International Journal of Control and Automation
Vol. 7, No. 11 (2014), pp. 35-42
http://dx.doi.org/10.14257/ijca.2014.7.11.04

# Multifractal-based Multiplicative Cascades Model for Network Applications

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***Abstract***

*A computer network is a telecommunications network that allows computers to share, exchange, and transmit data. Thus, the network flow rate is very critical. How to measure the flow rate attracted large number of attentions from both academia and practical parties. This paper proposes a multifractal-based multiplicative cascades model for network applications, especially to analyze the test the network flow rate. It is observed that the fitting is well so that the proposed model is able to work out the flow rate in the network applications. In addition, the model using the multifractal-based multiplicative cascades is able to illustrate the trends of the flow rate in the network applications.*

***Keywords:*** *Multifractal, Network, Multiplicative Cascades, Model*

## 1. Introduction

A computer network is a telecommunications network that allows computers to share, exchange, and transmit data [1]. In computer networks, devices pass data to each other along data connections such as wired or wireless channels. The connections between nodes are established using either cable media like TCP/IP or wireless media like WiFi, BlueTooth, *etc.*, [2, 3]. The best-known computer network is the Internet.

Computer networks support applications such as access to the World Wide Web, shared use of application and storage servers, use of email, and instant messaging applications. Computer networks differ in the physical media used to transmit their signals, the communications protocols to organize network traffic, the network's size, topology and organizational intent [4]. Thus, the network flow rate is very critical. How to measure the flow rate attracted large number of attentions from both academia and practical parties. For a high-performance networks, DevoFlow, a modification of the OpenFlow model, was proposed to achieve control and global visibility [5]. Quantitative information about velocity profiles, flow rate, and wall shear stress is essential so that the development the networks' topology and velocity field are modeled [6]. Mishra *et al.*, proposed a SEIQR (Susceptible, Exposed, Infectious, Quarantined, Recovered) model for the transmission of malicious objects in a computer network [7]. This paper analyzes the effect of quarantine on recovered modes as well as the behaviors are examined. Amiri *et al.*, introduced extremely narrow optical tweezers in generation of quantum photon information for computer network communication [8]. A nonlinear mathematical model was proposed to study the effect of malicious object on the immune response of the computer network to measure the flow rate [9]. This paper achieves criteria for local stability, instability, and global stability.

Within the above research, multifractal-based approach is important and studied by large number of scholars [10]. Based on the characteristics of network flow rate modeling, self-similarity theory and analysis models are widely investigated. However, there are still some further studies needed to improve the theory. Firstly, from the perspective of fractal process, long associated merely fractal behavior is one feature. Secondly, for a complex fractal process, the short-term behaviors have some fractal characteristics in long-term

ISSN: 2005-4297 IJCA

behaviors. Thirdly, such differences are so tiny so that it is very important to understand the effects on the analysis and measurements. Multifractal not only can examine the sudden flow of multi-scale, but also describe the burst phase for the time changes. Thus, the multifractal method with suitable models to analyze the network applications is very significant.

This paper proposes a multifractal-based multiplicative cascades model for network applications, especially to analyze the test the network flow rate. Section 2 reports on the definition and characteristics of multifractal. Section 3 illustrates the multifractal-based incremental model in terms of generating process and web division process.

## 2. Problem Description

### 2.1. Principle of Multifractal

Multifractal process includes all the processes of scale attribute, monofractal, and multifractal such as the self-similar or multiplicative cascades. If a stochastic process $\{X(t), t > 0\}$ has the stationary increment, and the increment matrix meets:

$$E[|X(\Delta t)|^q] = C(q)\Delta t^{\tau(q)+1} = C(q)\Delta t^{\tau_0(q)} \quad (1)$$

Then the process is multifractal process. Where, $q \in Q$ , $\tau(q)$ is the scale function or Holder coefficient, $C(q)$ is the matrix index, which is independent from time $t$ .

If $\tau(q)$ is the linear function of $q$ , the process is monofractal or single scale process. If $\tau(q)$ is the convex function of $q$ , when $\tau(q)$ has nonlinear relation of $q$ , the process is multifractal or multi-scale fractal process.

