

## Trajectory Tracking Control of Multi Degrees of Freedom Joints: Robust Fuzzy Logic-Based Sliding Mode Approach

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### Abstract

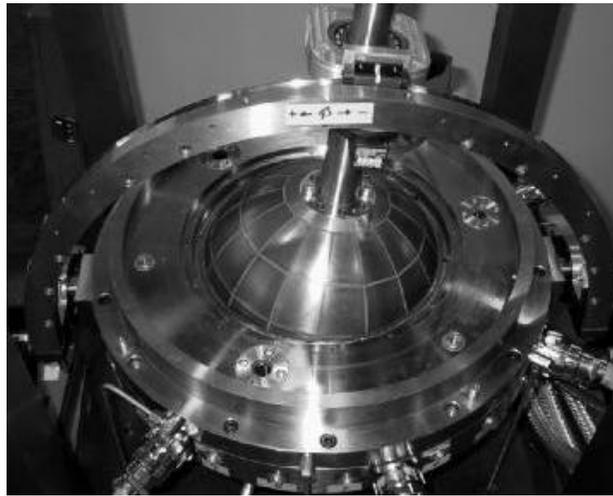
*The multi degrees of freedom actuator is an important joint, which has attracted worldwide developing interests for its medical, industry and aerospace applications. This paper addresses the problem of trajectory tracking of three dimensions joint in the presence of model uncertainties and external disturbances. An adaptive fuzzy sliding mode controller (AFLSMC) is proposed to steer a three dimension joint along a desired trajectory precisely. First, the dynamics model of a three dimension joint is formulated and the trajectory tracking problem is described. Second, a sliding mode controller (SMC) is designed to track a time-varying trajectory. The fuzzy logic system (FLS) is employed to approximate the uncertain model of the three dimension joint, with the tracking error and its derivatives and the commanded trajectory and its derivatives as FLS inputs and the approximation of the uncertain model as FLS output. And a fuzzy logic system is also adopted to attenuate the chattering results from the SMC. The control gains are tuned synchronously with the sliding surface according to fuzzy rules, with switching sliding surface as fuzzy logic inputs and control gains as fuzzy logic outputs. The stability and convergence of the closed-loop controller is proven using the Lyapunov stability theorem. Finally, the effectiveness and robustness of the proposed controller are demonstrated via simulation results. Contrasting simulation results indicate that the AFLSMC attenuates the chattering effectively and has better performance against the SMC.*

**Keywords:** *Trajectory tracking, sliding mode control, fuzzy rules, approximation, adaptive method, multi degrees of freedom joints*

### 1. Introduction

Advances in robotics, office automation and intelligent flexible manufacturing and assembly systems have necessitated the development of precision actuation systems with multiple degrees of freedom (DOF). In general, however, motion with several DOF is currently realised almost exclusively by using a separate motor/actuator for each axis, which results in complicated transmission systems and relatively heavy structures. This inevitably compromises the dynamic performance, owing to the effects of inertia, backlash, nonlinear friction and elastic deformation of gears, for example. Actuators which are capable of controlled motion in two or more degrees of freedom can alleviate these problems, while

being lighter and more efficient. A particular interesting configuration to perform these tasks is the spherical machine [1-3]. Figure 1 shows the multi-DOF actuators [4].



**Figure 1. Multi DOF Actuator**

There are two basic methods of dealing with control problems, the linear methodology and the nonlinear methodology. Strictly speaking, linear systems do not exist in practice, since all physical systems are nonlinear. However, the great majority of control algorithms are designed assuming linear behavior. To make linear methodologies work, the key assumption is to assume a linear characteristics over a small range of operation. This enables the designer to obtain a linearized version of a nonlinear system model. But, there are numerous control situations in which the linear control system fails to meet the requirements, for instance, systems with large parameter variations or when the state of the system is far from the linearization point. The parameter variations and nonlinearities can degrade system performance and possibly destabilize the system [5-6]. For multi DOF actuators, the linear control methods must essentially be viewed as approximate methods. This is because a multi DOF actuator dynamics is strongly nonlinear in nature due to the presence of Coulomb friction, backlash, payload variation, unknown disturbances, large dynamic coupling between different links, flexibility and time-varying parameters (*e.g.*, tear and wear) [7-10]. The linear approximation of a multi DOF actuator dynamics is valid only when a small workspace is required. Considering linear and nonlinear control methodologies, the nonlinear ones are more general since they can be successfully applied to linear systems whereas a linear controller might be insufficient for control of a nonlinear system. It is worthy to note that, more recently, there has been a growing interest among researchers in new types of controllers which can combine the approximation power of computational-intelligence techniques (*e.g.*, fuzzy logic and neural networks) with the simplicity of the most commonly used linear controller called three-term or proportional-integral-derivative (PID) controller. The complexity of nonlinear uncertain systems challenges a control system designer to come up with a unified systematic design procedure to meet the control objectives (*e.g.*, stability and robustness) and design specifications. Faced with such challenges, an investigation of different nonlinear uncertain systems has made it clear that the designer cannot expect one particular procedure be applicable to all nonlinear uncertain systems. The most common methodologies that have been proposed to solve the control problem of different

classes of the nonlinear uncertain systems consist of the several methodologies such as Feedback Linearization Control Methodology (Computed Torque Controller (CTC)), Passivity-Based Control Methodologies, Sliding Mode Control, Robust Lyapunov-Based Control Methodologies, Adaptive Control Methodologies and Computational-Intelligence-Based Control Methodologies including fuzzy logic, neural network and combination of neural network and fuzzy logic called neuro-fuzzy systems [11-14].

