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A Heuristic Algorithm for Alongside Replenishment Scheduling Problem of Ship Fleet with Time Constraint

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Abstract

The alongside replenishment scheduling problem with time constraint determining the partition of the ships, the order of replenishment and the allocation of time to the ships at the same time is analyzed. It is equivalent to a multi-stage flow shop scheduling problem with the object of maximizing the effectiveness value of ship fleet. The problem solving process is divided into three steps, and based on the analysis of the three steps, a heuristic algorithm is proposed. The algorithm firstly considers the time allocated to each ship, and then sequences the ships by heuristic rule combining greed with insertion, finally determines the ships partitioning to the port and standard side. Emulating example with different problems' scale and time constraints shows that the proposed heuristic algorithm is superior to some other algorithms.

Key words: *alongside replenishment scheduling, multi-stage flow shop scheduling, time constraint, heuristic algorithm*

1. Introduction

The alongside replenishment scheduling of ship fleet is a complicated problem. Especially at wartime, the ships in the ship fleet are large dispersed far away from the center of the formation where the carrier and supply ship are deployed, and the time available for replenishment is stochastic because the ship fleet may be attacked at any time, so how to arrange the replenishment process to maximize the effectiveness value of the ship fleet is a problem deserve to be discussed. It can be seen as an optimization problem which maximizes the effectiveness value of ship fleet by replenishing the ships within the available time between the beginning of replenishment and the attack from the enemy, and it is equivalent to the multi-stage flows shop scheduling problem with time constraints. The purpose of solving this problem is to give a plan about the reasonable partition of the ships into sets assigned to the two sides of the supply ship (port and standard sides), the optimal order in which the ships are replenished alongside the supply ship within each side, and the optimal allocation of time to each ship alongside the supply ship, to maximize the effectiveness value of ship fleet considering the available time's effect to the replenishment scheduling.

Nowadays, there are several main approaches to solve the multi-stage flow shop scheduling problem with time constraint including operational research method, heuristic algorithm, intelligence algorithm, *etc.*, [1-2]. NAWAZM etc. summarized and compared the current method of Gupta, Johnson, Palmer and NEH [3-4]. YANG D L and CHRN M S made a preliminary study on the two-stage flow shop scheduling problem with limited waiting time constraints, branch and bound method is used to solve the problem and optimal permutation schedule rules under several special conditions were provided [5]. SU L H built

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a integer programming model of hybrid two-stage flow shop scheduling problem with limited time constraints and batch process in the first stage, and a 2-opt heuristic algorithm was proposed [6]. XIAO Y J and LI T K built a constrained satisfaction optimization model for the hybrid flow shop scheduling problem with special time constraints, and a local search method was used to improve the convergence of the algorithm [7]. SUN Q Q and GAO K Z proposed a new heuristic algorithm-SDH heuristic algorithm to solve the no-wait flow shop scheduling problem with the objective of minimizing the total flow time, and the different heuristic algorithms are compared to show its advantage over the other algorithms [8]. TIEKE L and YAN L proposed a back tracking algorithm for hybrid flow shop scheduling problem with time constraint [9]. JEN-SHIANG C, JIN-SHAN Y studied the rule of modeling of the flow shop scheduling problem with time constraints [10]. Besides, several intelligence algorithms, such as genetic algorithm, tabu search and immune algorithm, were used to solve these problems [11-15].

In this paper, we present a heuristic algorithm for the alongside replenishment scheduling problem which is equivalent to the hybrid multi-stage flow shop scheduling problem. The alongside replenishment scheduling problem is described, and the model of the problem is given by three steps; The algorithm is discussed; and an example is given, and the results is worked out using the algorithm and analyzed; finally, we draw some conclusions.

2. Analysis of the Alongside Replenishment Scheduling Problem

2.1. Description of the Problem

The Problem can be defined as follows. The ship fleet is composed of several ships and one or two supply ships. The supply ship can replenish two ships simultaneously at port and standard side respectively, and even 3 ships using VERTREP. The supplies include oil, water, spare parts, ammunition, and daily supplies, *etc.*, the available replenishment time is the time between two raids, and it is a stochastic variable. The total replenishment time is composed of the time ships travel to the replenishment position, the time supplies are moved to the deck of supply ship, the time of delivering the supplies to the ships, and the time moving the supplies to usable positions. When the raid arrives, the replenishment process ends, the transfers in process are terminated. Only when the supplies are moved to the usable positions, a whole replenishment process is finished.

