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Analysis and Synthesis of the In-line Slider-Elliptic Crank Dwell Mechanism

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Abstract

Elliptic crank is a planet mechanism which is able to generate elliptic curve, with elliptic crank as the driven crank of slider-crank mechanism is equal to the mechanism which has a variable length and a variable speed along the elliptical moving crank, which changed the kinematic characteristics of the slider. This paper studies the motion characteristics of the in-line slider- elliptic crank mechanism. A slider mechanism, which has approximate dwell characteristics at the limit position, is proposed. The analysis and synthesis method of the mechanism was presented. The effect on dwell performance of mechanism parameters was analyzed. The adjusting method of the dwell range was put forward. In the end, the design example was given.

Keywords: *Combination mechanism; Slider-crank; Ellipse; Mechanism synthesis; Dwell*

1. Introduction

Slider-crank mechanism is one of the four bar mechanism with widely applications, in order to improve its kinematic characteristics whose combination mechanism with ellipse gear has been studied in some literatures, therewith some special motion law has been obtained [3-7]. However the defect of ellipse gear is the difficult production process, which limits its application. As we know the planetary mechanism is composed with a fixed central gear z_5, a tie bar 1 and a planetary gear wheel z_2, of which A at planetary gear is the cycloid [1-2]. As shown the dotted line in Figure 1, when the relative transmission ratio of the planetary gear z_2 and fixed central gear z_5 is $i = z_5/z_2 = 2$ relative to tie bar O_1B(tie bar 1) the cycloid degenerates to the elliptic curve, and such planetary mechanism is called elliptic crank or elliptic production mechanism. For the moving point A possesses different radius and different velocities at different locations, which could be used instead of crank drive block to change the motion law, thus obtain a slider mechanism, which has approximate dwell characteristics at the limit position . The methods for analysis and synthesis these combination mechanisms are given in this paper.

2. Mechanism Analysis

Figure 1 shows the schematic diagram of the In-line Slider-Elliptic Crank Dwell Mechanism Dwell Mechanism. In order to facilitate the analysis for the dwell mechanism, let “1” represents the length of original moving part (tie bar) O_1B, b represents the relative length of connecting rod BA, l represents the relative length of connecting rod AC and ϕ_1 represents the location of 1. When $\phi_1 = 0$ the location of

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planetary connecting rod is $\phi_0 = 180°$ at the initial time, namely the location of A_0 is between O_1 and B_0, which is shown as thin line in Fig.1.

2.1. Motion Analysis

The coordinates of moving point A is

$$\begin{cases} x = (1-b)\cos\phi_1 \\ y = (1+b)\sin\phi_1 \end{cases} \tag{1}$$

The angel β between connecting rod AC and x axis is

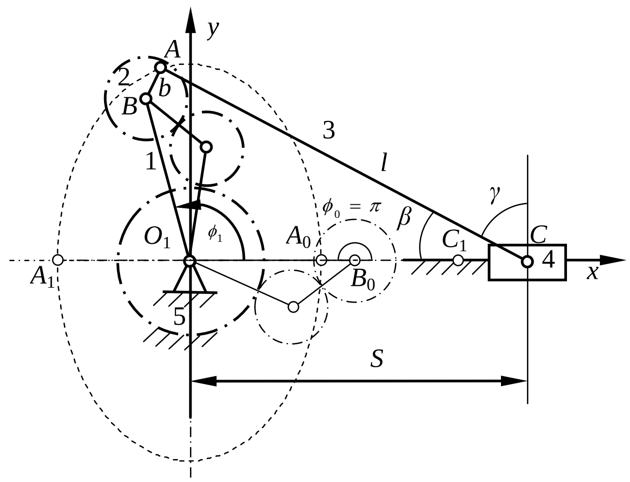

Figure 1 In-Line Slider-Elliptic Crank Dwell Mechanism

$$\beta = \arcsin\frac{y}{l} = \arcsin\frac{(1+b)\sin\phi_1}{l} \tag{2}$$

Slider displacement formula is

$$S = x + l\cos\beta \tag{3}$$

Taking the time derivative of formula (3) to get the velocity formula for the slider, which is,

$$S' = x' - l\sin\beta\beta' = x' - y\beta' \tag{4}$$

In which, x' and β' are respectively get by taking the time derivative of formulas (1) and (2),