A multitude of nonlinear control laws have been developed called sliding mode controller (SMC) in the multi DOF actuators literature. This controller incorporates the dynamic model of multi DOF actuators to construct model-based controller. The SMC have their root in feedback control methodology. The idea is to design a sliding mode controller which cancels the nonlinearities of a multi-DOF actuator. In this manner the closed-loop system becomes exactly linear or partly linear depending on the accuracy of the dynamic model, and then a linear controller such as PD and PID can be applied to control the multi-DOF actuator. This controller is a robust controller regarding to switching discontinuous part. However this part caused to design robust controller but chattering phenomenon are the main drawbacks of this design. The other potential difficulty encountered in the implementation of the SMC methodology is that the dynamic model of the system to be controlled is often not known accurately. In order to overcome these difficulties an intelligent version of the SMC has been proposed in this research [14-17].

As an attempt to deal with uncertainties and account for the concept of partial truth, one of the main method called fuzzy logic (FL) was proposed by Zadeh (1965). Fuzzy logic incorporates an alternative way of modelling, which allows modelling complex systems using a higher level of abstraction (*i.e.*, using linguistic variables such as, "slow", "medium" and "fast" ) originating from the knowledge and experience, for instance, input-output data of a system. Fuzzy logic provides approximation capabilities to capture uncertainties, which cannot be described by precise mathematical models. Precisely the modelling aspect of fuzzy has been employed by control community either to model a controller itself (*e.g.*, modelling the actions of a human operator that controlling a machine), or a systems to be controlled, may be precisely and rigorously [16-17]. Fuzzy systems can be used for modelling (approximation) of the behaviour of linear/nonlinear dynamic/static systems (*e.g.*, robot manipulator) or of a controller (*e.g.*, PID controller). If fuzzy logic is used to model a system it is called "*fuzzy model*" of the system while it is called "*fuzzy controller*" if it is used to model a control law [17-18].

In this research the new technique of sliding mode controller is recommended, namely, conventional adaptive fuzzy sliding mode controller. To modify the response of this controller, on-line tuning conventional method is recommended in this research.

This paper is organized as follows; section 2, is served as a modeling and formulation of spherical motor. Part 3, introduces and describes the control design for trajectory tracking. Section 4 presents the simulation results and discussion of this algorithm applied to a spherical motor and the final section describe the conclusion.

## 2. Modeling and Formulation

Since its inception, the field of multi-DOF actuator dynamics has presented many issues in refining both theory and operations; one of the most challenging areas of study has been the problem of computational efficiency in the dynamics of mechanisms. Many efficient algorithms in dynamics have been developed to address this problem.

The dynamics of multi-DOF actuators illustrate the relationship between force and motion. The generalized force for a multi-DOF actuator can be described as a second-order nonlinear

differential equation. The dynamic equation of multi-DOF actuators is derived using the Lagrangian. The Lagrangian is derived by subtracting potential energy from kinetic energy.

The dynamic equation of multi-DOF actuator governed by the following equation [1-3]:

$$H(q) \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{\gamma} \end{bmatrix} + B(q) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + C(q) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \quad (1)$$

- $\tau$  is actuation torque
- $H(q)$  is a symmetric and positive definite inertia matrix
- $B(q)$  is the matrix of coriolis torques
- $C(q)$  is the matrix of centrifugal torques.

The angular acceleration is found as to be:

$$\begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{\gamma} \end{bmatrix} = H^{-1}(q) \cdot \left\{ \tau - B(q) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} - C(q) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right\} \quad (2)$$

### 3. Control Design for Trajectory Tracking

Sliding mode control theory for control joint of robot manipulator was first proposed in 1978 by Young to solve the set point problem ( $\dot{q}_d = 0$ ) by discontinuous method in the following form;

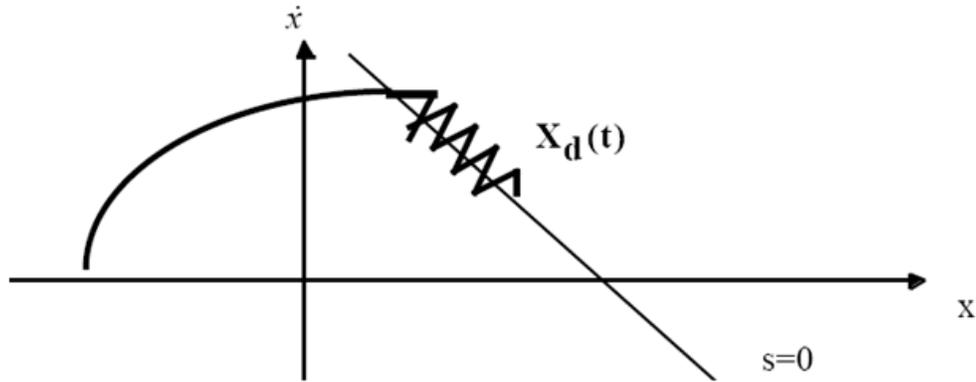
$$\tau_{(q,t)} = \begin{cases} \tau_i^+(q,t) & \text{if } S_i > 0 \\ \tau_i^-(q,t) & \text{if } S_i < 0 \end{cases} \quad (3)$$

where  $S_i$  is sliding surface (switching surface),  $i = 1, 2, \dots, n$  for  $n$ -DOF joint,  $\tau_i(q, t)$  is the  $i^{th}$  torque of joint. Sliding mode controller is divided into two main sub controllers:

- Corrective control ( $U_c$ )
- Equivalent controller ( $U_{eq}$ ).