Let T be the interval time between two raids, and it has a certain distribution. The ship fleet has R ships, n_r denotes the supplies' quantity of ship r need to be replenished, $r \in \{1,2,\cdots,R\}$. Let $t_{zz}^{(r)}(l)$ be the time the lth supply moved to the deck of the supply ship, $l \in \{1,2,\cdots,L\}$, L is the total number of supplies. Let $t_{bz}^{(r)}(l)$ be the time the lth supply transferred to ship r. Let $t_{xz}^{(r)}(l)$ be the time the lth supply moved to the specified position. Figure 1 shows the replenishment process.

To simplify the model, several assumptions are made as follows:

Assumption 1: The sequences the supplies moved to the deck of supply ship, transferred to the ships and moved away from the deck of ships are coincident.

Assumption 2: To ensure there are no supplies blocked at the deck of supply ship, assume that:

$$t_{bz}^{(r)}(l-1) < t_{zz}^{(r)}(l) \le t_{bz}^{r}(l+1),\ \ 2 \le l \le L-1$$

Assumption 5: The increasement of the effectiveness value is because of the replenishment, let $v_r(k)$ be the marginal effectiveness value of the kth supply delivered to ship r. It is only depend on its own state and has no matter with other ships.

Assumption 6: T has an exponential distribution with a known mean, τ , $F(t) = P\{t <= T\} = e^{-t/\tau}$.

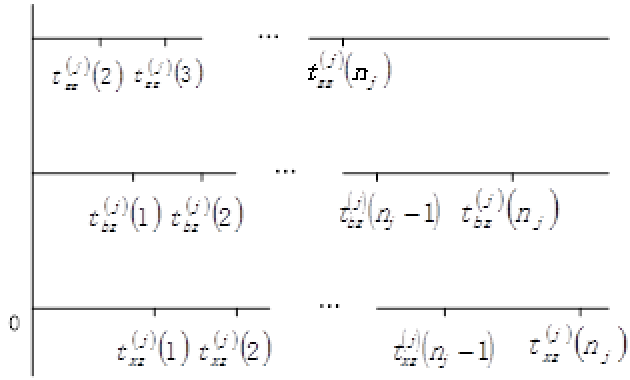

Figure 1. Replenishment Process

Assumption 3: To ensure there are no supplies blocked at the deck of ships, assume that:

$$t_{bz}^{(r)}(l-1) < t_{xz}^{(r)}(l) \le t_{bz}^{r}(l+1),\ \ 2 \le l \le L-1$$

Assumption 4: The effectiveness value of ship fleet is depend on the ships' effectiveness values, and it is additive,

$$V_G = \sum_{r=1}^{R} V_r$$

2.2 Analysis of the Problem

In this problem, the ships' quantity is the stage of the flow shop scheduling problem, the supplies are the jobs, and the time before the attack's coming is the time constraint. So the alongside replenishment scheduling problem can been as a multi-stage flow shop scheduling problem with the object of maximizing the effectiveness value of the ship fleet, it is a NP-hard problem, and can be described as $R/2/T/F/VG_{max}$, meaning R ships need be replenished at port and standard side within T to maximize the effectiveness value of ship fleet. For solving the problem, an effective heuristic method has to be designed. To design a targeted algorithm, the problem is analyzed through three steps. The first step, the ships are partitioned to the two sides, and calculate the effectiveness values of different partition plans; the second step, the replenishment is ordered, and effectiveness values of ship fleet in the single side with different orders are calculated; and finally, if the ships have been partitioned and the replenishment order is provided, the time can be allocated to the ships, and the total effectiveness value of ship fleet can be worked out. But in fact, as solving the problem, we often do it reversely.

