexity of the LM learning algorithm, a set of appropriate orthogonal basis functions are introduced into the input space of the process neural network to expand the input functions and the corresponding connection weight functions at the same time. Finally, the proposed prediction method is utilized to predict the fuel flow of some aircraft engine, the results indicate that the proposed prediction model seems to perform well

and appears suitable for using as a predictive maintenance tool, and the comparative results also indicate that the LM learning algorithm has a faster learning convergence speed and a higher prediction accuracy than the BP learning algorithm.

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