# Fuzzy Sliding Mode-PID Control for Space Manipulator Using Dynamically Equivalent Manipulator Model

Behrouz Kharabian

Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran

beh16819@yahoo.com

#### Abstract

In this paper, a sliding mode-PID control approach is proposed for free-floating space manipulator. Free-floating space manipulator has nonlinear dynamics and interactions between the manipulator and the spacecraft. In this paper, dynamic of space manipulator system is presented by dynamically equivalent manipulator (DEM) model. DEM is a fixedbase manipulator and its dynamics behavior under any control law is identical to dynamics behavior of space manipulator. Using sliding mode-PID control, sliding mode controller and PID controller properties are applied to the control system. In this case, high control gain leads to reduction of reaching time but increment of oscillation. Therefore fuzzy logic can be used as an intelligent approach in order to tune PID-like sliding surface gains. Simulation shows satisfactory results in tracking and error cancellation.

*Keywords:* Space manipulator, Free-floating, dynamically equivalent manipulator, sliding mode-PID control

### **1. Introduction**

Space manipulators have various applications in spacecraft and satellite missions, including assemblage, maintenance and repair. In free-flying space manipulators, Using reaction wheels and jets, inertia parameters of dynamic model of space manipulator can be linearly parameterized. But attitude control fuel consumption of reaction wheels and jets increases through motions on the spacecraft [1]. In order to conserve fuel or electrical power freefloating system is proposed. In free-floating approach, the base rotation due to the movements of joints has to be considered. Dubowsky and Torres [2] proposed disturbance map concept in order to minimize the base rotation. Since the manipulator and the base in the space manipulator system have complicated dynamics and nonlinear interactions, therefore control issue is a big problem. Papadopoulos and Dubowsky [1] showed that any control algorithm that can be applied to conventional fixed-base manipulators can be applied to space manipulators where exact knowledge of inertia parameters is available. Exact knowledge of the inertia parameter always is not available. In this paper, dynamic of space manipulator is presented by dynamically equivalent manipulator (DEM) model [3]. Using DEM model, this complicated structure maps to a fixed-base manipulator. In this model, dynamic and kinematic properties of space manipulator are preserved and dynamic equations can be linearly parameterized. Parlaktuna and Ozkan [4] applied an adaptive control method to space manipulator using DEM model. In this approach, unknown parameters such as mass and inertia tensor parameters are estimated by adaptive methods. In DEM model, uncertainty problem leads to unstable behavior. In order to handle uncertainties sliding mode control is a good choice. In sliding mode control as a robust approach, sensitivity to structured uncertainties and disturbances near sliding surface is an important issue, causes accuracy reduction and error increment. Considering space environment conditions, versatile and precise performance of space manipulator is a necessity. In this case, sliding mode control with other control method can be used to improve control system performance. Sliding mode controller whit PID-like sliding surface is one of these methods. Consequently, a hybrid sliding mode-PID control is proposed in order to use properties of both. Using high sliding surface gains leads to reaching time reduction but oscillation increases. Therefore, PID-like sliding surface gains is tuned by an on-line fuzzy approach. Fuzzy sliding mode-PID control as a robust and intelligent approach leads to smooth result with zero error. On the other hand, parametric uncertainties problem of model and disturbances can be handled entirely by this control approach.

This paper is organized as follows: Section 2 presents dynamic of free-floating space manipulator and DEM model. In Section 3, sliding mode-PID controller as a hybrid control is presented. PID gains are tuned by fuzzy approach in Section 4. Simulation result is shown in Section 5. Finally, conclusions are presented in Section 6.

# 2. Space Manipulator Dynamic and DEM Model

Consider a system with *n*-DOF rigid manipulator mounted on a free-floating base, as shown in Figure 1. Space manipulator system includes an n-link manipulator and its base. The base of space manipulator is denoted as link 1, the links of the manipulator as links 2 through n+1.



Figure 1. Free-floating space manipulator system

In Figure 1,  $(\phi, \theta, \omega)$  are Z-Y-Z Euler angles representing the orientation of the space manipulator's base,  $J_i$  is the joint connecting (*i*-1)th link and *i*th link,  $\theta_i$  is the rotation of the space manipulator's *i*th link around joint  $J_i$ .  $C_o$  is the space manipulator's total center of mass,  $C_i$  is center of mass of space manipulator's *i*th link,  $l_i$  is vector connecting  $C_o$  to  $C_i$ ,  $u_i$  is the rotation axis of  $J_i$ .



