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# A Study on the Performance Curves Analysis Using Stochastic Network Calculus

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***Abstract***

*This paper discusses the performance curves analysis based on stochastic network calculus under the diversified requirements from different applications in network. Such requirements need different quality of service (QoS), one of which is performance curve analysis. This paper proposes a theory model to investigate the performance curves in SNC. This model integrates the probability calculation into performance analysis model so as to present and analyze the statistical reuse characteristic of data stream over the Internet. The QoS model combines the above two analysis curves without using the min-plus algebra in order to avoid the noncooperation caused by Prb-MinPlus. Several innovations are included in this model. Firstly, the arrival curve indicates the stochastic data steam in the network by integrating the probability functions. Thus, the data stream is able to reflect the current situation in a network. Secondly, the model proposed in this paper integrates arrival curve and service curve to build up the model. As a result, the performance analysis could be carried out more precise. Thirdly, the proposed model has been compared with Jiang-SNC in terms of probability upper boundary and overall performance. It is observed that, the proposed model outperforms Jiang-SNC in QoS and can avoid the problem of Prb-MinPlus.*

***Keywords:*** *Stochastic Network; Calculus; Performance Analysis; Curves*

## 1. Introduction

With the fast development of network technologies, the connection modes into the Internet become more and more diversified such as heterogeneous methods, ubiquitous mechanism, and complex models [1-2]. Thus, the data stream over the Internet is changing from binary data into various complex steams mixed with texts, video, image, and voice [3]. The variousity of data needs different quality of service (QoS) for different applications [4-6]. It is necessary to control the network based on different QoS requirement. However, it is a challengeable task for working out the QoS since the complexity over Internet.

A major problem in network QoS is how to achieve the QoS mechanism designed to meet various QoS Control requirements while maximizing the utilization of network resources. This requires a full understanding of each data stream flow characteristics under the network in order to master a variety of network resources to provide data transmission services. It features an integrated optimization overall network performance. Previously, queuing theory

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was considered to be a comprehensive analysis of network performance. Queuing theory models and service models have been used for describing the data flow in the network [7]. And the network node provides data transmission services. Based on the effective performance analysis of queuing theory, it promoted the network from circuit-switched to packet switching pattern. However, by the 1990s, with the wide application of computer networks, the network data flow was presented by more characteristics [8]. Self-similar flow and mass systems commonly used in queuing theory are essentially different from the Poisson stream, which has shaken the queuing theory points in a computer network performance analysis as a basis for theoretical position.

In order to fulfill the blank of analysis theory of network performance, network calculus has been proposed [9]. Network Calculus was firstly introduced by Cruz. Two dimensions were then divided. One is deterministic network calculus (DNC) and the other is stochastic network calculus (SNC). Currently, both approaches are widely used in analyzing the QoS over the Internet through providing access control and resources management analysis in theory aspects [10-11]. However, research on SNC is relatively limited since it is much more complex due to the stochastic extension of DNC.

This paper proposes a theory model to investigate the performance curves in SNC. This model integrates the probability calculation into performance analysis model so as to present and analyze the statistical reuse characteristic of data stream over the Internet. This model will work out a group of curves to analyze the performance, providing QoS guarantee that improves the utilization of network resources.

The rest of this paper is organized as follows. In Section 2, the performance analysis model in SNC is presented. Section 3 illustrates the QoS performance analysis model. Section 4 discusses the experiments and data analysis. Section 5 concludes this paper by giving our findings and future work.

## 2. Performance Analysis Model in SNC

The performance analysis model adds probability calculation in the network. This model is based on the theorem with upper boundary probability. This model balances the noncooperation between rain-plus algebra and probability operations. Thus, two major curves are used for evaluating the QoS in SNC.

### 2.1. Arrival curve and service curve.

Arrival curve and service curve are based on min-plus convolution. They are extended from the DNC through stochastic extension by integrating the probability theory [12].

The arrival curve is defined as: Let $A(t)$ denotes a data stream, if there are two functions $\alpha(t) \geq 0$ (non-decreasing) and $f(x) \geq 0$ (non-increasing)

$\exists s$, $0 \leq s \leq t$, $\forall x \geq 0$

$$P\{A(s,t) - \alpha(t-s) > x\} \leq f(x)$$

 

Then, the curve is termed as stochastic arrival curve which is traffic-amount-centric. It could be expressed as $A(t) \sim_{ta} < f, \alpha >$. where $f(x)$ is the probability upper boundary function of arrival curve, $\alpha(t)$ is the arrival curve's upper boundary function of data stream.