### 2.2. Cascades Process

Cascades process is a division process which plays significant role in multifractal. Assume there is an interval with the unit range as $I \in [0,1]$ , which is with a homogeneous distribution of quality $M$ . The interval $I$ is divided into $d$ sub-intervals $\{I^1(k), k = 0,1,\ldots,d-1\}$ . Then, $M$ is distributed into each sub-interval $\{M^1(k), k = 0,1,\ldots,d-1\}$ . Each sub-interval is divided into $d$ sub-intervals and repeated the operations until the stop conditions are met.

In the cascades process, $M^i(k)$ is the multiplier factor which is labeled as $R$ . If it obeys the probability distribution $P_R(R)$ , then $E(R) = 1/2$ . From the layer 0, $t = 0$ , binary interval length $\Delta t_N = 2^{-N}$ . Multiplier factors obeys independent identical distribution (IID). According to the division processes, when the interval length is $\Delta t_N$ , the quality of the interval $\mu$ satisfies the power exponent trends:

$$E[|\mu(\Delta t_N)|^q] = (E(R^q))^N = \Delta t_N^{-lbE[R^q]} \quad (2)$$

## 3. Net Session in Network Applications

The net session in network is similar with the cascades process. The net session is divided into IP group until all the IP data packages are contained. Figure 1 shows the process of net session.

 

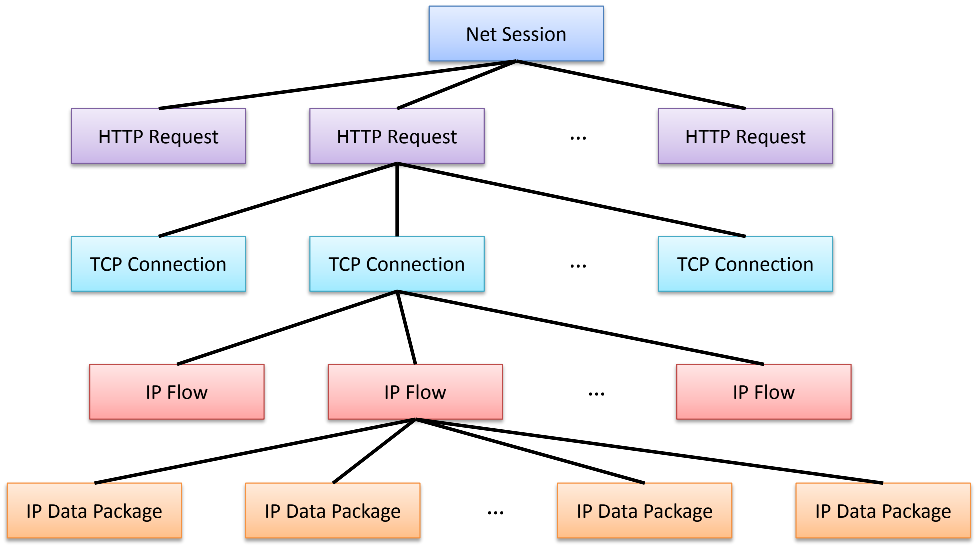


**Figure 1. Net Session Process in Network Applications**

In order to enable the flow rate model to reflect the detailed information about the network, it is necessary to use the combined information of scale function $\tau(q)$ and matrix factor $C(q)$. Thus, it is assumed that the multiplier factor obeys the cascades process of $P_R(R)$ distribution, generating the data sequence $\varphi(\Delta t_N)$ with the length of $2^N$. If there is a stochastic sequence $\{S_k \geq 0, k=1,2,\ldots,2^N\}$, the data sequence multiply with the stochastic sequence. Where, $S_k$ is consisted with the IID $s_k$ .and independent with $\varphi(\Delta t_N)$.