$$x' = -(1-b)\sin\phi_1\omega_1$$

$$y' = (1+b)\cos\varphi_1\omega_1$$

$$\beta' = \frac{y'}{l\cos\beta}$$

Taking the time derivative of formula (4) to get the acceleration formula for slider, which is

$$S'' = x'' - y'\beta' - y\beta'' \tag{5}$$

In which, x'' and β'' are respectively get by taking the second time derivative of formulas (1) and (2),

$$x'' = -(1-b)\cos\phi_1\omega_1^2$$
$$y'' = -(1+b)\sin\phi_1\omega_1^2$$
$$\beta'' = \beta'^2 \operatorname{tg}\beta + \frac{y''}{l\cos\beta}$$

Taking the time derivative of formula (5) to get the jerk formula, which is,

$$S''' = x''' - y''\beta' - 2y'\beta'' - y\beta''' \tag{6}$$

In which, x''' and β''' are respectively get by taking the third-order time derivative of formulas (1) and (2),

$$x''' = (1-b)\sin\phi_1\omega_1^3$$
$$y''' = -(1+b)\cos\phi_1\omega_1^3$$
$$\beta''' = 3\beta'\beta''\operatorname{tg}\beta + \beta'^3 + \frac{y'''}{l\cos\beta}$$

There are two parameters (b and l) for designing this in-line slider-elliptic crank dwell characteristics, in which parameter b determines the flattening results of elliptic.

2.2 The Minimum Transmission Angle $\gamma_{\min}$

As shown in Fig.1, transmission angles γ and β are complementary angles. When y at the maximum position, that is $y' = 0$, there is a minimum transmission angle value $\gamma_{\min}$. According to formula (4),

$$(1+b)\cos\phi_1 = 0$$

Then

$$\phi_1 = \phi_{\min} = 90° \tag{7}$$

And

$$\gamma_{\min} = \arccos\frac{1+b}{l} \tag{8}$$

2.3. The Minimum Length of Connecting Rod $l_{\min}$

Let the mechanism permitted transmission angle is $[\gamma]$. The transmission angle γ of mechanism must satisfy condition $\gamma_{\min} \geq [\gamma]$, that is $\cos\gamma_{\min} \leq \cos[\gamma]$. According to formula (8), we get

$$\cos\gamma_{\min} = \frac{1+b}{l} \leq \cos[\gamma]$$

So

$$l \geq \frac{1+b}{\cos[\gamma]} = l_{\min} \tag{9}$$

3. Synthesis of the Dwell Mechanism

3.1 Basic Parameters Relationships

As shown in Fig.1 the longer axis of ellipse and *y* axis coincide. Thus Slider 4 possesses approximate dwell characteristics at point C_1 when moving point *A* moves nearby point A_1 if length of connecting rod l just right equals to the radius of curvature of A_1, which is at the minor axis. Point C_1 is the centre of curvature of A_1 at ellipse necessarily. It can be easily proved that the left limit position of Slider 4 is point C_1, The minor axis semidiameter and major axis semidiameter of ellipse are 1-*b* and 1+*b* respectively. So the radius of curvature of A_1 is

$$l = \frac{(1+b)^2}{1-b} \tag{10}$$

Formula (10) indicates that for the two parameters of mechanism only the length *b* of planet connecting rod is independent, so *l* will be determined after selection of *b*. The value of *l* increases rapidly with the increase of *b*, thus leads to huge mechanism side, so *b*>0.6 is not recommended.

3.2 The Minimum Length of Planet Connecting Rod $b_{\min}$

The length of connecting rod *l* calculated by formula (10) still should satisfy the restriction for transmission angle in formula (9), which is

$$l = \frac{(1+b)^2}{1-b} \geq \frac{1+b}{\cos[\gamma]}$$

It is obtained that

$$b \geq \frac{1-\cos[\gamma]}{1+\cos[\gamma]} = b_{\min} \tag{11}$$

3.3. The Limit Position of Slider

The left and right limit positions of Slider 4 correspond to conditions $\phi_1 = \phi_{11} = 180°$ and $\phi_1 = \phi_{12} = 0°$ respectively. When the slider 4 at the left limit position the displacement C_1 is as following,