Figure 2. Coordinate frame attached to the space manipulator's links

Frames 1, ..., and n+1 are the coordinate frames attached to the center of mass of each space manipulator's link, as shown in Figure 2.  $L_i$  is vector connecting  $J_i$  to  $C_i$ ,  $R_i$  is vector connecting  $C_i$  to  $J_{i+1}$ . Assuming no external forces and torques act on the space manipulator, its center of mass  $C_o$  remains fixed in inertial space and can be selected as the origin of the inertial coordinate frame. The total kinetic energy of the system can be written as

$$T = \sum_{i=1}^{n+1} \left( \frac{1}{2} m_i \rho_i^T \rho_i + \frac{1}{2} \omega_i^T R_i^o I_i R_i^{o^T} \omega_i \right)$$
(1)

where  $\dot{\rho}_i$  is the translational velocity of the center of mass of the *i*th link,  $R_i^o$  is the rotation matrix that describes the coordinate frame *i* relative to frame *o*,  $\omega_i$  angular velocity of  $C_i$ ,  $I_i$  is inertia tensor of *i*th link. The DEM is a real fixed-base robot which can be physically built and experimentally used for studying the dynamic behavior of space manipulator. Assuming zero gravity, the system is not acted upon by any gravitational forces, therefore the potential energy is equal to zero, and the Lagrangian is equal to the kinetic energy. Vector of generalized coordinates are considered as

$$q = [\phi \quad \theta \quad \psi \quad \theta_2 \quad \dots \quad \theta_{n+1}]^T = [q_1 \quad \dots \quad q_{n+3}]^T$$

$$(2)$$

Therefore Lagrangian equation is written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_i}\right) - \frac{\partial T}{\partial q_i} = Q_i , \quad i=1,2,\dots,n+3$$
(3)

where  $Q_i$  is generalized force corresponding to the generalized coordinate  $q_i$ 

$$Q_i = 0$$
,  $i=1,2,3$  (4)

$$Q_i = \tau_{i-3}$$
,  $_{i=4,5,...,n+3}$   
and  $\tau_i$  is the torque exerted on the *i*th joint. Substituting (4) into (3), dynamic equation of

$$M(q)q + C(q,q)q = \tau \tag{5}$$

where  $M(q) \in R^{(n+3)\times(n+3)}$  is inertia matrix,  $C(q,q)q \in R^{(n+3)}$  is vector of the Coriolis and centrifugal forces,  $\tau$  is torque acting upon the joints of the space manipulator. Free-floating space manipulator can be mapped to a fixed-base manipulator that its first joint is passive spherical joint, so-called dynamical equivalent manipulator model. In this model, dynamic and kinematic properties of space manipulator are preserved. As in Figure 3 is shown DEM coordinate frames are parallel to the corresponding frames of the space manipulator and its base coincides with the total center of mass of the space manipulator.

In Figure 3, the axis of the space manipulator's *i*th joint is parallel to the axis of the DEM's *i*th joint. The displacement of each of the DEM's joints during motion is identical to the displacement of the corresponding space manipulator joint.



Figure 3. Space manipulator and its corresponding DEM

As in Figure 4 is shown, in DEM coordinate frames are respectively attached to the center of mass of the DEM links. In Figure 4,  $(\phi', \theta', \omega')$  are Z-Y-Z Euler angles representing the orientation of the DEM's passive spherical joint,  $J'_i$  is the joint connecting the DEM's (i+1) th link and *i*th link,  $\theta'_i$  is the relative rotation of the DEM's link around joint  $J'_{i,}$ ,  $l_{ci}$  is the vector connecting  $J'_i$  to  $C'_i$ ,  $\omega'_i$  is the angular velocity of  $C'_i$ .