Thus, we can get $E(\varphi(\Delta t_N)) = 2^{-N}$, thus,

$$\begin{aligned} & E[|X(\Delta t_N)|^q] = E[S^q]E[2^{Nq}]E[|\varphi(\Delta t_N)|^q] \\ & = E[S^q]2^{Nq}E[|\varphi(2^{-N}\Delta t_N)|^q] \\ & = E[S^q]2^{N(q+lbE(R^q))_{\Delta t} - lbE[R^q]} \end{aligned} \quad (3)$$

Where, $X(\Delta t)$ presents the flow rate in the network within unit time period. The stochastic variable $S$ and multiplier factor have the suitable possibility distribution which meets:

$$\begin{cases} -lbE[R^q] = \tau_0(q) \\ E[S^q]2^{N(q+lbE(R^q))} = C(q) \end{cases} \quad (4)$$

## 4. Proposed Model

### 4.1. $Beta(\alpha,\alpha)$ and Pareto Distribution

Based on the analysis of flow rate in network, $R$ should use symmetry distribution. It is studied that within [0, 1] for the $Beta(\alpha,\alpha)$ distribution, when $\alpha > 0$, $R$ could be fitted well. Thus, the multiplicative cascades process has the estimation of $\tau_0(q)$. According to (4), there is:

$$E[R^q] = \frac{\Gamma(\alpha+q)\Gamma(2\alpha)}{\Gamma(\alpha)\Gamma(2\alpha+q)} \quad (5)$$

Where, $\Gamma()$ is the Gamma function. In the proposed model, the independent Pareto distribution is selected for stochastic variable $S$. The probability density function is $p(x) = \beta k^{\beta} x^{-\beta-1}$. According to the characteristics of Pareto distribution, the $q$ degree matrix of $S$ is:

$$E(S^q) = \begin{cases} \infty & q \geq \beta \\ \dfrac{\beta k^q}{\beta - q} & q < \beta \end{cases} \tag{6}$$

Thus, we proposed the model which uses three parameters for the flow rate estimation in the network applications. They are $\alpha, \beta, k$, which meets:

$$\begin{cases} \hat{\tau}_0(q) = lb \dfrac{\Gamma(\alpha)\Gamma(2\alpha + q)}{\Gamma(\alpha + q)\Gamma(2\alpha)} \\ \hat{C}(q) = \dfrac{\beta k^q}{\beta - q} 2^{N(q - lb\frac{\Gamma(\alpha)\Gamma(2\alpha+q)}{\Gamma(\alpha+q)\Gamma(2\alpha)})} \end{cases} \tag{7}$$

### 4.2. Model Analysis

This section analyzes the proposed model from examining the statistics characteristics and other multifractal model comparison. The average value and STD of the model are:

$$E(X(\Delta t)) = E(S) = \frac{\beta k}{\beta - 1}, \beta > 1 \tag{8}$$

$$Var[X(\Delta t)] = E[S^2]2^{2N} E^N[R^2] - E^2[X(\Delta t)]$$
$$= \frac{\beta k^2}{\beta - 2}\left(\frac{\alpha + 1}{\alpha + 0.5}\right) - \frac{\beta^2 k^2}{(\beta - 1)^2}, \beta > 2 \tag{9}$$

According to the cascades process, the $Cov[X(\Delta t)_n, X(\Delta t)_{n+k}]$ could be got:

$$Cov[X(\Delta t)_n, X(\Delta t)_{n+k}] = \frac{\beta k^2 \alpha(\alpha + 1)^{N-1}}{\beta - 2(\alpha + 0.5)^N} k^{-lb\frac{\alpha+1}{\alpha+0.5}} - \frac{\beta k^2}{\beta - 2} \tag{10}$$

When the $N, k$ are big enough, the flow rate of the network (here is $Cov[X(\Delta t)_n, X(\Delta t)_{n+k}]$) will be determined by $k^{-lb\frac{\alpha+1}{\alpha+0.5}}$. Thus, the Hurst coefficient meets:

$$2H - 2 = -lb \frac{\alpha + 1}{\alpha + 0.5} \tag{11}$$

Then, $H = 1 - lb \dfrac{\alpha + 1}{2(\alpha + 0.5)}$ (12)

It is easily observed that, for $\alpha > 0$, $H \in [0.5, 1]$. Therefore, when $N$ and $k$ are large enough, the flow rate from the model is prone to monofractal.

