A Low Complexity Multi-Packet Reception Technique for Wireless Ad Hoc Networks

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Abstract

In this paper, a practical multi-packet reception technique is proposed for wireless ad hoc networks in which concurrent transmissions generally occur. In practical systems, these transmissions are performed without pre-compensation of time and frequency offsets before transmission due to decentralized nature of ad hoc networks. We first derive an optimal asynchronous joint detection (OAJD) technique for concurrent transmissions by considering time and frequency offsets among the multiple received packets. Furthermore, we propose a suboptimal asynchronous joint detection (SAJD) technique with log-likelihood ratio by utilizing the fact that practical pulse-shaping filters including raised cosine filter roll down beyond the number of symbol times. The proposed SAJD yields better performance than that of the successive interference cancellation (SIC) technique which successively decodes a packet with the strongest signal and cancels a decoded packet from multiple received packets.

Keywords: Wireless ad hoc network, multiple packet reception, time and frequency offset, successive interference cancellation (SIC)

1. Introduction

In wireless ad hoc networks, nodes transmit and receive in a distributed manner. There exist various medium access (MAC) protocols such as ALOHA and carrier sense multiple access with collision avoidance (CSMA/CA). When multiple packets are received at a node simultaneously, the receiver declares a collision and the transmitters retransmit the packet according to a pre-defined procedure. Most MAC protocols were proposed to avoid these packet collisions. To increase the performance of the MAC protocols, multiple packet reception (MPR) techniques have been proposed in [1–3], where the multiple packets are distinguished and decoded by using multiple antennas. In previous studies on MPR, timing and frequency offsets among received packets were assumed to be perfectly pre-compensated, and thus, the received multiple packets are aligned in both time and frequency domain. In practice, however, this assumption is not feasible because of the decentralized nature of wireless ad hoc networks.

Several techniques have been proposed for practical implementation of MPR considering this timing and/or frequency offset. In [4], the successive interference cancellation (SIC) technique was used for decoding collided packets. When two or

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more packets collide, a receiver first decodes the packet with the strongest signal, subtracts it from the received signal and decodes the weaker one from the remained signal. This process is repeated to decode all the collided packets. SIC does not necessarily utilize multiple antennas at the receiver. The biggest limiting factor of SIC is its requirement of significant signal strength differences among overlapped signals. Other techniques including [5–7] have adopted similar interference cancellation techniques for decoding the randomly arriving packets with multiple frequency offsets (MFOs).

In this paper, we propose a practical MPR technique for asynchronous ad hoc networks where the timing and frequency offsets are not pre-compensated before transmission. This kind of system can be easily found in distributed ad hoc networks, which have nodes with their own clock mismatch [8]. When all the received packets have different timing and frequency offsets, the detection performance of the conventional single user detection (SUD) technique degrades significantly due to the inter-symbol interference (ISI) resulting from the practical pulse-shaping filters. This problem cannot be solved even if the timing and frequency offsets are perfectly known at the receiver because the compensation of the timing and frequency offset of a specific packet cannot correct the mismatch for all the remaining packets. To solve the asynchronous problem, we first derive the optimal asynchronous joint detection (OAJD) technique. Assuming that the $K$ number of packets with $(2N+1)$ symbols are simultaneously received at a receiver, the OAJD technique decodes all $K(2N+1)$ symbols simultaneously. This induces a computational complexity of $O(M^{(2N+1)K})$ where $M$ is the modulation order. Hence, the computational complexity of the OAJD technique increases exponentially as $K$ and $N$ increase. To reduce the computational complexity of OAJD, we also propose a suboptimal asynchronous joint detection (SAJD) technique based on the log-likelihood ratio (LLR). We observe that when the timing offset occurs, the ISI is not significantly affected by the apart symbols from the current symbol because the practical pulse-shaping filter falls down beyond the number of symbol times. Thus, the proposed SAJD reduce the computation complexity from $O(M^{(2N+1)K})$ to $O(M^{\Delta K})$ where $\Delta$ is the number of contributing symbols. Simulation results show that the proposed SAJD enhances the performance of the MPR under the various channel conditions compared to the conventional SUD and SIC. The performance is especially significant when the strengths of the received signals are comparable.