Figure 4. Fixed-base robot manipulator with a passive spherical joint at the base

Assuming zero gravity, the potential energy is equal to zero, and Lagrangian is equal to kinetic energy, and written as [3]

$$T' = \sum_{i=1}^{n+1} \left(\frac{1}{2}m_i v_{ci}^{'T} v_{ci}^{'} + \frac{1}{2}\omega_i^{'T} R_i^{o} I_i R_i^{o^T} \omega_i^{'}\right)$$
(6)

where  $m'_i$  is the mass of the DEM's *i*th link,  $\omega'_i$  is the angular velocity of  $C'_i$ . The vector of generalized coordinates for DEM are considered as

$$q' = [\phi' \quad \phi' \quad \psi' \quad \theta'_2 \quad \dots \quad \theta'_{n+1}]^T = [q'_1 \quad \dots \quad q'_{n+3}]^T$$
 (7)

and Lagrange equation is written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_{i}}\right) - \frac{\partial T}{\partial q_{i}} = Q_{i}$$
(8)

where

$$Q_i = 0, \quad i = 1, 2, 3$$
 (9)

$$Q_i = \tau_{i-3}, \quad i = 4,5,...,n+3$$
  
therefore dynamic equation of DEM is obtained as  
 $M'(q')q' + C'(q',q')q' = \tau'$  (10)

where  $\tau' = \begin{bmatrix} 0 & 0 & 0 & \tau'_2 & \dots & \tau'_{n+1} \end{bmatrix}$  is the torque exerted on the DEM's joints.  $M'(q') \in R^{(n+3)\times(n+3)}$  is inertia matrix,  $C'(q',q')q' \in R^{(n+3)}$  is vector of the Coriolis and centrifugal forces. In Eq (10), M'(q') is symmetric and positive-definite matrix. Also M'(q') and  $M'^{-1}(q')$  are uniformly bounded. Dynamic and kinematic parameters of space manipulator can be mapped to DEM, and written as

$$m_{i}^{'} = \frac{m_{i}(\sum_{k=1}^{n+1} m_{k})^{2}}{\sum_{k=1}^{i-1} m_{k} \sum_{k=1}^{i} m_{k}}, \quad i = 2,...,n+1 ,, \\ I_{i}^{'} = I_{i}, \quad i = 1,...,n+1, \\ W_{1} = \frac{R_{1}m_{1}}{\sum_{k=1}^{n+1} m_{k}}, \\ W_{i} = R_{i}(\frac{\sum_{k=1}^{i} m_{k}}{\sum_{k=1}^{i-1} m_{k}}) + L_{i}(\frac{\sum_{k=1}^{i-1} m_{k}}{\sum_{k=1}^{n+1} m_{k}}), \quad i = 2,..., n+1, \\ \sum_{k=1}^{i} m_{k} \sum_{k=1}^{i-1} m_{k}, \\ I_{c_{i}} = L_{i}(\frac{\sum_{k=1}^{i-1} m_{k}}{\sum_{k=1}^{n+1} m_{k}}), \quad i = 2,...,n+1$$
(11)

In this model, mapping uncertainties including in masses  $(dm'_i)$ , links' length  $(dW_i)$ , location of center of mass  $(dl_{ci})$  and inertia tensor  $(dI'_i)$  due to error in  $m_i$ ,  $L_i$ ,  $R_i$  and  $I_i$  are considered in the dynamic model of DEM. Therefore, dynamic equation estimation is written as

$$\hat{M}'(q') \dot{q} + \hat{C}'(q', q') \dot{q}' = \tau'$$
(12)

The DEM is dynamically equivalent to the space manipulator. In the other words, if exerted torque on the space manipulator's joints is equal to exerted torque on the DEM's joints, starting from equal initial condition, DEM's joints and space manipulator's joints track similar paths (*i.e.*, if  $\tau(t) = \tau'(t)$  and  $q(t_0) = q'(t_0)$  then q(t) = q'(t)).

#### 3. Sliding Mode- PID Controller

In order to minimize error and modify tracking, PID controller, as a capable and simple control approach in transient and steady state can be used in designing of sliding mode controller to improve stability and performance of system.  $q'_d$  is considered as desired joint

displacement. Control law  $\tau$  is designed so that joint displacement q' tracks the desired trajectory. Tracking error is defined by

$$e' = q_d - q' \tag{13}$$

In order to benefit by PID control approach in the sliding mode control, a PID-like sliding surface is considered as

$$S = K_D e' + K_P e' + K_I \int e' dt \tag{14}$$

and sliding condition is given as

$$\frac{1}{2}\frac{d}{dt}(S^{T}S) \leq -\eta |S| \tag{15}$$

where  $\eta$  is a positive constant.