## 5. Experiments and Discussion

This paper uses the network flow rate under different time scale for analyzing the package and byte. The following figure 2 illustrates the DEC-PKT captured by LBNL, using the model proposed in this paper under 0.1 s and 1 s, where ts presents the time scale.

 

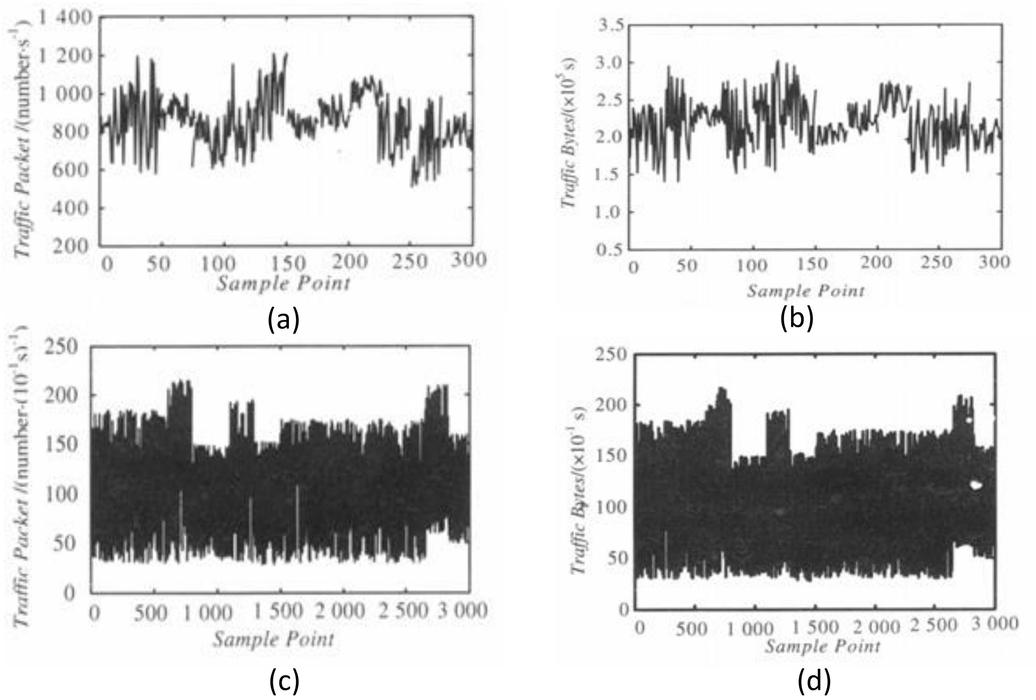


**Figure 2. Flow Rate Curves under Different Time**

From the above Figure 2, it is observed that under different time, the trends are almost the same. Figure 2 (a) and (b) show the flow rate of package and Byte when $ts = 1$ s in the network. However, in some specific time slot, the changes are sharply. Figure (c) and (d) show the flow rate of package and Byte when $ts = 0.1s$ . The deviations are almost the same. It is observed that, the model using the multifractal-based multiplicative cascades is able to illustrate the trends of the flow rate in the network applications.

Figure 3 (a) and (b) shows the trend of the scaling function and moment factor logarithmic plot extraction with three groups of traffic data at different time scales. The impact of scaling function flow characteristic time scale of the chart can be concluded that with the increase of the degree of polymerization of $q$ , the scaling function differences at different time scales under the same set of data is larger. For example, DEC-PKT-1 packet traffic in 0.1s time scale and scaling function packet traffic in the 1s time scale has obvious changes, which illustrate the effect of the scaling function that can be well described by the time scale of the flow characteristics described as an important feature of the short-term behavior of network traffic. Figure 3 (b) shows the moment of impact factor of a packet length of flow characteristics, e.g. DEC-PKT-1 1s in the byte and packet traffic flow scale corresponding factors are significant differences in the moment. And with the degree of polymerization increases, the moment difference factor is increased, indicating the degree of torque factor can indicate sudden changes in traffic on the same time scale. Take DEC-PKT-1 data for example, the experiment extracted through the model parameter scaling function and moment factor required for multi-fractal flow mode, the model fitting comparison, the choice of the time scale of 1s. From the scale function in Figure 3 (a), the multiplier factor from the cascades process of the Byte flow rate obeys the Beta distribution which has the parameter $\alpha_1 = 20$ , and the parameter for package level is $\alpha_2 = 36.5$ . From the matrix factor, Pareto distribution parameters are $k = 2, \beta = 9.25$ . Figure (c) and (d) show the evaluation results from using the real data from the network for examining the data from this model. It is observed that the fitting is well so that the proposed model is able to work out the flow rate in the network applications.