The organization of this paper is as follows. Section 2 presents the system model. In Section 3, the asynchronous joint detection schemes are proposed. In Section 4, simulations are performed to compare the proposed technique with the conventional ones. In Section 5, conclusions are drawn.

1 Note that the OAJD technique under perfect timing and frequency offset pre-compensation at the receiver has been recently proposed in [9].
2. System Model

We assume that \( K \) users transmit their packets simultaneously to a common receiver. Each packet may be asynchronous in the symbol level as depicted in Figure 2. In the frequency domain, there is also frequency offset due to local oscillator mismatch so that the received signals are rotated independently. The receiver is assumed to perfectly estimate both frequency and timing offsets of the \( K \) transmitters. This assumption can be feasible by using the known preamble and postamble signals in transmitted packets [8]. At an arbitrary time \( m \), the received signal \( y[m] \) is given as

\[
y[m] = \sum_{k=1}^{K} h_k e^{j\omega_m} \sum_{n=0}^{N} s[m - nT - \tau_{k}] x_k[n] + \omega[m]
\]

(1)

where \( K = \{1, \ldots, K\} \), \( h_k \) is a one-tap channel coefficient for the \( k \)-th packet and is assumed to be unchanged within a packet transmission, \( i.e., \) a narrow-band slowly fading channel is assumed, and \( s[m] \) is the combined transmitter and receiver pulse shaping filter. Moreover, \( x_k[n] \) represents the transmitted symbol with index \( n \) of the packet \( k \). Each packet consists of \( (2N + 1) \) symbols, which are indexed as \( x_k[-N], \ldots, x_k[0], \ldots, x_k[N] \). The received signal in (1) contains both frequency and timing offsets for each packet. Let \( f_{\tau} \) and \( \tau_{\tau} \) denote the frequency offset [rad/sec] and the timing offset [sec] of the \( k \)-th packet,
respectively. Note that, in Eq. (1), the frequency offset and the timing offset are modeled as a rotation of the received constellation and a time shift of the received waveform, respectively. We denote by \( \omega[m] \) the additive white Gaussian noise at time \( m \) with noise variance of \( \sigma^2 \) i.e., \( \omega[m] \sim CN(0, \sigma^2) \).

In this paper, unlike the previous studies in [10, 11] that assumed an ideal rectangular pulse-shaping filter, we consider a practical pulse-shaping filter such as the root-raised-cosine (RRC) filter. The RRC filter is applied to both transmitter and receiver sides; therefore, the net response of the raised-cosine (RC), \( s[m] \) is expressed as:

\[
s[m] = \sin \left( \frac{\pi m}{T} \right) \cos \left( \frac{\pi \beta m}{T} \right) \frac{\pi m \left(1 - \frac{4 \beta^2 m^2}{T^2}\right)}{T}
\]

where \( \beta(0 \leq \beta \leq 1) \) is the roll-off factor. Since the impulse response \( s[m] \) is not equal to zero at \( m \neq nT \) for integer \( n \), any sample \( y[m] \) of asynchronous signals at the receiver is affected by all symbols of the \( K \) transmitters as in (1).

### 3. Multi-packet Reception with Joint Detection

In this section, we propose a multi-packet reception technique that jointly decodes packets arriving at the receiver randomly (or fully asynchronously) when a practical RRC filter is used to mitigate inter-symbol interference (ISI). We first derive the OAJD technique to obtain the optimal bit-error rate (BER) performance. The OAJD considers the following maximum likelihood (ML) detection problem:

\[
(\hat{x}_1, \cdots, \hat{x}_K) = \arg\max_{x_1, \cdots, x_K} f(y | x_1, \cdots, x_K)
\]