The service curve is defined as: a data stream $A(t)$ pass a system for specific service, its output is $D(t)$, if $\exists \beta(t) \geq 0$ (non-decreasing) and $g(x) \geq 0$ (non-increasing), $\forall t \geq 0, x \geq 0$, s.t.

$$P\{A \otimes \beta(t) - D(t0 > x\} \leq g(x)$$

It implies that this system provides weak stochastic service curve. It could be expressed as $S(t) \sim_{ws} < g, \beta >$, where $g(x)$ and $\beta(t)$ represent the upper boundary and low boundary function of the service curve respectively.

### 2.2. Performance analysis using curves.

The performance in SNC is described by the randomly selected samples [13]. Thus, arrival and service curves play significant roles in indicating the performance of SNC. In order to evaluate the performance by the above curves, an equal definition is worked out for expressing the arrival curve, which is based on min-plus convolution.

Since the probability upper function $f(x)$ of arrival curve $< f, \alpha >$ is a non-creasing and not less than 0. When $x \to +\infty$, the probability of $A(s,t) - \alpha(t-s) > x$ is 0. Thus, $f(x)$ could associate with a truncated distribution of $X$, $P\{X > x\} = f(x)$. Therefore, the arrival curve is equal as:

$$P\{A(s,t) - \alpha(t-s)\} \leq P\{X > x\}$$

When two random variables $X_1$ and $X_2$ have the sequence $\leq_{st}$, $X_1 \leq_{st} X_2$ could be defined when $P\{X_1 > x\} \leq \{X_2 > x\}$. Thus, the stochastic arrival curve could be equally defined as:

$$A(s,t) - \alpha(t-s) \leq_{st} X$$

Since $\alpha(t-s)$ is a real-valued function, $A(s,t) \leq_{st} \alpha(t-s) + X$ is established. Let random process $\alpha'(t) = \alpha(t) + X$ denotes as a stochastic arrival curve, which could be expressed as:

$$A(s,t) \leq_{st} \alpha'(t-s)$$

Then for the service curve, the probability upper function is $g(x)$, which associates with a truncated distribution of random variable $Y$. Thus, $g(x) = P\{Y > x\}$ could be obtained. The definition of service curve could be expressed as equal as:

$$A \otimes \beta(t) - D(t) \leq_{st} Y$$

### 2.3 QoS analysis

The QoS model combines the above two analysis curves without using the min-plus algebra in order to avoid the noncooperation caused by Prb-MinPlus. $\forall X, Y$ are two stochastic variables which are not less than 0. Their truncated distribution are $F_X^{'}(x) = P\{X > x\}$ and $F_Y^{'}(x) = P\{Y > x\}$, if $x > 0$, then

$$P\{X + Y > x\} \leq (F_X^{'} \otimes F_Y^{'})(x)$$

Where $\otimes$ is the operation of min-plus convolution. If non-increasing functions $f(x) \geq 0$ and $g(x) \geq 0$ exist, they may meet $F_X^{'}(x) \leq f(x)$ and $F_Y^{'}(x) \leq g(x)$. Then, $P\{X + Y > x\} \leq (f \otimes g)(x)$ is established. If $X$ and $Y$ are independent, we can get $P\{X + Y > x\} = 1 - (F_X \times F_Y)(x)$, where $F_X(x) = P\{X \leq x\}$, $F_Y(x) = P\{Y \leq x\}$. Let $f'(x) = 1 - \max[1, f(x)]$ and $g'(x) = 1 - \max[1, g(x)]$, then the following equation could be obtained:

$$P\{X + Y > x\} = 1 - (F_X \times F_Y)(x) \leq 1 - (f' \times g')$$

Where $f_1 \times f_2(x)$ indicates the Stieltjes convolution:

$$f_1 \times f_2(x) = \int_{-\infty}^{+\infty} f_1(x - y) df_2(y).$$

If the stochastic sequence has the characteristics of algebraic calculation, $D(t) \geq_{st} A \otimes \beta(t) - Y$ could be got. $Y$ is independent with $t$, we can get:

$$A \otimes \beta(t) - Y = \inf_{0 \leq s \leq t}\{A(t) + \beta(t - s)\} - Y = \inf_{0 \leq s \leq t}\{A(t) + \beta(t - s) - Y\} = [A \otimes (\beta - Y)](t)$$

Let service curve is $\beta'(t) = \beta(t) - Y$, the performance of service curve could be expressed as:

$$D(t) \geq_{st} A \otimes \beta'(t)$$

Assume that, in a system, a data stream $A(t)$ in the stochastic network may undertake the service process $S(t)$. If $A(t)$ has the arrival curve $< f, \alpha >$, $S(t)$ has the service curve $< g, \beta >$, the backlog of the data stream in the system could be:

$$P\{Back\log(t) > x\} \leq f \otimes g(x - \alpha \Theta \beta(0))$$

 

Proof.