$$S_1 = l - (1-b) = \frac{4b}{1-b}$$

When Slider 4 at the right limit position the displacement C_1 is as following,

$$S_2 = l + (1 - b) = \frac{2 + 2b^2}{1 - b}$$

Relative stroke length of Slider 4 is

$$H = S_2 - S_1 = 2(1 - b) \tag{12}$$

3.4 Dwell Range ϕ_d

Fig.2 shows the motion curve of In-line Slider-Elliptic Crank Dwell Mechanism. It is shown that the first derivative, second derivative and third derivative of displacement S are all zero when at A_1 (ϕ_1 = 180°) and the motion curve is flat, so it possesses approximate dwell characteristics at the A_1 position. As shown in Fig.3, the displacement variation of Slider 4 is only 0.005 during ϕ_1 varying from 149.43° to 210.57°, which is considered as dwell range and represented with ϕ_d .

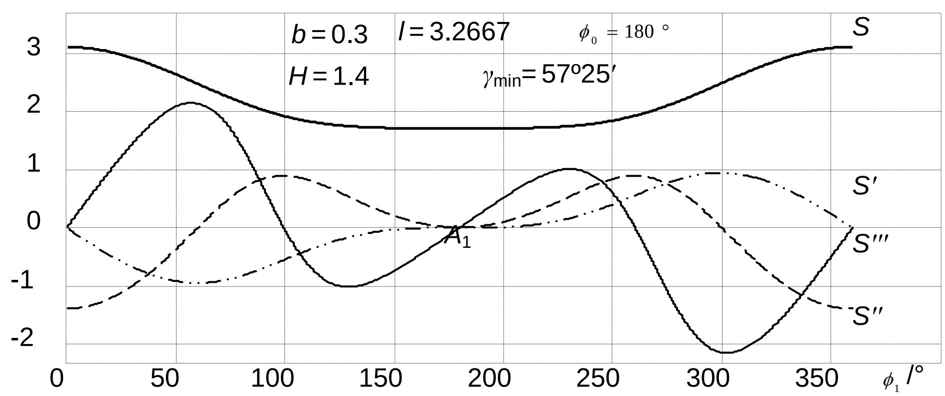

Figure 2 Motion Curve of In-Line Slider-Elliptic Crank Dwell Mechanism

With reference to Fig.3 and curve S' in Figure 2 the displacement curve S is a convex curve and there is no strict flat base theoretically, that is no dwell motion theoretically.

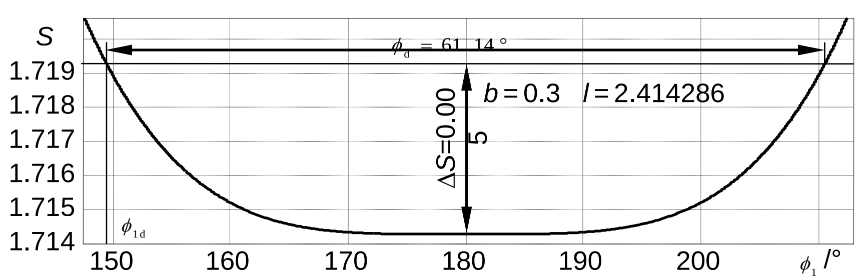

Figure 3 Schematic Diagram of Dwell Range ϕ_d

However, take the components elasticity in to account there is gap between the kinematic pair during the practical application process, so generally mechanisms will show certain dwell motion at the limit position. The more flat the curve (higher order derivative is zero) is, the more close it is to flat bottom, the greater the dwell range will be. Dwell range is affected by gap between the kinematic pair and components elasticity, so theoretically there is no precise computing method. As it is shown in figure3, it is moved upwards ΔS=0.005 from theoretical curve bottom (point A_1) to determine the dwell range. The figure 0.005 is only an assumed value, not a standard for design.

As shown in Fig.3, assume that the displacement compensation is ΔS, when at initial position $\phi_1 = \phi_{1d}$, the slider displacement is $S = l - (1-b) + \Delta S$, which is plug into formula (3)

$$4b(1-b)\cos^2\phi_{1d} - 2(1-b)(\Delta Sb - 4b - \Delta S)\cos\phi_{1d} - 4b^2 - 8\Delta Sb + \Delta S^2 b + 4b - \Delta S^2 = 0$$

Get

$$\phi_{1d} = \arccos\left(\frac{1+b}{4b}\sqrt{\frac{8\Delta Sb}{1-b} + \Delta S^2} - \frac{\Delta S(1-b)}{4b} - 1\right) \tag{13}$$

