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Slider Crank Mechanism Design with Time Ratio and Minimum Transmission Angle

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Abstract

The size parameters of slider crank mechanism are directly treated as design variable. The analytical synthesis method of the mechanism with time ratio and the selecting range of design variable are presented. The mechanism synthesis method which simultaneously satisfies the conditions of time ratio and minimum transmission angle is presented. The synthesis problem of the slider crank mechanism, which makes it have maximum minimum transmission angle when time ratio is given, is completely solved. This method can not only judge the feasibility of mechanism synthesis, but also determine the parameters of mechanism one time. The iteration and checking are avoided, so it makes the synthesis of mechanism fast and accurately.

Keywords: *Slider crank mechanism; Mechanism synthesis; Time ratio; Transmission angle*

1. Introduction

Slider-crank mechanism is widely applied in engineering. Offset slider-crank mechanism has characteristics of quick return, and its design problems generally attributed to design size parameters according to the stroke of the slide, the time ratio and transmission angle etc. Transmission angle is one of important index to measure the performance of force transmission. The designer must consider a question is how to make the mechanism get the best transmission performance in the precondition of kinematic requirements. There are a lot of the literature to research synthesize method of the slider-crank mechanism according to the time ratio k. But for a given time ratio k, synthesize a mechanism to satisfy the allowable transmission angle $[\gamma]$ or to make minimum transmission angle $\gamma_{\min}$ achieve to maximum, is not very good to solve, the mechanism satisfy the requirements can be obtained often need to pass iterative and checking, or the proof it is no solution. The crank length is directly treated as design variable, and the analytic design method of the plane slider-crank mechanism is given with the time ratio k and satisfy allowable transmission angle $[\gamma]$ at the same time, in this paper. This method not only can judge the feasibility of the mechanism synthesis, but also can once determine mechanism parameters, to avoid the iteration and check, make the mechanism synthesis fast and accurate, has certain practical value.

2. Designing Slider-crank Mechanism According to Time Ratio

As shown in Figure 1, in the slider-crank mechanism ABC, offset is e, the length of the crank AB is a, the length of the connecting rod BC is b, the slider stroke is s. Points C_1 and C_2 are the limiting positions of the slider C. The mechanism has minimum transmission angle $\gamma_{\min}$ when the slider on the point C'. θ is the angle

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between one position of crank and another when slider locates on the limiting position C_1 and C_2. The formula of time ratio k is

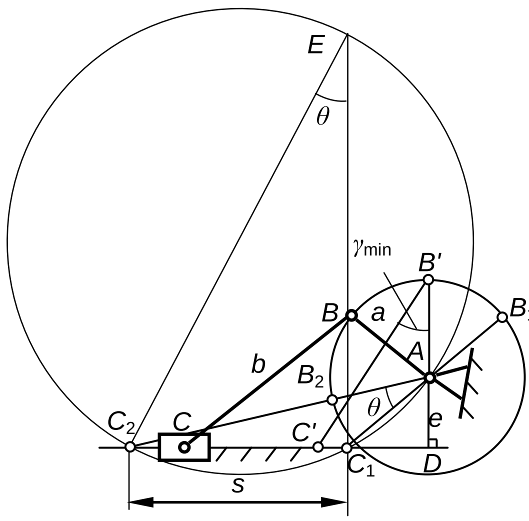

Figure 1. Slider-crank Mechanism

$$k = \frac{\pi + \theta}{\pi - \theta} \tag{1}$$

In order to facilitate calculation, the relative size is used, that is, take the slider stroke $s = 1$, the following a, b and e are the relative length when $s = 1$. At this time, the mechanism has three size parameters, the slider storke $s = 1$ and the known time ratio k each provides a constraint, so the mechanism only one size parameters can be chosen.