As the definition of backlog, we can get $Back\log(t) = A(t) - D(t)$ . As $D(t) = S(t) - S(t_0) + A(t_0)$ , where $t_0$ is the value of minimal $\inf_{0 \le s \le t}[A(s) + S(s,t)]$ ( $\min(\inf_{0 \le s \le t}[A(s) + S(s,t)]) = s$ ). Thus,

$$Back\log(t) = A(t) - [S(t) - S(t_0) + A(t_0)] = A(t_0,t) - S(t_0,t)$$

$$= [A(t_0,t) - \alpha(t-t_0)] + [\beta(t-t_0) - S(t_0,t)] + [\alpha(t-t_0) - \beta(t-t_0)]$$

$$\le [A(t_0,t) - \alpha(t-t_0)] + [\beta(t-t_0) - S(t_0,t)] + \sup_{s \ge 0}[\alpha(s) - \beta(s)]$$

$$= [A(t_0,t) - \alpha(t-t_0)] + [\beta(t-t_0) - S(t_0,t)] + \alpha\Theta\beta(0)$$

If $Back\log(t) > x$, then $[A(t_0,t) - \alpha(t-t_0)] + [\beta(t-t_0) - S(t_0,t)] > x - \alpha\Theta\beta(0)$ .

Based on the probability relationship, we can get the following result:

$$P\{Back\log(t) > x\} \le P\{[A(t_0,t) - \alpha(t-t_0)] + [\beta(t-t_0) - S(t_0,t)] > x - \alpha\Theta\beta(0)\}$$

According to the definition of arrival curve and service curve, we can get:

$$P\{[A(t_0,t) - \alpha(t-t_0)] > x - \alpha\Theta\beta(0)\} \le f(x - \alpha\Theta\beta(0))$$

$$P\{[\beta(t-t_0) - S(t_0,t)] > x - \alpha\Theta\beta(0)\} \le g(x - \alpha\Theta\beta(0))$$

Thus,

$$P\{Back\log(t) > x\} \le f \otimes g(x - \alpha\Theta\beta(0))$$

End.

## 3. Experiments and Discussions

In this section, a specific network example is used for demonstrating the analysis model proposed in this paper, comparing the results from the method of Jiang-SNC [12]. The major difference of the proposed model is using the min-plus algebra in order to avoid the noncooperation caused by Prb-MinPlus.

Firstly, the theoretical analysis is carried out for discussing the performance results. If the stochastic network is able to provide strict service curve $< g, \beta >$ , the performance analysis results from Jiang-SNC is:

$$P\{Back\log(t) > x\} \le f^{\theta} \otimes g(x - \alpha\Theta\beta(0))$$

Where, $f^{\theta}(x) = f(x) + \frac{1}{\theta}\int_x^{\infty} f(y)dy$ , $\theta > 0$ and $\theta$ meets the condition that the mean speed of $\alpha(t) + \theta t$ is less than $\beta(t)$. Since $f^{\theta}(x) > f(x)$, then the results from the proposed model outperforms the approach of Jiang-SNC. The main reason is that Jiang-SNC uses the increasing item of upper probability boundary $f^{\theta}(x)$. Therefore, it causes the noncooperation of Prb-MinPlus. While, in this paper's model, $D(t)=\inf_{0\le s\le t}[A(s)+S(s,t)]$ has been replaced by $D(t)=S(t)-S(t_0)+A(t_0)$, avoiding the noncooperation problem.

Secondly, we take the Markov stochastic network for example to carry out the experiments to compare both methods. Specifically, markov modulated on-off process is used with transfer probability $\mu$ on->off, $\lambda$ off->on. $\mu=0.25(ms^{-1})$, $\lambda=0.02(ms^{-1})$, $P=1.0(Mbps)$