For good tracking, states have to reach to sliding surface and remain on it, and sliding condition guaranteed, consequently error becomes zero. Therefore, control law is designed so that system stability is satisfied according to lyapunov conditions. Dynamic of system can be rewritten based on sliding surface as

$$M'S + C'S = P - K_D \tau'$$
(16)

where

$$P = M'(K_D q_d + K_D e + K_P e + K_P e + K_I e + K_I \int e' dt) + C'(K_D q_d + K_P e' + K_I \int e' dt)$$
(17)

Considering the sensitive and complicated dynamic of space manipulator system and sliding condition, control law is designed according to system model as

$$\tau' = M'(q')\sigma_r + C'(q',q')\sigma_r + k_1 S - k_2 \operatorname{sgn}(S)$$
(18)

where  $\dot{\sigma}_r$  and  $\ddot{\sigma}_r$  are given as

$$\sigma_r = \lambda_1 q_d + \lambda_2 e' + \lambda_3 \int e dt \tag{19}$$

$$\ddot{\sigma}_r = \lambda_1 q_d + \lambda_2 e + \lambda_3 e$$
<sup>(20)</sup>

where  $\lambda_{1,2,3} \in \mathbb{R}^{(n+1)\times(n+1)}$  and  $k_{1,2} \in \mathbb{R}^{(n+1)\times(n+1)}$  are diagonal positive-definite matrices.

*Remark 3.1*: avoiding oscillation, entries of  $\lambda_3$  are considered very small.

*Remark 3.2*: In order to eliminate the input chattering problem the sgn(S) is replaced by the  $sat(\frac{S}{s})$  function, and defined as

$$sat(\frac{S}{\delta}) = \begin{cases} 1, & \frac{S}{\delta} \ge 1 \\ \frac{S}{\delta}, & -1 \le \frac{S}{\delta} \le 1 \\ -1, & \frac{S}{\delta} \le -1 \end{cases}$$
(21)

where  $\delta$  is boundary layer.

*Remark 3.3*:  $k_2$  has to be considered such that

$$k_2(i,i) \ge |\Delta f_i|, \qquad k_2 = diag\{k_2(i,i)\}, \quad i = 1,2,...,n+1$$
(22)

and

$$\Delta f = \begin{bmatrix} \Delta f_1 & \Delta f_2 & \dots & \Delta f_{n+1} \end{bmatrix}^T$$
where
$$(23)$$

$$\Delta f = (\hat{M}' - M') \hat{\sigma}_r + (\hat{C}' - \hat{C}') \hat{\sigma}_r$$
(24)

Lyapunov function for system is proposed as

$$v = \frac{1}{2}S^T RS \tag{25}$$

where  $R \in R^{(n+1)\times(n+1)}$  is adjustable positive-definite matrix. It is considerable that *R* is identity matrix. Therefore, lyapunov function of system is written as

$$v = \frac{1}{2}S^T S \tag{26}$$

Control law has to satisfy lyapunov stability conditions written as below

$$v = \frac{1}{2}S^T S > 0 \tag{27}$$

$$\dot{v} = \frac{d}{dt} \left(\frac{1}{2}S^T S\right) = S^T \dot{S} \le 0$$
<sup>(28)</sup>

Substituting (16) into (28), obtains

$$\dot{v} = -S^T \dot{M}' \dot{C} S + S^T \dot{M}' (P - K_D \tau')$$
(29)

And substituting (18) into (29), obtains

$$v = -S^{T} \hat{M}^{-1} \hat{C}^{S} S$$

$$+S^{T} \hat{M}^{-1} (P - K_{D} (\hat{M}^{T} \sigma_{r} + \hat{C}^{T} \sigma_{r} + k_{1} S - k_{2} sat (\frac{S}{\delta})))$$
(30)

where  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$ ,  $k_1$  and  $k_2$  can be selected such that

$$v \le -\mu \sum_{i=1}^{n+1} |S_i| \le 0$$
 (31)

where  $\mu$  is a positive constant.