Discontinuous controller causes an acceptable tracking performance at the expense of very fast switching. Conversely in this theory good trajectory following is based on fast switching, fast switching is caused to have system instability and chattering phenomenon. Fine tuning the sliding surface slope is based on nonlinear equivalent part [10-11]. However, this controller is used in many applications but, pure sliding mode controller has two most important challenges: chattering phenomenon and nonlinear equivalent dynamic formulation in uncertain parameters[12].

Chattering phenomenon (Figure 2) can cause some problems such as saturation and heat the mechanical parts of joints or drivers.



**Figure 2. Chattering as a Result of Imperfect Control Switching [1]**

Design a robust controller for multi-DOF-joints is essential because these joints have highly nonlinear dynamic parameters. Consider a nonlinear single input dynamic system is defined by:

$$\mathbf{x}^{(n)} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})\mathbf{u} \quad (4)$$

Where  $\mathbf{u}$  is the vector of control input,  $\mathbf{x}^{(n)}$  is the  $n^{th}$  derivation of  $\mathbf{x}$ ,  $\mathbf{x} = [\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}, \dots, \mathbf{x}^{(n-1)}]^T$  is the state vector,  $\mathbf{f}(\mathbf{x})$  is unknown or uncertainty, and  $\mathbf{b}(\mathbf{x})$  is of known *sign* function. The main goal to design this controller is train to the desired state;  $\mathbf{x}_d = [\mathbf{x}_d, \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d, \dots, \mathbf{x}_d^{(n-1)}]^T$ , and trucking error vector is defined by [14]:

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{\mathbf{x}}, \dots, \tilde{\mathbf{x}}^{(n-1)}]^T \quad (5)$$

A time-varying sliding surface  $\mathbf{s}(\mathbf{x}, \mathbf{t})$  in the state space  $\mathbf{R}^n$  is given by:

$$\mathbf{s}(\mathbf{x}, \mathbf{t}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{\mathbf{x}} = \mathbf{0} \quad (6)$$

where  $\lambda$  is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows:

$$\mathbf{s}(\mathbf{x}, \mathbf{t}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \left(\int_0^t \tilde{\mathbf{x}} dt\right) = \mathbf{0} \quad (7)$$

The main target in this methodology is kept the sliding surface slope  $\mathbf{s}(\mathbf{x}, \mathbf{t})$  near to the zero. Therefore, one of the common strategies is to find input  $\mathbf{U}$  outside of  $\mathbf{s}(\mathbf{x}, \mathbf{t})$ .

$$\frac{1}{2} \frac{d}{dt} \mathbf{s}^2(\mathbf{x}, \mathbf{t}) \leq -\zeta |\mathbf{s}(\mathbf{x}, \mathbf{t})| \quad (8)$$

where  $\zeta$  is positive constant.

$$\text{If } \mathbf{S}(0) > 0 \rightarrow \frac{d}{dt} \mathbf{S}(t) \leq -\zeta \quad (9)$$

To eliminate the derivative term, it is used an integral term from  $t=0$  to  $t=t_{reach}$

$$\int_{t=0}^{t=t_{reach}} \frac{d}{dt} \mathbf{S}(t) \leq - \int_{t=0}^{t=t_{reach}} \eta \rightarrow \mathbf{S}(t_{reach}) - \mathbf{S}(0) \leq -\zeta(t_{reach} - 0) \quad (10)$$

Where  $t_{reach}$  is the time that trajectories reach to the sliding surface so, suppose  $\mathbf{S}(t_{reach} = 0)$  defined as

$$\mathbf{0} - \mathbf{S}(0) \leq -\eta(t_{reach}) \rightarrow t_{reach} \leq \frac{\mathbf{S}(0)}{\zeta} \quad (11)$$

And

$$\text{if } S(0) < 0 \rightarrow 0 - S(0) \leq -\eta(t_{reach}) \rightarrow S(0) \leq -\zeta(t_{reach}) \rightarrow t_{reach} \leq \frac{|S(0)|}{\eta} \quad (12)$$

Equation (12) guarantees time to reach the sliding surface is smaller than  $\frac{|S(0)|}{\zeta}$  since the trajectories are outside of  $S(t)$ .

$$\text{if } S_{t_{reach}} = S(0) \rightarrow \text{error}(x - x_d) = 0 \quad (13)$$

suppose  $S$  is defined as

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right) \tilde{x} = (\dot{x} - \dot{x}_d) + \lambda(x - x_d) \quad (14)$$

The derivation of  $S$ , namely,  $\dot{S}$  can be calculated as the following;

$$\dot{S} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \quad (15)$$

suppose the second order system is defined as;

$$\ddot{x} = f + u \rightarrow \dot{S} = f + U - \ddot{x}_d + \lambda(\dot{x} - \dot{x}_d) \quad (16)$$