where \( y = \{y[m_1], \cdots, y[m_L]\} \) is the set of received samples at the receiver, \( L \) is the number of received samples, \( x_k = \{x_{k[-N]}, \cdots, x_k[N]\} \) are the transmitted symbols of the \( k \)-th packet, and \( \hat{x}_k \) is the estimate of \( x_k \). The function \( f(y | x_1, \cdots, x_K) \) is a likelihood function defined as follows:

\[
f(y | x_1, \cdots, x_K) = \frac{1}{\pi^d \det(\Sigma)} \exp \left(-\frac{(y - \tilde{y})^T \Sigma^{-1} (y - \tilde{y})}{\sigma^2} \right)
\]

where \( \Sigma = \sigma^2I \) and \( \tilde{y} \) is represented by

\[
\tilde{y} = \{\tilde{y}[m_1], \cdots, \tilde{y}[m_L]\}^T
\]

where

\[
\tilde{y}[m] = \sum_{k=1}^{K} h_k \sum_{n=-N}^{N} s[m - nT - \tau_k] x_k[n]
\]

Since OAJD exhaustively compares every possible symbol combination of \( K(2N + 1) \) symbols from all the \( K \) packets and maximizes the likelihood value, it estimates
the transmitted symbols $x_i$ with optimal BER performance. However, the complexity of OAJD in (3) is $O(M^{k(x+1)})$. This extremely high complexity is fundamentally attributable to the fact that the impulse response of the RRC filter $s[m]$ has non-zero values at $m \neq nT$ for integer $n$ so that all the symbols of the all asynchronously received packets contribute to the all received samples. Thus, it is not appropriate for real-time receiver design.

In order to reduce the complexity of the OAJD technique to a practical level, we propose a SAJD technique. The proposed SAJD technique utilizes a fact that the impulse response of RRC filter $s[m]$ is close to zero when $|m|$ is greater than several multiples of $T$. In the proposed scheme, we assume that only $\Delta$ neighboring symbols contribute to the current received sample $y[m]$. For example, in Fig. 2, upon sampling $y[0]$, we limit the number of contributing symbols per packet to two (i.e., $2\Delta = 2$). Thus, only six ($\Delta \times K = 2$) dotted symbols ($x[-1], x[0], x_1[-1], x_1[0], x_2[-1], x_2[0]$) are assumed to contribute to $y[0]$ and the other symbols are treated as ISI at the receiver. With this assumption, the received signal $y[m]$ is rewritten as follows:

$$y[m] = \sum_{i \in \mathcal{N}_i} h_i e^{j\omega_i} \sum_{m \in h_i} s[m - nT - \tau_i] x_i[n] + \omega_i[m] + \omega_{i}(m, \Delta)$$

(7)

where $\omega_{i}(m, \Delta)$ is a residual interference and $\mathcal{N}_i = \left\{ n \mid \left| m - nT - \tau_i \right| < \frac{\Delta}{2} \right\}$ is the neighbor symbol set for the $k$-th packet. The residual interference term $\omega_{i}(m, \Delta)$ can be expressed as the sum of an additional additive ISI and all the symbols from a packet, except $\Delta$ nearby symbols:

$$\omega_{i}(m, \Delta) = \sum_{i \in \mathcal{N}_i} \omega_{i,i}(m, \Delta)$$

(8)

where

$$\omega_{i,i}(m, \Delta) = h_i e^{j\omega_i} \sum_{n \in \mathcal{N}_i} s[m - nT - \tau_i] x_i[n]$$

(9)

In (9), $\omega_{i,i}(m, \Delta)$, which is the sum of $(2N + 1 - \Delta)$ independent symbols, can be approximated as Gaussian noise with noise variance $\sigma_{i,i}(m, \Delta)$. The Gaussian approximation is justified by the further simulation results.