# 4. Fuzzy Tuning of Sliding Mode-PID Controller

High gain is used for tracking improvement, reduction of controller sensitivity to model uncertainty, external disturbances rejection and reaching time reduction. Despite having these preferences, using high gain leads to oscillation increment. Therefore, PID gains are tuned by fuzzy intelligent approach. In fuzzy approach, PID gains are tuned such that error reduces and states approach sliding surface. In the other words, fuzzy sliding mode-PID is a doubled approach in order to improve performance and stability. In order to simplify gains tuning, they can be normalized between 0 and 1 as

$$K_{PN} = \frac{K_P - K_{P\min}}{K_{P\max} - K_{P\min}}$$
(32)

$$K_{DN} = \frac{K_D - K_{D\min}}{K_{D\max} - K_{D\min}}$$
(33)

International Journal of Control and Automation Vol.7, No.2 (2014)

$$K_{IN} = \frac{K_I - K_{\text{Im}\,in}}{K_{\text{Im}\,ax} - K_{\text{Im}\,in}} \tag{34}$$

where *min* gains and *max* gains are considered properly. Integral and derivative time constants are respectively defined by

$$T_i = \frac{K_P}{K_I} \tag{35}$$

$$T_d = \frac{K_D}{K_P} \tag{36}$$

Assuming relation between integral and derivative time constant as below

$$(37)$$

therefore  $K_I$  is obtained as

$$K_I = \frac{K_P^2}{\alpha K_D}$$
(38)

Membership functions for errors and gains can be considered as shown in Figures (5), (6) and (7). The gains are logically tuned according to error and error derivative. Domains of

variation of e'(t) and e'(t) are  $[e'_{M^-}, e'_{M^+}]$  and  $[e'_{M^-}, e'_{M^+}]$ , respectively. If error is big and error derivative is small then  $K_{PN}$  is big,  $K_{DN}$  is small and  $\alpha$  is small. If error is small and error derivative is big then  $K_{PN}$  is small,  $K_{DN}$  is big and  $\alpha$  is big. The gains are formulated by using product inference engine, singleton fuzzifier, and center average defuzzifier. Therefore the gains are written as

$$K_{PN}(t) = \frac{\sum_{l=1}^{N} y_{P}^{-t} \mu_{A^{l}}(e^{t}(t)) \mu_{B^{l}}(e^{t}(t))}{\sum_{l=1}^{N} \mu_{A^{l}}(e^{t}(t)) \mu_{B^{l}}(e^{t}(t))}$$
(39)

$$K_{DN}(t) = \frac{\sum_{l=1}^{N} y_{D}^{-l} \mu_{A^{l}}(e^{i}(t)) \mu_{B^{l}}(e^{i}(t))}{\sum_{l=1}^{N} \mu_{A^{l}}(e^{i}(t)) \mu_{B^{l}}(e^{i}(t))}$$
(40)

$$K_{\alpha}(t) = \frac{\sum_{l=1}^{N} \overline{y} \mu_{A^{l}}(e^{\prime}(t)) \mu_{B^{l}}(e^{\prime}(t))}{\sum_{l=1}^{N} \mu_{A^{l}}(e^{\prime}(t)) \mu_{B^{l}}(e^{\prime}(t))}$$
(41)

where  $y_{P}$ ,  $y_{D}$  and  $y_{\alpha}$  are respectively the center of membership functions of  $K_{PN}$ ,  $K_{DN}$ and  $\alpha$ , N is the number of fuzzy rules,  $A^{l}$  and  $B^{l}$  are the membership functions of e'(t) and e'(t), respectively.



Fuzzy rules are selected such that transient response and steady state response improve.





Figure 6. Membership functions for  $K_{PN}$  and  $K_{DN}$ 



Figure 7. Membership functions for  $\alpha$ 

Fuzzy rules for gains tuning are given as

Table 1. Fuzzy rules for  $K_{PN}$  tuning (N=49)

		$\dot{e}'(t)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB	s	В	В	B	B	В	s
e'(t)	NM	s	В	В	В	В	В	s
	NS	s	s	в	В	В	s	s
	zo	s	s	s	в	s	s	s
	PS	s	s	В	В	В	s	s
	PM	s	В	в	В	В	В	s
	PB	s	В	в	В	В	В	s

		$\dot{e}'(t)$						
		NB	NM	NS	ZO	PS	PM	PB
	NB	В	S	s	s	S	s	В
	NM	В	В	s	s	s	В	В
	NS	в	В	в	s	В	В	В
<i>e</i> ′(t)	zo	в	В	В	В	В	В	В
	PS	в	В	В	s	В	В	В
	PM	в	В	s	s	s	В	В
	PB	в	s	s	s	s	s	В

Table 2. Fuzzy rules for  $K_{DN}$  tuning (N=49)