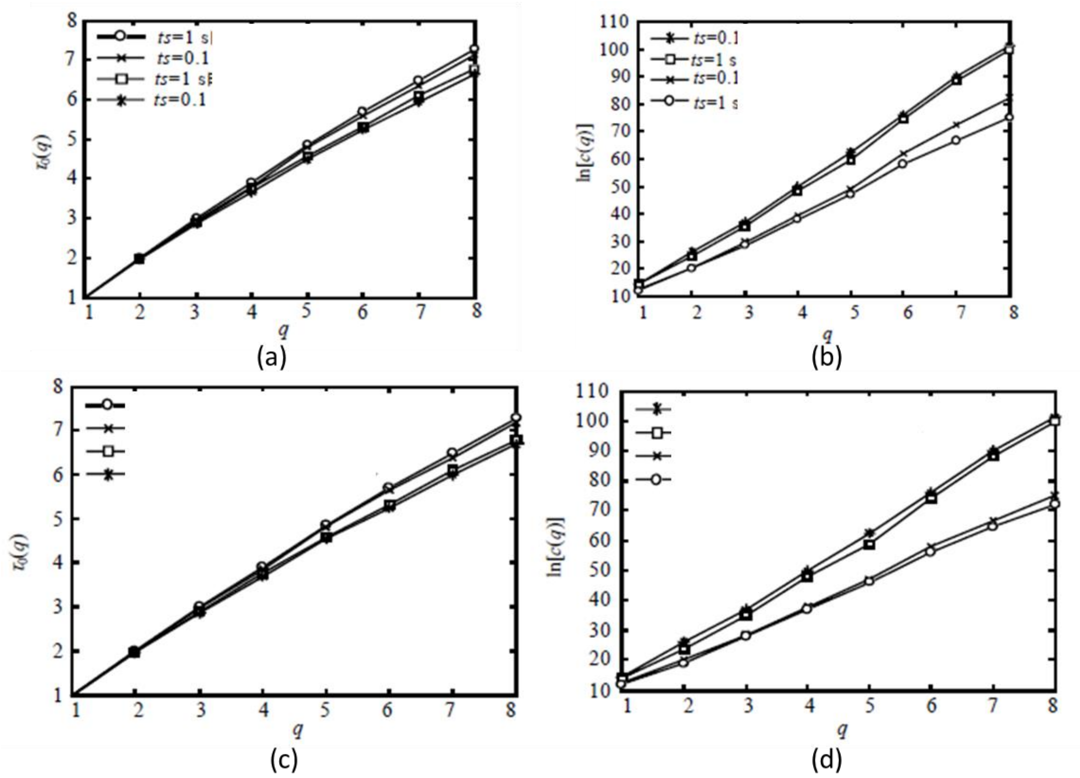


**Figure 3. Experimental Results**

## 6. Conclusion and Remarks

This paper is based on the multifractal multiplicative cascades principle for introducing a model to work out the flow rate under the network applications. This model uses the Beta and Pareto distribution to establish the multifractal process, which is used for fitting the network flow rate with a good performance. Even by the multiplicative cascades with the current process is a network packet switching, packet switching transmission is similar to the process in which the multiplier factor and random variables are with the same functions. According to different network characteristics using a variety of fitting methods, it is to achieve different types of multi-fractal network model established.

The future work will be carried out into two aspects: first, it is important to improve the precise of the model when the network is with huge quantity of data transmission where the network is very complex. Secondly, the variables in this model could be extended so that the feasibility and practicality could be improved. Thus, the application in real-life cases could be achieved easily.

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