Where  $f$  is the dynamic uncertain, and also since  $S = 0$  and  $\dot{S} = 0$ , to have the best approximation,  $\hat{U}$  is defined as

$$\hat{U} = -\hat{f} + \ddot{x}_d - \lambda(\dot{x} - \dot{x}_d) \quad (17)$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$U_{dis} = \hat{U} - K(\tilde{x}, t) \cdot \text{sgn}(s) \quad (18)$$

where the switching function  $\text{sgn}(S)$  is defined as

$$\text{sgn}(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \quad (19)$$

and the  $K(\tilde{x}, t)$  is the positive constant. Suppose by (7) the following equation can be written as,

$$\frac{1}{2} \frac{d}{dt} s^2(x, t) = \dot{S} \cdot S = [f - \hat{f} - K \text{sgn}(s)] \cdot S = (f - \hat{f}) \cdot S - K|S| \quad (20)$$

and if the equation (11) instead of (10) the sliding surface can be calculated as

$$s(x, t) = \left(\frac{d}{dt} + \lambda\right)^2 \left(\int_0^t \tilde{x} dt\right) = (\dot{x} - \dot{x}_d) + 2\lambda(\dot{x} - \dot{x}_d) - \lambda^2(x - x_d) \quad (21)$$

in this method the approximation of  $U$  is computed as

$$\hat{U} = -\hat{f} + \ddot{x}_d - 2\lambda(\dot{x} - \dot{x}_d) + \lambda^2(x - x_d) \quad (22)$$

Based on above discussion, the sliding mode control law for multi-DOF-joints is written as:

$$U = U_{eq} + U_c \quad (23)$$

where, the model-based component  $U_{eq}$  is the nominal dynamics of systems and calculated as follows:

$$U_{eq} = \left[ H^{-1}(q) \left( B(q) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + C(q) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{S} \right] H(q) \quad (24)$$

and  $U_c$  is computed as;

$$U_c = K \cdot \text{sgn}(S) \quad (25)$$

By (24) and (25) the sliding mode control of multi-DOF-joint is calculated as;

$$\begin{bmatrix} \widehat{\tau}_\alpha \\ \widehat{\tau}_\beta \\ \widehat{\tau}_\gamma \end{bmatrix} = \begin{bmatrix} \mathbf{H}^{-1}(\mathbf{q}) \left( \mathbf{B}(\mathbf{q}) \begin{bmatrix} \dot{\alpha}\dot{\beta} \\ \dot{\alpha}\dot{\gamma} \\ \dot{\beta}\dot{\gamma} \end{bmatrix} + \mathbf{C}(\mathbf{q}) \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{\gamma}^2 \end{bmatrix} \right) + \dot{\mathbf{S}} \end{bmatrix} \mathbf{H}(\mathbf{q}) + \mathbf{K} \cdot \text{sgn}(\mathbf{S}) \quad (26)$$

**Proof of Stability:** The lyapunov formulation can be written as follows,

$$V = \frac{1}{2} \mathbf{S}^T \cdot \mathbf{H} \cdot \mathbf{S} \quad (27)$$

the derivation of  $V$  can be determined as,

$$\dot{V} = \frac{1}{2} \mathbf{S}^T \cdot \dot{\mathbf{H}} \cdot \mathbf{S} + \mathbf{S}^T \mathbf{H} \dot{\mathbf{S}} \quad (28)$$

the dynamic equation of multi-DOF actuator can be written based on the sliding surface as

$$\mathbf{H} \dot{\mathbf{S}} = -\mathbf{V} \mathbf{S} + \mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \quad (29)$$

it is assumed that

$$\mathbf{S}^T (\dot{\mathbf{H}} - 2\mathbf{B} + \mathbf{C}) \mathbf{S} = \mathbf{0} \quad (30)$$

by substituting (29) in (30)

$$\dot{V} = \frac{1}{2} \mathbf{S}^T \dot{\mathbf{H}} \mathbf{S} - \mathbf{S}^T \mathbf{B} + \mathbf{C} \mathbf{S} + \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} \mathbf{S}) = \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} \mathbf{S}) \quad (31)$$

suppose the control input is written as follows

$$\widehat{\mathbf{U}} = \mathbf{U}_{\text{Nonlinear}} + \widehat{\mathbf{U}}_c = [\widehat{\mathbf{H}}^{-1}(\mathbf{B} + \mathbf{C}) + \dot{\mathbf{S}}] \widehat{\mathbf{H}} + \mathbf{K} \cdot \text{sgn}(\mathbf{S}) + \mathbf{B} + \mathbf{C} \mathbf{S} \quad (32)$$

by replacing the equation (32) in (25)

$$\dot{V} = \mathbf{S}^T (\mathbf{H} \dot{\mathbf{S}} + \mathbf{B} + \mathbf{C} - \widehat{\mathbf{H}} \dot{\mathbf{S}} - \widehat{\mathbf{B}} + \mathbf{C} \mathbf{S} - \mathbf{K} \text{sgn}(\mathbf{S})) = \mathbf{S}^T (\widetilde{\mathbf{H}} \dot{\mathbf{S}} + \widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S} - \mathbf{K} \text{sgn}(\mathbf{S})) \quad (33)$$

and

$$|\widetilde{\mathbf{H}} \dot{\mathbf{S}} + \widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S}| \leq |\widetilde{\mathbf{H}} \dot{\mathbf{S}}| + |\widetilde{\mathbf{B}} + \mathbf{C} \mathbf{S}| \quad (34)$$

the Lemma equation in multi-DOF actuator can be written as follows

$$\mathbf{K}_u = [|\widetilde{\mathbf{H}} \dot{\mathbf{S}}| + |\mathbf{B} + \mathbf{C} \mathbf{S}| + \eta]_i, i = 1, 2, 3, 4, \dots \quad (35)$$

and finally;

$$\dot{V} \leq - \sum_{i=1}^n \eta_i |S_i| \quad (36)$$

Design and tuning the coefficients in sliding mode controller is the main challenge to design stable and robust controller. To have stable and robust controller, this research focuses on the design model-free adaptive scheme to tune the discontinuous part. Based on above discussion, compute the best value of sliding surface slope and gain updating factor coefficients have played important role to improve system's tracking performance especially when the system parameters are unknown or uncertain. This problem is solved by tuning the surface slope coefficient ( $\lambda, \mathbf{K}$ ) of the sliding mode controller continuously in real-time. In this methodology, the system's performance is improved with respect to the classical sliding mode controller. Based on the following formulations to adjust the sliding surface slope and gain updating coefficients we define  $\hat{f}(x|\lambda, K)$  as the model-free tuning.