The proposed SAJD algorithm is described in Algorithm 1. The receiver first estimates the timing and frequency offsets for the $K$ number of arriving packets and initializes the LLRs for all the symbols from all the packets to zeros. Then, for each received sample, it calculates the neighbor symbol set $\mathcal{N}_i$ for all packets and calculate partial LLRs for the neighboring symbols. Finally, the receiver updates the final LLRs by the partial LLRs and turns to the next received sample. The output of the algorithm is the $K$ sets of final packet LLRs $\{\overline{\Lambda}_1, \ldots, \overline{\Lambda}_K\}$ that are used to decode each bit stream from $K$ transmitters.
Algorithm 1 LLR Computation Algorithm for the Proposed Suboptimal Asynchronous Joint Detection

The LLR of the $n$-th bit in the $k$-th packet, $\Lambda_{n,k}$, is calculated in Algorithm 1. For simplicity, we assume that $K$ users transmit binary phase-shift keying (BPSK) symbols. Let us assume that the receiver samples at time $m$. Then, $\Lambda_{n,k}$ is expressed as

$$\Lambda_{n,k} = \log \left( \sum_{i} \exp \left\{ \frac{- (y[m] - B^{-})^2}{\pi (\sigma^2 + \sigma_i^2)} \right\} \right)$$

$$\sum_{i} \exp \left\{ \frac{- (y[m] - B^{-})^2}{\pi (\sigma^2 + \sigma_i^2)} \right\} \approx \log \left( \exp \left\{ \frac{\min_{i} \left[ (y[m] - B^{-})^2 \right]}{\pi (\sigma^2 + \sigma_i^2)} \right\} \right)$$

where the condition $\epsilon_1$ is represented as

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2 Extension to other constellations such as QPSK and 16QAN is straight-forward
\[ c1: x_i[n] \in \{1, -1\} \text{ for } n \in \mathcal{N}_i, \quad k \in K \setminus k' \text{ and } n \in \mathcal{N}_i \setminus n' \]  

(12)

and \( B^+ \) and \( B^- \) are defined as the term \( \sum_{e \in \mathcal{E}} h_e e^{j\phi_e} \sum_{m \in \mathcal{M}} s[m - nT - \tau_e] x_i[n] \) in (7) conditioned on \( x_i[n'] = 1 \) and \( x_i[n'] = -1 \), respectively. The numerator (denominator) of the LLR in (10) is derived as the sum of the probability of all the possible symbol combination given that \( x_i[n'] = 1 \) (\( x_i[n'] = -1 \)). The approximation from (10) to (11) results from a well-known exponential approximation [12]

\[ \sum A_i \exp(B_i) \approx \max_i A_i \exp(B_i) \]  

(13)

In order to accurately calculate the LLR in (11), the receiver should estimate the additional noise variance \( \sigma_i^2 = \sum_{k \in K} \sigma_{i,k}^2(m, \Delta) \) precisely. With the Gaussian assumption for the ISI, the noise variance \( \sigma_{i,k}^2(m, \Delta) \) can be modeled as a sum of noise variances of ISI from the \( (2N + 1 - \Delta) \) symbols. Without loss of generality, we assume that average transmitted symbol energy is equal to 1. Then, the noise variance \( \sigma_{i,k}^2(m, \Delta) \) can be calculated as follows:

\[ \sigma_{i,k}^2(m, \Delta) = E\left[|o_{i,k}(m, \Delta)|^2\right] \]

\[ = |h_i|^2 \sum_{s \in \mathcal{S}_{\mathcal{M}}^k} s^* \left[m - nT - \tau_s\right] \]

\[ = |h_i|^2 \left( \sum_{s \in K} s^* \left[m - nT - \tau_s\right] - \sum_{s \in K} s^* \left[m - nT - \tau_s\right] \right) \]

\[ = |h_i|^2 \left( \frac{\beta}{4} \cos\left(\frac{2\pi \tau_s}{T}\right) + 1 - \frac{\beta}{4} \right) \\sum_{s \in K} s^* \left[m - nT - \tau_s\right] \]

(14)

(15)

The infinite power sum of RC filter \( \sum_{s \in K} s^* \left[m - nT - \tau\right] = \frac{\beta}{4} \cos\left(\frac{2\pi \tau}{T}\right) + 1 - \frac{\beta}{4} \) (from [13]) is used in the approximation from (15) to (16).