2.1. Basic Design Formula of Slider-crank Mechanism

In AC_2C_1, supposing that the angle AC_2C_1 is β, the length of AC_2 is $b+a$ and the length of AC_1 is $b-a$. By the cosine law [1], we have

$$1^2 = (b+a)^2 + (b-a)^2 - 2(b+a)(b-a)\cos\theta$$

That is

$$1 = 2b^2 + 2a^2 - 2(b^2 - a^2)\cos\theta \tag{2}$$

By the sine law[1], we have

$$\frac{b-a}{\sin\beta} = \frac{1}{\sin\theta}$$

That is

$$\boldsymbol{e} = (a+b)\sin\beta = (b^2 - a^2)\sin\theta \tag{3}$$

Simultaneous equations (2) and (3), we can obtain the design formula of Slider-crank mechanism. That is

$$\begin{cases} b = \sqrt{\dfrac{1 - 2a^2(1 + \cos\theta)}{2(1 - \cos\theta)}} \\ e = \dfrac{1 - 4a^2}{2(1 - \cos\theta)}\sin\theta \end{cases} \tag{4}$$

Because of *b* and *e* is the positive of real Numbers, it must to be

$$\frac{1 - 2a^2(1 + \cos\theta)}{2(1 - \cos\theta)} \geq 0 \text{ , } 1 - 4a^2 \geq 0$$

This means $a \leq 0.5$.

By the Grashof conditions: $b - a > e$. Taking formula (4) into the inequality, we can obtain the range of crank relative length *a* is

$$\frac{\sin\theta}{2(\cos\theta + 1)} < a \leq 0.5 \tag{5}$$

2.2. Slider-crank Mechanism Design with *k* and make γ_{min} to Achieve Maximum

The best transmission mechanism is a mechanism which has maximum minimum transmission angle $(\gamma_{min})_{max}$ under the conditions that the time ratio *k* is given.

As shown in Figure 1. The equation of minimum transmission angle of slider-crank mechanism is

$$\cos\gamma_{min} = \frac{a + e}{b} \tag{6}$$

The extreme value of γ_{min} necessarily in the place of $(\cos\gamma_{min})' = 0$, Taking the *a* derivative of formula (6) to get

$$(1 + e')b - b'(a + e) = 0$$

In which, e' and b' are taking the *a* derivative of *e* and *b* from the formula (4). We obtain

$$4a^3(1 + \cos\theta)\sin\theta + a\sin\theta(\cos\theta - 3) - \cos\theta + 1 = 0 \tag{7}$$

With the method of undetermined coefficients for 3 times equation (7) decomposed factoring to get

$$(2a^2(1 + \cos\theta) + a\sin\theta - 1)(2a\sin\theta + \cos\theta - 1) = 0$$

So, the roots for 3 times equation (7) are

$$a_1 = \frac{1 - \cos\theta}{2\sin\theta} = \frac{\sin\theta}{2(\cos\theta + 1)}$$

$$a_2 = \frac{-\sin\theta + \sqrt{\sin^2\theta + 8(1+\cos\theta)}}{4(\cos\theta + 1)}$$

$$a_3 = \frac{-\sin\theta - \sqrt{\sin^2\theta + 8(1+\cos\theta)}}{4(\cos\theta + 1)}$$

Because of $a_3 < 0$, it cannot be used as the relative length of a crank. a_1 don't satisfy formula (5), it also cannot be used as the relative length of a crank. In fact, when $a = a_1$ and $b = a + e$, the minimum transmission angle of the mechanism is 0 °, $\cos\gamma_{min}$ get maximum 1. So the root for three equation (7) is

$$a = \frac{-\sin\theta + \sqrt{\sin^2\theta + 8(1+\cos\theta)}}{4(\cos\theta + 1)} \tag{8}$$

When given the time ratio k, we can synthesis a slider-crank mechanism which has maximum minimum transmission angle $(\gamma_{min})_{max}$ by the equation (8) and (4).

Taking formula (8) into the formula (4), get b and e after another, taking them into the formula (6) to get

$$(\gamma_{min})_{max} = \arccos\left(\frac{d - 2\sin\theta}{1+\cos\theta}\sqrt{\frac{2\sin\theta}{d - \sin\theta}}\right) \tag{9}$$

Where $d = \sqrt{\sin^2\theta + 8(1+\cos\theta)}$.

Formula (9) is mainly used for the feasibility judgment of the mechanism synthesis, such as given the time ratio $k = 1.31$, and ask the allowable transmission angle for $[\gamma] = 40°$. From the formula (1) calculated to get $\theta = 24.16°$, and from formula (9) get $(\gamma_{min})_{max} = 38.77° < [\gamma]$, this shows that the mechanism synthesis has no solution.