$$\hat{f}(x|\lambda, K) = \lambda^T \zeta(x) \quad (37)$$

If minimum error ( $\lambda^*$ ) is defined by;

$$\lambda^* = \text{arg min} [(\text{Sup}) \hat{f}(x|\lambda) - f(x)] \quad (38)$$

Where  $\lambda^T$  is adjusted by an adaption law and this law is designed to minimize the error's parameters of  $\lambda - \lambda^*$ .

$$\lambda^T = [-e^2 - \frac{(x-x_a)}{1+|e|} + x_a] \times Ksgn(s) \quad (39)$$

$$x = \frac{\ddot{e}_k}{\dot{e}_{(t)}} \quad (40)$$

Adaption law in sliding mode controller is used to adjust the sliding surface slope and gain updating coefficients. Model-free tuning part is supervisory controllers based on reduce the error.

Sliding mode controller is divided into two sub-parts: discontinuous part and equivalent part. Discontinuous part is worked based on switching function to improve robustness. Equivalent part is based on system's dynamic formulation which these formulations are nonlinear; MIMO and some of them are unknown. Regarding to above discussion to reduce/eliminate the chattering online tuning applied to the sliding mode method. Multi-DOF actuator's dynamic formulations are highly nonlinear and some of parameters are unknown therefore design a controller based on dynamic formulation is complicated. To solve this challenge fuzzy logic methodology is applied to sliding mode controller. In this method fuzzy logic method is used to estimate some dynamic formulation that they are used in equivalent part.

In this method; dynamic nonlinear equivalent part is replaced by performance/error-based fuzzy logic controller. In fuzzy error-based sliding mode controller; error based Mamdani's fuzzy inference system has considered with two inputs, one output and totally 49 rules instead of the dynamic equivalent part. The fuzzy-based sliding mode controller's output is written;

$$\hat{U} = U_{eq\_fuzzy} + U_{Modified} \quad (41)$$

Based on fuzzy logic methodology

$$f(x) = U_{fuzzy} = \sum_{l=1}^M \theta^T \zeta(x) \quad (42)$$

where  $\theta^T$  is adjustable parameter (gain updating factor) and  $\zeta(x)$  is defined by;

$$\zeta(x) = \frac{\sum_i \mu(x_i) x_i}{\sum_i \mu(x_i)} \quad (43)$$

Where  $\mu(x_i)$  is membership function.  $U_{fuzzy}$  is defined as follows;

$$U_{fuzzy} = \sum_{l=1}^M \theta^T \zeta(x) + U_{uncertain} = [H^{-1}(B + C) + \dot{S}]H \quad (44)$$

Design of fuzzy-based sliding mode controller divided into 4 steps:

- Fuzzification
- fuzzy rule base and rule evaluation
- aggregation of the rule output (fuzzy inference system)
- defuzzification.

**Fuzzification:** the first step in fuzzification is determine inputs and outputs which, it has two inputs ( $e, \dot{e}$ ) and one output ( $\tau_{fuzzy}$ ). The inputs are error ( $e$ ) which measures the difference between desired and actual output, and the change of error ( $\dot{e}$ ) which measures the difference between desired and actual rate of errors, and output is fuzzy equivalent torque. The second step is chosen an appropriate membership function for inputs and output which, to simplicity in implementation because it is a linear function with regard to acceptable performance triangular membership function is selected in this research. The third step is chosen the correct labels for each fuzzy set which, in this research namely as linguistic variable. Based on experience knowledge the linguistic variables for error ( $e$ ), change of error and output are; Negative Big (NB), Negative Medium (NM), Negative Small (NS),

Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and based on experience knowledge it is quantized into thirteen levels represented by: -1, -0.83, -0.66, -0.5, -0.33, -0.16, 0, 0.16, 0.33, 0.5, 0.66, 0.83, 1.

**Fuzzy Rule Base and Rule Evaluation:** the first step in rule base and evaluation is to provide a least structured method to derive the fuzzy rule base which, expert experience and control engineering knowledge is used because this method is the least structure of the other one and the researcher derivation the fuzzy rule base from the knowledge of system operate and/or the classical controller. Design the rule base of fuzzy inference system can play important role to design the best performance of fuzzy sliding mode controller, that to calculate the fuzzy rule base the researcher is used to heuristic method which, it is based on the behavior of the control of robot manipulator suppose that two fuzzy rules in this controller are;

$$\begin{aligned} \mathbf{F.R}^1: & \text{IF } e \text{ is NB and } \dot{e} \text{ is NB, THEN } \tau \text{ is PB.} \\ \mathbf{F.R}^2: & \text{IF } e \text{ is PS and } \dot{e} \text{ is NS THEN } \tau \text{ is Z} \end{aligned} \quad (45)$$

Rule evaluation focuses on operation in the antecedent of the fuzzy rules in fuzzy-based sliding mode controller. This part is used *AND/OR* fuzzy operation in antecedent part which *AND* operation is used.