Proposition 1: To the "two-machine" flow shop scheduling problem, assume that the partitioning plan P divides ships to two sets c and $\tilde{c}$, the ships in c is replenished in port side, and the ships in $\tilde{c}$ is replenished in standard side, If $V_r^*(c)$ and $V_{R-r}^*(\tilde{c})$ which is the optimal effectiveness value of each single have been obtained, Let $V(P^*)$ be the optimal effectiveness value of the ship fleet, and the P^* be the optimal partition, the optimal expected effectiveness value over all possible partition is

$$V_r(P^*) = \max[V_r^*(c) + V_{R-r}^*(\tilde{c})]$$

There are 2^R partitions, however the partitioning $(c, \tilde{c})$ and $(\tilde{c}, c)$ are symmetric, only 2^{R-1} partitions need be considered.

Then, assume that the ships have already been partitioned, and then the replenishment orders in each side need be considered. Let stage *r* denote that *r* ships need be replenished. For example, the ship fleet has 3 ships, and stage 2 shows that there still are 2 ships need be replenished, let s denote the possible states, *s* can be{1, 2}, {1, 3}, {2,3}. We can prove that in stage *r*, it has $\binom{R}{r}$ states. Let S_r denote the set including all states in stage *r*, in the above example, $S_2 = \{\{1,2\}, \{1,3\}, \{2,3\}\}$.

Proposition 2: For a single side, assume that *r* ships need be replenished, the maximum expected effectiveness value is

$$V_r^*(s) = \max_{j \in s}\left[\max_{k=0,1,\cdots,n_j}\left[V_j(k) + F\left(t_{bz}^r(k)\right)V_{r-1}^*\left(s \setminus \{j\}\right)\right]\right]$$

for all $j \in s$, $r \in \{2,\cdots,R\}$; and $V_1^*(j) = V_j(n_j)$, for $j \in \{1,\cdots,R\}$; $F\left(t_{bz}^r(k)\right)$ denotes the probability the raid arrival time is larger than $t_{bz}^r(k)$; $V_j(k) = \sum_{l=0}^{k} v_r(l)F\left(t_{xz}^r(l)\right)$ denotes that the effectiveness value of ship *j* when *k* supplies are received, where $v_j(0) = 0$, $t_{xz}^r(0) = 0$.

Finally, when the partitioning of ships and the orders in each side are fixed, the time allocated to each ship should be proposed to determine which supplies should be replenished.

Proposition 3: For a single side of supply ship, and the fixed order of ships, the maximum expected effectiveness value with r ship need be replenished is

$$V_r^* = \max_{k=0,1,\cdots,n_r}\left[V_r(k) + F\left(t_{bz}^r(k)\right)V_{r-1}^*\right],\quad r = 2,\cdots,R$$

Obviously, $V_1^* = V_1(n_1)$.

3. Algorithm Steps

3.1 Description of Algorithm

According to the analysis above, a heuristic algorithm combined greed with insertion is given. And the main thinking is described as follows:

(1) Assuming that there is only one side of the supply ship is occupying in the replenishment, calculate the expected effectiveness value. Iterate from stage 1 to stage *R*, calculate the expected effectiveness values of corresponding states in the stage, using the heuristic rule combined greed with insertion, the maximum expected effectiveness values of all states can be worked out, and the first replenished ship in each state at certain stage and the time allocated to ships can be given. For example, $V_r^*(s) = V_{j_r}(k_r) + F\left(t_{bz}^r(k_r)\right)V_{r-1}^*\left(s \setminus \{j_r\}\right)$ is the optimal expected value, so the ship j_r is the first ship to be replenished, and the number of supplies replenished to ship *j* is k_r.

(2) According to the maximum expected effectiveness value of the whole ship fleet, the sequence of the ship fleet and its allocating of time can be worked out using the method of recursion and backward insertion. For example, V_R^* is the maximum

expected effectiveness value in stage R, the optimal expected effectiveness value in stage R-1 V_{R-1}^* can be deduced. Be similar to V_R^* , $V_{R-1}^*(s') = V_{j_{R-1}}(k_{R-1}) + F(t_{bz}^r(k_{R-1}))V_{r-1}^*(s' \setminus \{j'\})$, so in stage R-1, the first ship replenished is ship j', and the supplies' number allocated to ship is k'.Then the ship replenished firstly and the allocated supplies' number to it from stage R to 1 can be worked out. For single side, the order $j_R, j_{R-1}, \cdots, j_1$ and $k_R, k_{R-1}, \cdots, k_1$ are the final order of ships and the number of supplies allocated for each ship.