According to symmetry characteristic of mechanism, the slider dwell range is

$$\phi_d = 2(180° - \phi_{1d}) \tag{14}$$

3.5 Parameters Effects on Dwell Range ϕ_d

Fig.4 and Fig.5 show the effects on dwell range ϕ_d by parameters b and ΔS. It is indicated that ΔS makes greater influence on dwell range, it is monotone increasing function relation between them; length of planet connecting rod b makes less influence

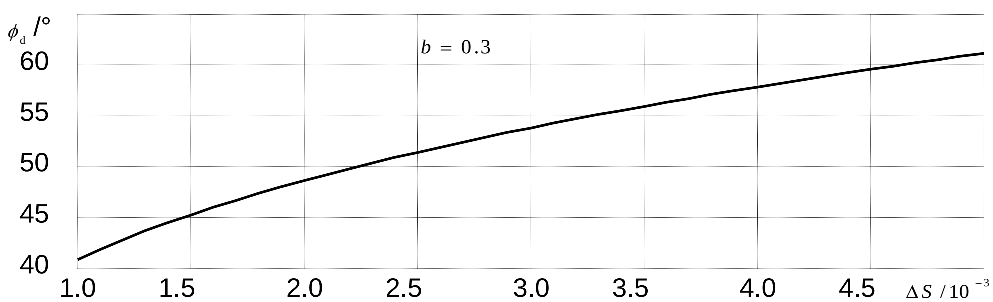

Figure 4 Relation Between Dwell Range ϕ_d And Compensation ΔS

on dwell range ϕ_d, it shows a trend which declines and then climbs up. The minimum position is at $d\phi_d / db = 0$, take b derivation of formula(14),

$$(\Delta S - 9)b^2 + 2(3 - \Delta S)b - 1 + \Delta S = 0$$

Get

$$b = 1 - \frac{2}{3 - \sqrt{\Delta S}} \tag{15}$$

3.6. Adjustment of Dwell Range ϕ_d

The bottom of displacement curve will come to a point when the length of connecting rod l slightly greater than the value get by formula (10), and the dwell range will get smaller with it. So the dwell range can be adjusted by making use of this characteristic.

As shown in Figure 6 an under the double convex will appear at the bottom of

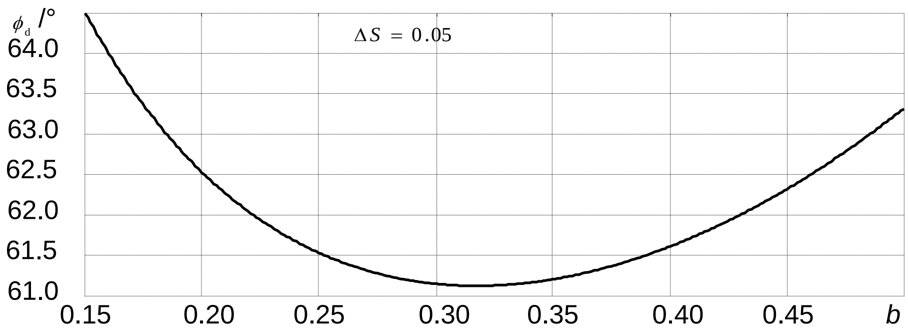

Figure 5 Relation Between Dwell Range ϕ_d and Length Of Planet

displacement curve when the length of connecting rod l is slightly smaller than the value get by formula (10). As b=0.3, the displacement curve will bring out protruding sunken fluctuations phenomenon when the value of l decreases 0.226346. The dwell range could be expanded as long as the fluctuation quantity ΔS is in compensation range. In this paper, Slider 4 keeps dwell moving during φ_1 varying from 137.38° to 222.61°, the dwell range is enlarged 24.09°, and the smaller l is, the large dwell range will be. By making use of this characteristic the dwell range can be adjusted wider.