**Aggregation of the Rule Output (Fuzzy Inference):** Max-Min aggregation is used in this work. The aggregation can be used to calculate the output fuzzy logic and two most common methods are Max-min aggregation method and Sum-min aggregation method.

Max-min aggregation:

$$\mu_U(x_k, y_k, U) = \mu_{\cup_{i=1}^r FR^i}(x_k, y_k, U) = \max \left\{ \min_{i=1}^r \left[ \mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U) \right] \right\} \quad (46)$$

Sum-min aggregation:

$$\mu_U(x_k, y_k, U) = \mu_{\cup_{i=1}^r FR^i}(x_k, y_k, U) = \sum \min_{i=1}^r \left[ \mu_{R_{pq}}(x_k, y_k), \mu_{p_m}(U) \right] \quad (47)$$

**Defuzzification:** The last step to design fuzzy inference in our fuzzy sliding mode controller is defuzzification. This part is used to transform fuzzy set to crisp set, therefore the input for defuzzification is the aggregate output and the output is a crisp number. Centre of gravity method (*COG*) and Centre of area method (*COA*) are two most common defuzzification methods. *COG* method used the following equation to calculate the defuzzification:

$$\mathbf{COG}(x_k, y_k) = \frac{\sum_i U_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)}{\sum_i \sum_{j=1}^r \mu_u(x_k, y_k, U_i)} \quad (48)$$

*COA* method used the following equation to calculate the defuzzification:

$$\mathbf{COA}(x_k, y_k) = \frac{\sum_i U_i \cdot \mu_u(x_k, y_k, U_i)}{\sum_i \mu_u(x_k, y_k, U_i)} \quad (49)$$

Where  $\mathbf{COG}(x_k, y_k)$  and  $\mathbf{COA}(x_k, y_k)$  illustrates the crisp value of defuzzification output,  $U_i \in U$  is discrete element of an output of the fuzzy set,  $\mu_u(x_k, y_k, U_i)$  is the fuzzy set membership function, and  $r$  is the number of fuzzy rules.

Based on online tuning of these two coefficient;

$$\lambda_{new} = \lambda \cdot \alpha \text{ and } K_{new} = K \cdot \alpha \quad (50)$$

The dynamic equation of robot manipulator can be written based on the sliding surface as;

$$H\dot{S} = -VS + H\dot{S} + VS - \tau \quad (51)$$

It is supposed that

$$S^T(\dot{H} - 2V)S = 0 \quad (52)$$

it can be shown that

$$H\dot{S} + (V + \lambda)S = \Delta f - K \quad (53)$$

where  $\Delta f = [H(q)\ddot{q} + V(q, \dot{q})\dot{q}] - \sum_{l=1}^M \theta^T \zeta(x)$

as a result  $\dot{V}$  is became

$$\begin{aligned} \dot{V} &= \frac{1}{2} S^T \dot{H} S - S^T V S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= S^T (-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - K_j)] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - ((\theta_j^*)^T \zeta_j(S_j)))] - S^T \lambda S + \sum (\frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} \zeta_j(S_j) S_j + \dot{\phi}_j]) \end{aligned}$$

where  $\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j)$  is adaption law,  $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} S_j \zeta_j(S_j)$ ,

consequently  $\dot{V}$  can be considered by

$$\dot{V} = \sum_{j=1}^m [S_j \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j))] - S^T \lambda S \quad (54)$$

the minimum error can be defined by

$$e_{mj} = \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j)) \quad (55)$$

$\dot{V}$  is intended as follows

$$\begin{aligned} \dot{V} &= \sum_{j=1}^m [S_j e_{mj}] - S^T \lambda S \\ &\leq \sum_{j=1}^m |S_j| |e_{mj}| - S^T \lambda S \\ &= \sum_{j=1}^m |S_j| |e_{mj}| - \lambda_j S_j^2 \\ &= \sum_{j=1}^m |S_j| (|e_{mj}| - \lambda_j S_j) \end{aligned} \quad (56)$$

For continuous function  $g(x)$ , and suppose  $\varepsilon > 0$  it is defined the fuzzy logic system

$$\text{Sup}_{x \in U} |f(x) - g(x)| < \varepsilon \quad (57)$$

the minimum approximation error ( $e_{mj}$ ) is very small.

$$\text{if } \lambda_j = \alpha \text{ that } \alpha |S_j| > e_{mj} (S_j \neq 0) \text{ then } \dot{V} < 0 \text{ for } (S_j \neq 0) \quad (58)$$

Figure 3 shows the new proposed method with application to multi-DOF-actuators.