(3) On the basis of considering the replenishment scheduling problem occupying in the single side and its states in different stages, the ship fleet can be partitioned into two sets, and according to the derived data in (2) and (3), the expected effectiveness values of different partitions can be worked out in which we can find out the maximum effectiveness value, and also we can know the optimal partition, order and the allocation.

3.2 Detailed Algorithm Steps

Step 1: Initialization. Input the data required: the number of the ships, expected attack time τ , the total number of supplies required by ship *j*: *N*[*j*], marginal effectiveness values of supplies on receiver $v_j(k)$, transfer completion time of supplies on ships, and the strike down completion time.

Step 2: Calculate the cumulative expected effectiveness values as Figure 2.

Step 3: calculate the expected effectiveness value in state *s* at stage *r as* Figure 3.

```
for j=1 to R
    for k=1 to N[j]
        V_j(k) = V_j(k - 1) + v_j(k) * exp(- t_bz^j(k) / τ)
    endfor
endfor
```

Figure 2. The Pseudo Code of Step 2

```
/*for each stage of the scheduling problem*/
for r=2 to R
   /*for each state in this stage*/
   for i=1 to NS[r]   /*NS[r] denotes the number of
                        states in stage r*/
      /*for each ships in state s*/
      for j=1 to r
         /*for each supply requested by ship*/
         for k=1 to N[j]
            VOPT[r] [s] = V_j[k] + VOPT[r - 1] [sm]
            * exp(- t_xz^j(k) / τ)
            /* VOPT[R] [S] is the optimal expected
            effectiveness value in state s at stage r, sm is
            the available state at stage r-1*/
         end for
      end for
   end for
end for
```

Figure 3. The Pseudo Code of Step 3

Step 4: calculate the expected effectiveness values of possible partitions as Figure 4.
Step 5: Output the results.

4. Example Analysis

The algorithm presented in part 4 was coded in Visual C++ run on a Pentium (R) Dual-Core / CPU 3.2GHz / RAM 4G computer. To test the effect on the replenishment scheduling of different variables, based on the heuristic algorithm proposed in this paper, the different scales of ship fleet and expected attack arrival times were settled, and the effects on the maximum expected effectiveness value and the replenishment plan of these variables would be discussed as follows. Among the results, the result maximum expected effectiveness value $V^*(P)$ never increase obviously is the steady result.

```
for r=1 to int (R/2)
   rc=R-r
     /*for each state in this stage*/
     for i= 1 to NS[r]
           VOPT=VOPT[r][s]+VOPT[rc][sc]
           /*rc, sc is the complement of r, c
     endfor
endfor
```

Figure 4. The Pseudo Code of Step 4

4.1 Data Design

The main variables need be considered including: the scale of ship fleet, the interval attack arrival time, the categories and the quantities of the supplies, the time conducted replenishment, etc. Three groups are settled shown in table 1. The times conducting the replenishment can be created by some uniform distributions and they are known. The interval attack arrival time has an exponential distribution, and its expected values can be settled to show the rule of the effect on replenishment scheduling of *T*.

Table 1. The Data Designed of the Example

Group	Scale	The interval attack time	Time to conduct replenishment
Group 1	*R*=4	$\tau \in [1, 50]$	$t_{bz}^r(k) \in [0.1, 0.15]$
Group 2	*R*=6	$\tau \in [1, 50]$	$t_{bz}^r(k) \in [0.1, 0.15]$
Group 3	*R*=8	$\tau \in [1, 50]$	$t_{bz}^r(k) \in [0.1, 0.15]$

There are different maximum expected effectiveness values if the expected interval attack arrival time is different, the detailed result are shown in Table 2.