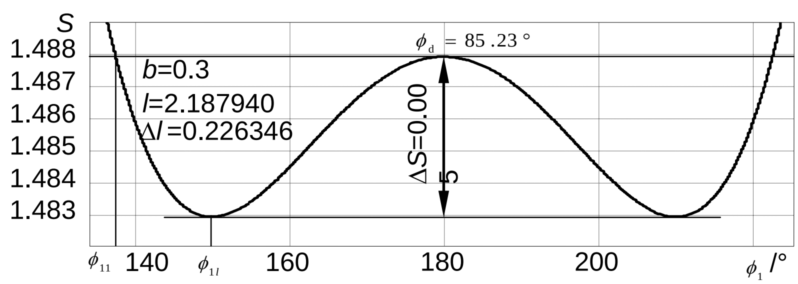

Figure 6 The Amplification of Dwell Range ϕ_d

As shown in Figure 6, with the decrease of l the left limit position of the slider is not at $\phi_1 = 180°$, but at $\phi_1 = \phi_{1l}$. The extremum conditions of displacement S is $S' = 0$, and it is get from formula (4)

$$yy' - x'\sqrt{l^2 - y^2} = 0$$

That is

$$(\cos^2 \phi_{1l} - 1)[4b(1+b)^2 \cos^2 \phi_{1l} - l^2(1-b)^2 + (1-b^2)^2] = 0$$

Get

$$\phi_{1l} = \pm \arccos \frac{b-1}{2(1+b)} \sqrt{\frac{l^2-(1+b)^2}{b}} \tag{16}$$

Plug $\phi_1 = \phi_{1l}$ into formula (3)

$$S_1 = \frac{2\sqrt{b(l^2-(1+b)^2)}}{1+b} \tag{17}$$

Relative stroke length of Slider 4 is

$$H = l + (1-b) - S_1 \tag{18}$$

Fluctuation quantity of slider displacement ΔS is

$$\Delta S = l - (1-b) - S_1 \tag{19}$$

Plug formula (17) into formula (19)

$$(1-b)^2 l^2 + 2(1+b)^2 (b-1-\Delta S) l + (1+b)^2 [(b-1-\Delta S)^2 + 4b] = 0$$

Get

$$l = \frac{(1-b+\Delta S)(1+b)^2 - 2(1+b)\sqrt{b\Delta S(2-2b+\Delta S)}}{(1-b)^2} \tag{20}$$

Selecting b and ΔS, the length of connecting rod can be calculated by formula (20).

As shown in Fig.6, dwell range is symmetrical regard to $\phi_1 = 180°$, the displacements at the initial point $\phi_1 = \phi_{11}$ and the intermediate point $\phi_1 = 180°$ are the same, which is $S = l - (1-b)$, according to formula (3),

$$-2b\cos^2\phi_{11} + (b-1)(l-1+b)\cos\phi_{11} + (b-1)l + 1 + b^2 = 0$$

Get

$$\phi_{11} = \arccos \frac{b(l+b) - l + 1}{2b} \tag{21}$$

So the dwell range ϕ_d is

$$\phi_d = 2(\pi - \phi_{11}) = 2\pi - 2\arccos \frac{b(l+b) - l + 1}{2b} \tag{22}$$

That is

$$l = \frac{1 + b^2 + 2b\cos(\phi_d/2)}{1-b} \tag{23}$$

According to formula (23) which is for mechanism design, when b is determined, l can be calculated by referring to dwell range ϕ_d . Therewith H and ΔS can be calculated by (17), (18) and (19).

3. Design Examples

Design a Slider-Elliptic Crank Dwell Mechanism, and the requirements are as following: stroke length of slider is 200mm; permitted transmission angle is [40°]; dwell range is about 80° .

Firstly, it is calculated that the minimum length of planet connecting rod b_{min}=0.1325 based on formula (11); secondly, let b=0.2, it is calculated that l=1.68302222 and l_{min}=1.5665 based on formula (23) and (9) respectively. It is indicated that l satisfy the transmission angle requirements. Then it is calculated that S_1= 0.87957180 ; H=1.60345042 ; ΔS=0.00345042 based on formula (17), (18) and (19) respectively, it is indicated that value of ΔS is smaller so it can be compensated; Finally, it is calculated that γ_{min}=44.5° based on formula (8). And l_{O1B}=200mm/H=124.7310mm, and the mechanism actual size l_{AB}=bl_{O1B}=24.95mm, l_{AC}=ll_{O1B}=209.93mm, so in dwell range the practical fluctuation quantity of Slider 4 is ΔSl_{O1B}=0.43mm.

4. Conclusion

The dwell motions of In-line Slider-Elliptic Crank Mechanism are approximate, and the dwell range is about 60°. Only one of which is optional parameter, the smaller length of planet connecting rod is suitable by considering the structure size.

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