4. Numerical Examples

We perform simulations to evaluate the proposed SAJD technique. In our simulations, two users transmit their own BPSK-modulated packets of length of \( N = 600 \) simultaneously to a common receiver. Lack of timing and frequency synchronization among the three nodes is modeled as \( \{\tau_i, \tau_s\} \) and \( \{f_i, f_s\} \), and they are assumed to be random and uniformly distributed with intervals \( \tau_i \in [0, T) \) and \( f_i \in [-10^{-4}, 10^{-4}] \), respectively. The symbol duration, \( T \), is set to \( 10^{-4} \) sec and the roll-off factor, \( \beta \), is set to a typical value of 0.35. We set \( L = 2N + 1 \). We assume uncoded modulation. The wireless channel coefficients \( \{h_i, h_s\} \) are assumed to be independent Rayleigh distributed and unchanged during a packet reception [15].

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3 As \( N \) and \( K \) increase, the OAJD cannot be simulated because the computational complexity is huge. Hence, in the simulation, the proposed SAJD is only shown. With \( N = 5 \) and \( K = 2 \), the simulation result indicates that the performance of the OAJD and SAJD is nearly the same, which is not shown in this paper.
We compare the proposed SAJD technique with the SUD and the SIC. In the SUD, a receiver simply decodes one packet while considering other packets as additive Gaussian noise. In the SIC, the receiver decodes the strongest packet first, subtracts it from the combined signal, and decodes the second strongest packet, and so on [14].

Figure 3(a) shows the BER performance of the proposed SAJD technique compared with the SUD and the SIC techniques for varying signal-to-noise ratio (SNR) in a symmetric channel environment. In a symmetric channel environment, the received channel strengths of the two packets are the same on average, i.e., \( E[\|h_1\|^2] = E[\|h_2\|^2] \). The BER performance is calculated by averaging each BER of the two packets. In Figure 3(a), it is clearly seen that the proposed SAJD scheme outperforms SIC and SUD in all SNR ranges. Note that the BER of SAJD for \( \Delta = 2 \) is roughly a tenth of that of the SIC at maximum. Moreover, the BER of SAJD (\( \Delta = 4 \)) is less by a half of that of the SAJD for \( \Delta = 2 \) at the high SNR.

Interestingly, it is observed that the BERs of the three techniques are bounded in high SNR region. This is explained by the influence of the residual interference at the receiver. In the SUD, when decoding one packet, the sum of the other packets is the residual interference and it results in high BER even in the high SNR region. For the SIC, the sum of the weaker packets is the residual interference, which is smaller than or equal to that of the SUD. On the
other hand, the SAJD receiver only has $\omega$, as the residual interference and it is smaller than that of the SIC in symmetric channel environments; thus, the BER performance of the SAJD scheme is bounded at much lower BER than the other technique.

Figure 3(b) shows the BER performance in an asymmetric channel environment when $E[|h|^2] = 10^6 E[|h|^2]$. In this case, the SIC notably achieves low BER compared with the previous symmetric channel environment. This is because the BER of decoding the stronger packet is generally good enough to perfectly reconstruct the transmitted stronger packet. The SIC achieves even better performance than the SAJD for $\Delta = 2$ when the SNR is greater than 20dB. However, the SAJD for $\Delta = 4$ still achieves the best BER for all observed SNR ranges.

5. Conclusion

We propose a novel and practical joint detection technique for distributed ad hoc networks in which the timing and frequency offset cannot be pre-compensated. The proposed SAJD technique works well for practical pulse-shaping filters such as the RRC filter with a reasonable complexity of $O(M)$ at the receiver. We also derived an analytical model and an algorithm to compute accurate bit LLR values for multiple packets. The numerical results show that the proposed SAJD technique outperforms the conventional SUD and SIC techniques in terms of the BER at receiver. The proposed SAJD technique with a single antenna can be extended to multiple antennas at the receiver, which remains for our further study.

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References


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