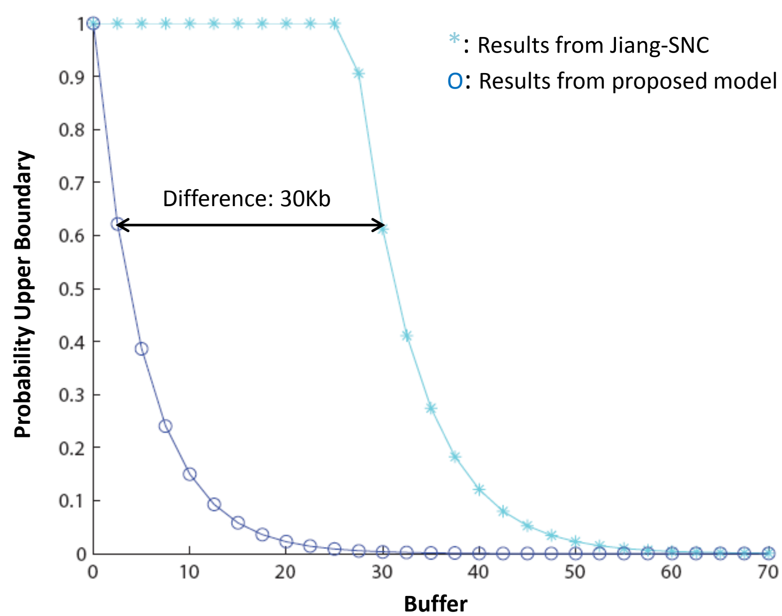


**Figure 1. Comparison of two approaches**

From the Figure 1, it could be observed that, the model proposed in this paper outperforms Jiang-SNC approach. From the comparison, the probability upper boundary has the similar parts which is decreasing function of main element $ae^{-bx}$. The improved part is $(1+\frac{1}{\theta b})$ which is used in Jiang-SNC model for addressing the Prb-MinPlus problem [14-15]. While, it is not included in our model, that avoid the extra boosts of the probability upper boundary. In our model, $\alpha(t)=\rho(b)t$, $f(x)=e^{-bx}$, where, $b\ge 0$ is a free parameter, and $\rho(b)$ meets:

$\rho(b)=\frac{1}{2b}(Pb-\mu-\lambda+\sqrt{(Pb-\mu+\lambda)^2+4\lambda\mu})$. That means the stochastic arrival curve is presented in the data stream in the network system. The service speed is $C=1.0(Mbps)$ as a static service network system.

 

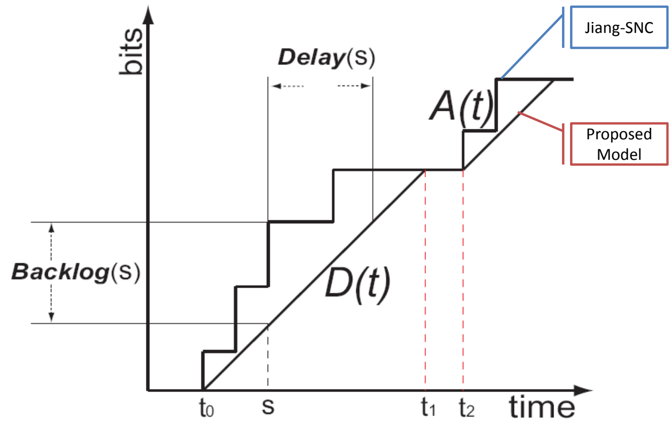


**Figure 2: Performance Analysis**

Thirdly, the QoS performance analysis is compared between the two approaches. It can be seen from Figure 2, the outperformance could be obtained from several dimensions. In the first place, the input and output relation is insistent. As $D(t) \le A(t)$, the output stream $D(t)$ is not bigger than the input $A(t)$ at the time $t$. That is significant for control the data stream in stochastic network since the accumulated data may cause data traffic problem within the limited bandwidth. Moreover, the backlog relationship with the input/output is optimized. As $Back\log(t) = A(t) - D(t)$, for time $t$, the backlog could be calculated by the difference of $A(t)$ and $D(t)$. That is the vertical distance in Figure 2, which could be marked as $v(A(t), D(t))$. Therefore, it could be visualized that the backlog of stochastic network could be calculated easily. Finally, the delay of transfer associates with input/output. Their relationship could be expressed as $Delay(t) = \inf_{0 \le s \le t} \{\tau : A(t) \le D(t+\tau)\}$. That means, at time $t$, the tasks arrive at the system have to wait for the service. The delay time is $Delay(t)$, which could be got from the horizontal distance between $A(t)$ and $D(t)$. The delay time is marked as $h(A(t), D(t))$. For the tasks arrive at time $t$, they will be output at the time $t + Delay(t)$.

## 4. Conclusion

This paper proposes a model to carry out the performance analysis in stochastic network by integrating arrival curve and service curve. It uses the probability calculation in presenting the stochastic arrival and service within a network system. The definitions of arrival curve and service curve are different from the traditional method so as to avoid the problem of Prb-MinPlus.

Several research findings are significant. Firstly, the arrival curve indicates the stochastic data steam in the network by integrating the probability functions. Thus, the data stream is able to reflect the current situation in a network. Secondly, the model proposed in this paper integrates arrival curve and service curve to build up the model. As a result, the performance analysis could be carried out more precise. Thirdly, the proposed model has been compared with Jiang-SNC in terms of probability upper boundary and overall performance. It is observed that, the proposed model outperforms Jiang-SNC in QoS and can avoid the problem of Prb-MinPlus.