4.2 Results and Analysis

The results are shown in Table 2, the blacked data in the column of Vmax are the optimal result, CPU indicates the computing time using the algorithm in this paper and the results by heuristic algorithm Gupta and NEH are presented.

From Table 2, we can draw the conclusions as follows:

(1) When the scale of the problem is large, the algorithm in this paper is superior to other algorithms, but when the scale is not big large, it is hard to distinguish which algorithm is the best.

(2) As τ increasing, the expected effectiveness value of the problem increases, but the increasing rate will become smaller and smaller. The relationship between τ and V_{max} are shown in Figure 5.

(3) In the simulation process, the efficiencies of the heuristic algorithms are almost same. The larger the scale of the problem is, the greater the cpu time is. But the times in Table 2 smaller than 10 seconds, so the algorithm can satisfy the computing of the problem.

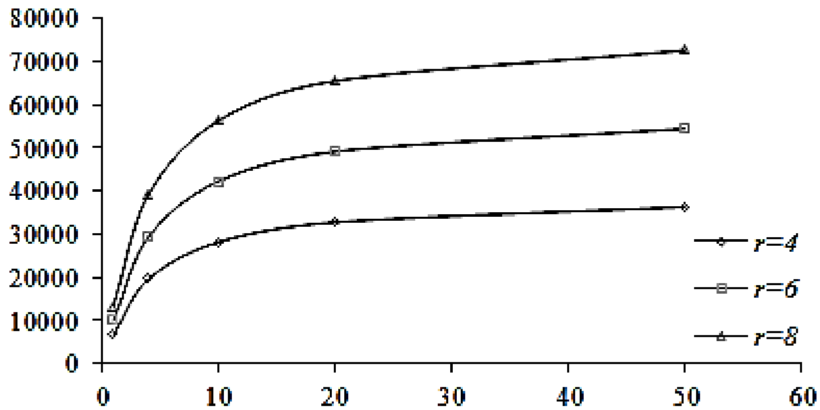

Figure 5. The Relationship between τ and V_{max}

Table 2. The Results with Different Scale of Ship Fleet and τ

R	τ	CPU/s	Vmax		
			GI	Gupta	NEH
4	1	1.25	**6649.37**	6076.05	5918.23
	4	1.31	**19585.5**	18025.43	17994.62
	10	1.48	28129.6	**29211.51**	29106.56
	20	1.26	**32726.6**	31600.87	31756.92
	50	1.45	36281	**36378.79**	35470.52
6	1	2.81	**9963.25**	9213.55	9575.44
	4	3.15	**29368.56**	27308.57	28361.77
	10	3.36	42198.4	39403.92	**42368.48**
	20	2.94	**49084.50**	45346.65	46534.96
	50	3.01	**54428.5**	51164.90	53223.85
8	1	7.19	**13289.57**	8999.23	9804.78
	4	8.23	**39168.35**	27560.16	28128.74
	10	7.47	**56262.2**	37960.65	41171.88
	20	9.02	**65443.26**	45763.16	48162.24
	50	8.07	**72562.49**	49655.16	52896.42

But to the commander of the fleet, the schedule about partition of ships, the order ships are replenished and the time allocated to ships is more important. So the result should be proposed similar to the result of the ship fleet which has 4 ships and $\tau = 4.0$ shown in Figure 6.

The Largest Combat value is: 19585.5.

Port side	Receiver Number of Supplies	Time Alongside
3	11	1.42
2	23	3.20

Standard side	Receiver Number of Supplies	Time Alongside
4	8	1.00
1	41	5.12

Figure 6. Resulting of Alongside Replenishment Schedule

5. Conclusion

This paper describes the alongside replenishment scheduling of navy ship fleet with time constraints which can be seen as a multi-stage flows shop scheduling problem with time constraints. The problem was analyzed, and the mathematical model of the problem was given by three steps, and the object was to optimal partition the ships into two sets, sequence the ships at each side and allocate the time to the ships. A heuristic algorithm combining greed with insertion is proposed. The simulation result shows that the alongside replenishment scheduling problem can be solved effectively, and it is more effective when the scale of the problem becomes larger.

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