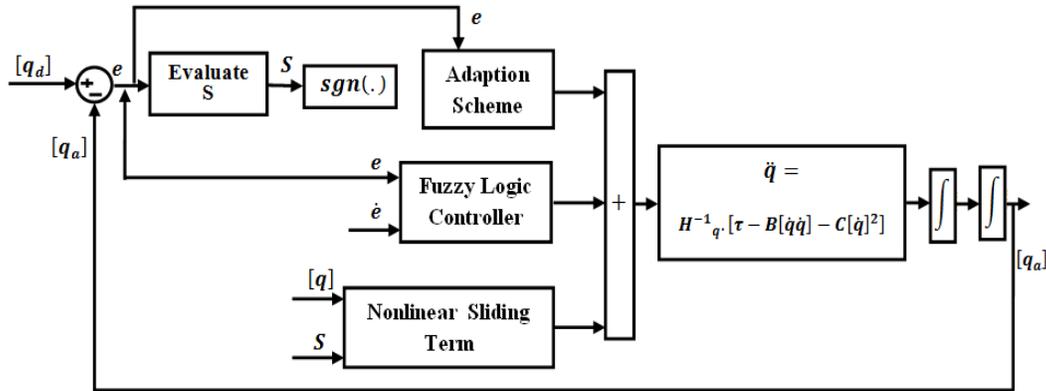


Figure 3. Proposed Intelligent Robust Sliding Mode Control

#### 4. Results and Discussion

To evaluate the designed controller, repetitive simulation tests were performed via numerical simulation.

**Coefficient Optimization:** Figure 4 illustrates the coefficient optimization in sliding mode controller. According to the following Figure researchers should to do much iteration.

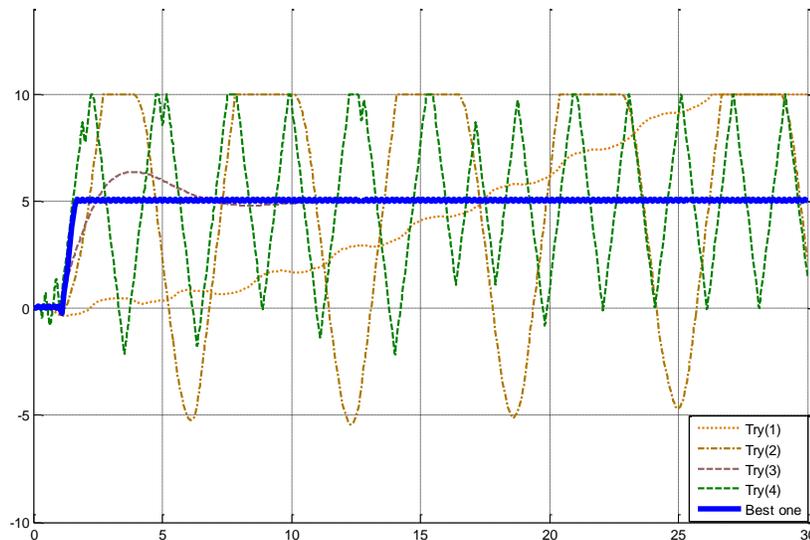
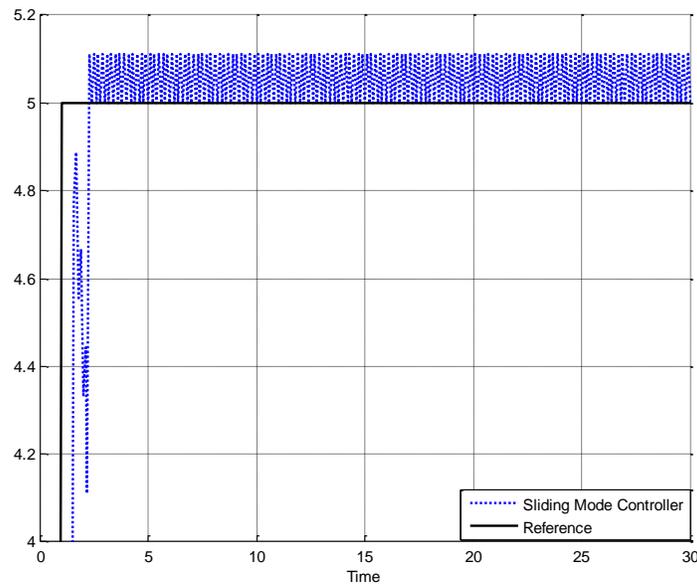


Figure 4. Sliding Mode Controller's Coefficient Tuning

Regarding to above Figure, however sliding mode controller has a good tracking but chattering phenomenon is caused to challenge in this control technique.

**Chattering Phenomenon:** one of the main objectives in this research is reduce/eliminate the chattering. In this design sliding mode controller has chattering in certain/uncertain

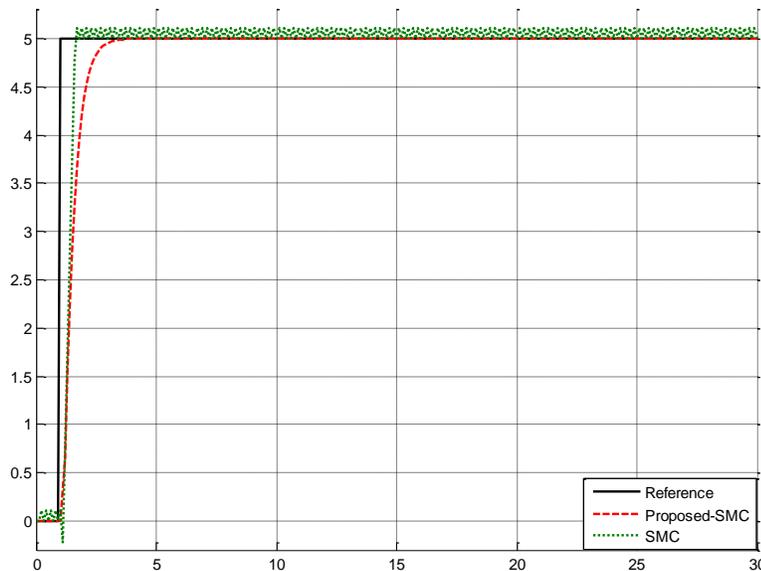
situation. Figure 5 shows chattering phenomenon in sliding mode controller which applied to multi-DOF-joint. This challenge caused to motor vibration and oscillation.