Future research directions could be carried out from several aspects. First is the extension of both arrival curve and service curve which may consider more impact factors within a network such as routines, disturbances, etc. Second is the compatibility, independability, and solidarity of this model with other performance dimensions like data accuracy, data privacy and so on could be further investigated.

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## References

[1] F. Ciucu, A. Burchard and J. Liebeherr, “A network service curve approach for the stochastic analysis of networks”, Paper presented at the ACM SIGMETRICS performance evaluation review, ACM, vol. 33, **(2005)**, pp. 279-290.
[2] R. Y. Zhong, Q. Y. Dai, T. Qu, G. J. Hu and G. Q. Huang, “RFID-enabled Real-time Manufacturing Execution System for Mass-customization Production”, Robotics and Computer-Integrated Manufacturing, vol. 29, no. 2, **(2013)**, pp. 283-292.
[3] F. Ciucu, A. Burchard and J. Liebeherr, “Scaling properties of statistical end-to-end bounds in the network calculus”, Information Theory, IEEE Transactions on, vol. 52, no. 6, **(2006)**, pp. 2300-2312.
[4] Q. Y. Dai, R. Y., Zhong, G. Q. Huang, T. Qu, T. Zhang and T. Y. Luo, “Radio frequency identification-enabled real-time manufacturing execution system: a case study in an automotive part manufacturer”, International Journal of Computer Integrated Manufacturing, vol. 25, no. 1, **(2012)**, pp. 51-65.
[5] Y. Liu, C. -K. Tham and Y. Jiang, “A calculus for stochastic QoS analysis”, Performance Evaluation, vol. 64, no. 6, **(2007)**, pp. 547-572.
[6] R. Y. Zhong, Q. Y. Dai, K. Zhou and X. B. Dai, “Design and Implementation of DMES Based on RFID”, Paper presented at the 2nd International Conference on Anti-counterfeiting, Security and Identification, Guiyang, **(2008)** August 20-23, pp. 475-477.
[7] J. -L. Scharbarg, F. Ridouard and C. Fraboul, “A probabilistic analysis of end-to-end delays on an AFDX avionic network”, Industrial Informatics, IEEE Transactions on, vol. 5, no. 1, **(2009)**, pp. 38-49.
[8] R. Y. Zhong, G. Q. Huang, Q. Y. Dai and T. Zhang, “Estimation of Lead Time in the RFID-enabled Real-time Shopfloor Production with a Data Mining Model”, Proceeding of The 19th International Conference on Industrial Engineering and Engineering Management, **(2012)** October 27-29, ChangSha, P.R. China.
[9] R. Y. Zhong, G. Q. Huang, Q. Y. Dai and T. Zhang, “Mining SOTs and Dispatching Rules from RFID-enabled Real-time Shopfloor Production Data”, Journal of Intelligent Manufacturing, DOI: 10.1007/s10845-012-0721-y, **(2013)**.
[10] J. P. Georges, T. Divoux and E. Rondeau, “Confronting the performances of a switched ethernet network with industrial constraints by using the network calculus”, International journal of communication systems, vol. 18, no. 9, **(2005)**, pp. 877-903.
[11] Y. Gao and Y. Jiang, “Performance analysis of a cognitive radio network with imperfect spectrum sensing”, Paper presented at the INFOCOM IEEE Conference on Computer Communications Workshops, **(2010)**.
[12] Y. Jiang, Q. Yin, Y. Liu and S. Jiang, “Fundamental calculus on generalized stochastically bounded bursty traffic for communication networks”, Computer Networks, vol. 53, no. 12, **(2009)**, pp. 2011-2021.
[13] R. Y. Zhong, Z. Li, A. L. Y. Pang, Y. Pan, T. Qu and G. Q. Huang, “RFID-enabled Real-time Advanced Planning and Scheduling Shell for Production Decision-making”, International Journal of Computer Integrated Manufacturing, vol. 26, no. 7, **(2013)**, pp. 649-662.
[14] A. Bose, X. Jiang, B. Liu and G. Li, “Analysis of manufacturing blocking systems with network calculus”, Performance Evaluation, vol. 63, no. 12, **(2006)**, pp. 1216-1234.
[15] Y. Jiang, “Stochastic network calculus for performance analysis of Internet networks—An overview and outlook”, Paper presented at the 2012 International Conference on Computing, Networking and Communications (ICNC), **(2012)**, pp. 638-644.

 

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