2.3. Slider-crank Mechanism Design with *k* and $\gamma_{min} \geq [\gamma]$

From formula (6) and (3), cancellation e gets

$$b = \frac{\cos\gamma_{min} \pm \sqrt{\cos^2\gamma_{min} - 4a\sin\theta(1 - a\sin\theta)}}{2\sin\theta}$$

Taking b into the formula (2) get

$$a^4 - 4c_1a^3 + c_2a^2 + c_1a + +c_0 = 0 \tag{10}$$

Where

$$c_0 = \frac{1}{16}\left(1 - \frac{1+\cos 2\gamma_{min}}{1+\cos\theta}\right)$$

$$c_1 = \frac{\sin\theta}{4(1+\cos\theta)}$$

$$c_2 = \frac{1}{8}\left(\cos 2\gamma_{min} - 5 + \frac{4}{1+\cos\theta}\right).$$

To solve the four times equations (10) by the method of literature [8], and the coefficients are analyzed and simplified. We can get

$$a_{1,2} = \frac{1}{2}(2c_1 + c_6 \mp \sqrt{(2c_1 + c_6)^2 - 4c_7}) \tag{11}$$

Where $c_7 = c_5 + c_1(4c_5 + 1)/2c_6$

$c_6 = \sqrt{2c_5 + 4c_1^2 - c_2}$;

$c_5 = 2\sqrt{\frac{-c_3}{3}}\cos(\frac{1}{3}\arccos\sqrt{\frac{27c_4^2}{-4c_3^3}}) + c_1/6$;

$c_4 = -c_1^2(2c_0 + c_2/6 + 1/8) + c_0c_2/3 - c_2^3/108$;

$c_3 = -c_1^2 - c_0 - c_2^2/12$

When the time ratio k and allowable transmission angle $[\gamma]$ is given, angle θ is calculated by formula (1), taking θ and $\gamma_{\min} = [\gamma]$ into the formula (11) determine a, from the formula (4) determine the other two parameters of mechanism, get a slider-crank mechanism its minimum transmission angle $\gamma_{\min}$ equal to the allowable transmission angle $[\gamma]$.

Usually, if the time ratio k and allowable transmission angle $[\gamma]$ is reasonable, the mechanism has the solution, the formula (11) provides the range of relative length a of crank. If choose a in the section $[a_1, a_2]$, minimum transmission angle $\gamma_{\min}$ is not less than the allowable transmission angle $[\gamma]$.

3. Design Examples

Design a slider-crank mechanism, the slider stroke $s = 200$ mm, the time ratio $k = 1.2$, allowable transmission angle $[\gamma] = 40°$.

From the formula (1) obtained $\theta = 16.36°$.

From the formula (9) calculated $(\gamma_{\min})_{\max} = 47.2° > [\gamma] = 40°$ .The design is feasible.

From the formula (11) :

$a_1 = 0.40259427$ $a_2 = 0.49271336$.

$c_0 = 0.02506531$ $c_1 = 0.03594457$

$c_2 = -0.34812593$ $c_3 = -0.03645663$

$c_4 = -0.00266929$ $c_5 = 0.16236201$

$c_6 = 0.82341849$ $c_7 = 0.19836358$

The range of a is [0.40259427, 0.49271336]. Take $a = 0.45$, from the formula (4)

$b = 1.59617422$ $e = 0.66073951$.

From the formula (6) $\gamma_{\min} = 45.9° > [\gamma] = 40°$

The mechanism size is

$l_{AB} = as = 90$ mm

$l_{BC} = bs = 319.235$ mm

$l_{AD} = es = 132.148$ mm.

The best transmission mechanism design

From the formula (8), a = 0.47047427.
From the formula (4), b = 1.27910526; e = 0.39858514.
From the formula (6), $\gamma_{\min}$ = 47.2°.
The mechaniam size is

$$l_{AB} = as = 94.095 \text{ mm};$$

$$l_{BC} = bs = 255.821 \text{ mm}$$

$$l_{AD} = es = 79.717 \text{ mm}.$$

4. Conclusion

The design method of plane slider-crank mechanism is proposed with the time ratio and allowable transmission angle, in this paper, the theoretical analysis and design examples show that the theory is correct, reliable, high precision. This method directly used to the size parameters of the mechanism as design variables, it is a kind of practical strong design method.

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