**Figure 5. Chattering Phenomenon in Sliding Mode Control**

Regarding to Figure 5 pure sliding mode controller has chattering in certain/uncertain conditions. This problem caused to oscillation and motor vibration. To reduce/eliminate this challenge in certain/uncertain condition proposed methodology is used.

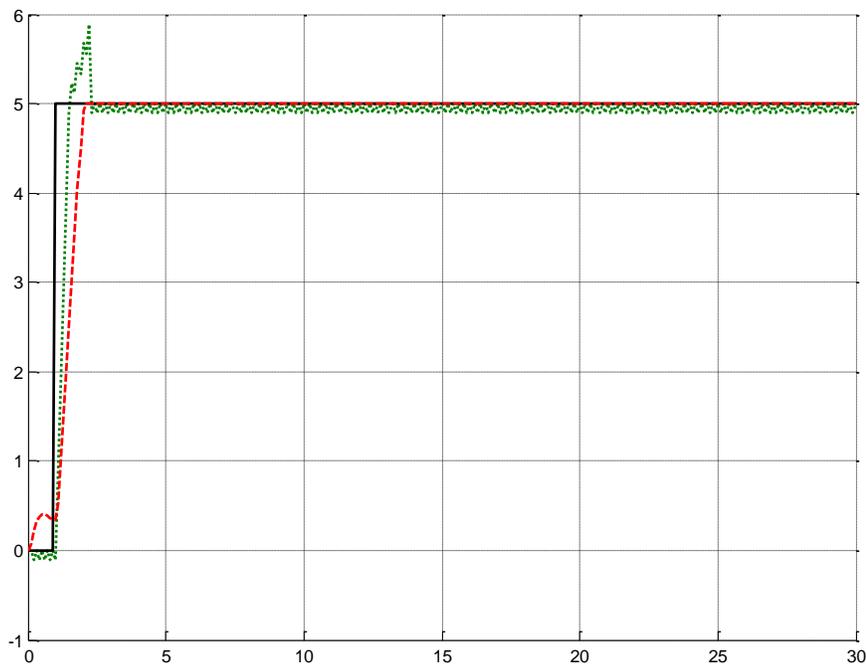
**Certain Trajectory Tracking:** Figure 6 shows trajectory tracking in two types of controllers, sliding mode controller and proposed method. According to this Figure, clearly, sliding mode controller in certain condition has acceptable performance but it has chattering.



**Figure 6. Trajectory Tracking Proposed Method and SMC**

Regarding to above Figure, conventional sliding mode controller has chattering as well undershoot but the rate of trajectory tracking is better than proposed method in certain condition. In the next step robustness should be test in proposed methodology in presence of uncertainty.

**Uncertain Trajectory Tracking:** Figure 7 illustrates the power of disturbance rejection in proposed method compare to sliding mode controller. According to the following Figure, proposed method is more robust than sliding mode controller because it has two important positive points; the first one is on-line tuning to tune the coefficients and the second one is intelligent fuzzy-based method to improve the power of disturbance rejection. Regarding to below Figure, sliding mode controller have a transient error in uncertain condition and the percentage of chattering increase. Regarding to above discussion, proposed methodology is more robust and stable.



**Figure 7. Uncertain Trajectory Tracking Proposed Method and SMC**

## 5. Conclusion

In this project, trajectory tracking control of multi degrees of freedom joints: robust fuzzy logic-based sliding mode approach, research team examined and studied the use of new control methodology to trajectory tracking of three dimension motor (spherical motor) with application to medical industry. Multi-DOF actuators are widely used in a number of industries (such as aerospace, automotive industry and medical industries). researchers designed and modeled a new high precision, stable and robust intelligent control unit to reduce the rate of oscillation from 30% in conventional to 0.01% in proposed method, improved the rate of motion precision up to 40% and lowered the disturbance from 0.12 to 0.0001562 inch, reduced the rate of energy consumption from 8% to 0.88%, and at last reduced the rate of errors by about 38%. This method is much simpler and more compact in design than the sliding mode methodologies.

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Iranian center of Advance Science and Technology (IRAN SSP) is one of the independent research centers specializing in research and training across of Control and Automation, Electrical and Electronic Engineering, and Mechatronics & Robotics in Iran. At IRAN SSP research center, we are united and energized by one mission to discover and develop innovative engineering methodology that solve the most important challenges in field of advance science and technology. The IRAN SSP Center is instead to fill a long standing void in applied engineering by linking the training a development function one side and policy research on the other. This center divided into two main units:

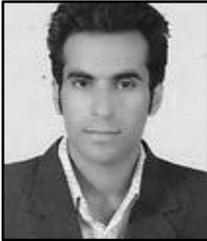
- Education unit
- Research and Development unit

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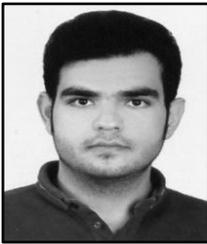
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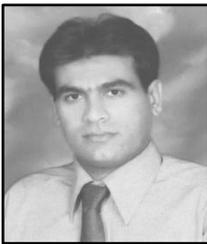
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