Table 3. Fuzzy rules for $\alpha$ tuning (N=49
--

		$\dot{e}'(t)$						
		NB	NM	NS	zo	PS	PM	PB
	NB	1	1	1	1	1	1	1
<i>e</i> ′(t)	NM	2	2	1	1	1	2	2
	NS	3	2	2	1	2	2	3
	zo	3	3	2	2	2	3	3
	PS	3	2	2	1	2	2	3
	PM	2	2	1	1	1	2	2
	PB	1	1	1	1	1	1	1

# 5. Simulation Result

In this section, a 2-DOF space manipulator has been supposed which according to DEM model, a fixed-base manipulator with three joints is obtained such that first joint is passive spherical. Therefore, results are considered for two active joints. Kinematic and dynamic parameters of the space manipulator present in Table 4. Corresponding parameters of the DEM present in Table 5.

Table 4. Dynamic parameters of 2-link space manipulator

link	$L_i(m)$	$R_i(m)$	$m_i(kg)$	$I_i(kg.m^2)$
base	-	0.5	4	0.4
2	0.5	0.5	1	0.1
3	0.5	0.5	1	0.1

link	$W_i(m)$	$l_{ci}(m)$	$m_i(kg)$	$I_i(kg.m^2)$
1	0.333	0	4	0.4
2	0.750	0.333	1.8	0.1
3	0.917	0.417	1.2	0.1

Table 5. Dynamic parameters of 3-link DEM

Dynamic equation of 2-link space manipulator is written as

$$M(q) \ddot{q} + C(q, q) \dot{q} = \begin{bmatrix} 0 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$
(42)

Corresponding dynamic equation of 2-active joints DEM with three links is written as

$$\hat{M}'(q') \dot{q}' + \hat{C}'(q', q') \dot{q}' = \begin{bmatrix} 0 \\ \tau'_2 \\ \tau'_3 \end{bmatrix}$$
(43)

In simulation  $\lambda_2$ ,  $k_1$  and R have been considered identity matrices. Also  $\lambda_1$ ,  $\lambda_3$  with small entries and  $k_2$  are considered as

$$\lambda_1, \lambda_3 = \begin{bmatrix} 10^{-6} & 0 & 0 \\ 0 & 10^{-6} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}, k_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

Membership functions for error and error derivative are Gaussian (Variance = 0.3 and  $[e_{M^-}, e_{M^+}] = [-1,1]$ ,  $[e_{M^-}, e_{M^+}] = [-1,1]$ ). The *min* and *max* gains are considered as below

Gains fuzzy estimates are shown in Figure 8.



Figure 8. Gains fuzzy estimates for (a) second and (b) third entries of sliding surface





pulse) trajectories for (a) joint 2 and (b) joint 3

The rest of simulations are for step signal as desired trajectory. Tracking error are shown in Figure 10. Control signals as exerted torque are shown in Figure 13.

International Journal of Control and Automation Vol.7, No.2 (2014)



Figure 10. Joint angle error for (a) joint 2 and (b) joint 3



Figure 11. Exerted torque on (a) joint 2 and (b) joint 3

Second and third entries curve of sliding surface vector are shown in Figure 12.



Figure 12. (a) Second entry and (b) third entry curve of sliding surface vector

It has been mentioned that joint 2 and joint 3 are active and first joint is passive spherical joint, therefore, lyapunov function is defined by

$$v = \frac{1}{2} \begin{bmatrix} S_2 \\ S_3 \end{bmatrix}^T \begin{bmatrix} S_2 \\ S_3 \end{bmatrix}$$
(44)

Lyapunov function and lyapunov function derivative curve are shown in Figures 13 and 14 respectively.



Figure 14. Lyapunov function derivative curve

Control system performance in the presence of unit step disturbance in 20th second on exerted torque is assessed as shown in Figure 15.



Figure 15. Disturbance rejection in (a) joint 2 and (b) joint 3

# 6. Conclusions

In this paper, an effective control approach has been proposed to improve performance and handling of uncertainties for free-floating space manipulator. Considering complicated dynamic of the space manipulator, it is big challenge to design a control approach to achieve desired performance. In this paper, space manipulator's dynamic has been mapped to a fixed-base manipulator's dynamic, so-called dynamically equivalent manipulator (DEM). Proposed controller benefits from preferences of PID control and sliding mode control. In addition, to

achieve more smooth response fuzzy approach has been used to tune the PID gains. Control law has been designed so that error cancellation, good state trajectory tracking and lyapunov stability conditions are satisfied. Simulation result shown that control system has proper performance.

### References

- [1] E. Papadopoulos and S. Dubowsky, "On the nature of control algorithms for free-floating space manipulators", IEEE Trans. on Robotics and Automation, vol. 7, (1991), pp. 750-758.
- [2] S. Dubowsky and M. A. Torres, "Path Planning for Space Manipulators to Minimize Spacecraft Attitude Disturbance", Proc. IEEE Int. Conference on Robotics and Automation Sacramento, California, (1991).
- [3] B. Liang, Y. Xu and M. Bergerman, "Mapping a space manipulator to a dynamically equivalent manipulator", ASME Journal of Dynamic Dynamic Systems, Measurement and Control, vol. 120, (**1998**), pp. 1-7.
- [4] O. Parlaktuna and M. Ozkan, "Adaptive control of free-floating space manipulators using dynamically equivalent manipulator model, Robotics and Autonomous Systems, vol. 46, (**2004**), pp. 185-193.
- [5] D. Muro-Maldonado, A. Rodriguez-Angeles and C. A. Cruz-Villar, "Sliding PID Control for Trajectory Tracking of a 2 DOE Robot Manipulator: simulations and Experiments", 4th Int. Conference on Electrical and Electronics Engineering (ICEEE), (2007).
- [6] A. Rhif, "A High Order Sliding Mode Control with PID Sliding Surface: Simulation on a Torpedo", Int. Journal of Information Technology, Control and Automation (IJITCA), vol. 2, no. 1, (**2012**).
- [7] D. Hernandez, W. Yu and M. A. Moreno-Armendariz, "Neural PD control with second-order sliding mode compensation for robot manipulators", Proc. Int. Joint Conference on Neural Networks, San Jose, California, USA, (2011).
- [8] Q. P. Ha, D. C. Rya and H. F. Durrant-Whyte, "Fuzzy moving sliding mode control with application to robotic manipulators, Automatica, vol. 35, (**1996**), pp. 607-616.
- [9] Y. Fenga, X. Yu and Z. Manc, "Non-singular terminal sliding mode control of manipulators", Automatica, vol. 38, (2002), pp. 2159-2167.
- [10] Y. Li and Q. Xu, "Adaptive Sliding mode Control With Perturbation Estimation and PID Sliding Surface for Motion Tracking of a Piezo-Driven Micromanipulator", IEEE Trans. on Control System Technology, vol. 18, no. 4, (2010).
- [11] L. Sciavicco and B. Sociliano, "Modeling and Control of Robot Manipulators", McGraw-Hill, New York, (1996).
- [12] V. Parra-Vega, S. Arimoto, Y. H. Liu, G. Hirzinger and P. Akella, "Dynamic Sliding PID Control for Tracking of Robot Manipulator: Theory and Experiments, IEEE Trans. on Robotics and Automation, vol. 19, no. 6, (2003).
- [13] J. Ohri, D. R. Vyas and P. N. Topno, "Comparison of Robustness of PID Control and Sliding Mode Control of Robotic Manipulator", Int. Symposium on Devices MEMS, Intelligent Systems & Communication (ISDMISC), (2011).
- [14] T. H. Tran, Q. P. Ha and H. T. Nguyen, "Robust Non-Overshoot Time Responses Using Cascade Sliding Mode-PID Control", Journal of Advanced Computational Intelligence, vol. 11, no. 10, (2007).
- [15] Y. S. Lu and C. M. Cheng, Design of a Non-Overshooting PID Controller with an Integral sliding Perturbation Observer for Motor Positioning Systems, JSME Int. Journal, series C., vol. 48, no. 1, (2005).
- [16] S. Abiko and G. Hirzinger, "Computational Efficient Algorithms for Operational Space Formulation of Branching Arms on a Space Robot", IEEE/RSJ Int. Conference on Intelligent Robots and Systems, Acropolis Convention Center Nice, France, (2008).
- [17] T. C. Kuo and Y. J. Huang, "A Sliding Mode PID Controller Design for Robot Manipulators", IEEE Int. Symposium on Computational Intelligence in Robotics and Automation, (2005), pp 27-30.

International Journal of Control and Automation Vol.7, No.2 (2014)