International Journal of Control and Automation
Vol.7, No.10 (2014), pp.99-108
http://dx.doi.org/10.14257/ijca.2014.7.10.10

Upper Bound for Average Outage Probability of BT-ADF relay networks Over Quasi-Static Rayleigh Fading channels

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Abstract

In this paper, we propose the analytical approach for burst transmission (BT) adaptive decode-and-forward (ADF) relaying schemes consisting of burst transmission over quasi-static Rayleigh fading channels. At first, we focus on the error-event at relay nodes for symbol burst transmission, whereas previous researches considered one data symbol transmission so that they showed the best performance bounds. By considering symbol burst for ADF relay systems, we derive an exact outage probability expression which can be the performance of practical systems. Furthermore, the average outage probability is derived in approximated closed-form upper bound for an arbitrary link SNR. The derived analytical approach is verified based on the number of relays and burst symbols. Its accuracy is confirmed by comparison with simulation results.

Keywords: *burst transimission, ADF relay, Quasi-static Rayleigh fading channels, outage probability*

1. Introduction

Cooperative diversity networks have recently been widely discussed for wireless systems [1, 2]. Effects of cooperative diversity in radio communication have been studied [3, 4]. In decode-and-forward (DF) relay networks, the relay detects the received signal and then retransmits a regenerated signal [1, 2]. At the destination, the receiver can employ a variety of diversity combining techniques to benefit from the multiple signal replicas available from the relays and the source. Use of DF schemes presumes the incorporation of a cyclic redundancy check (CRC) codes from a higher layer (*e.g.*, data link layer) in order to detect errors. The advantages of general cooperative diversity schemes come at the expense of the spectral efficiency since the source and all the relays must transmit on orthogonal channels (*i.e.*, different time slots or frequency bands) in order to avoid interfering with each other as well [2]. Recent researches are carried out for relay-selection schemes in which only two channels are need (one for the direct link and the other one for best relay link) [5-11]. However, they need to additional process or feedback information for channel states.

The authors in [11] have shown the general approach applicable for both DF and ADF relaying and derived an exact bit error rate (BER) as well-known tractable forms, which can explain how an erroneous detection at each relay affects both the received signal-to-noise ratio (SNR) and the average BER. Note that even if it can give exact results [11], previous researches have been carried out under the assumption that relay nodes can detect symbol-by-symbol error [9-11]. It means that at each relay, transmission mode or sleep mode (*i.e.*, no-transmission) can be determined per symbol-by-symbol. It is not practical and the performance based on this implies only an achievable lower bound. So far as we know, the practical approach covering the burst transmission (BT) for ADF relay systems has not been addressed in the literature yet. Furthermore, no one has expressed the approximated outage probability expression as well-known tractable forms, which can explain how an erroneous detection at each relay affects both the received SNR and the average outage probability.

At first, we consider not symbol-by-symbol but burst-by-burst error detection for ADF relay systems. Based on this, we focus on the error-event at relay nodes for symbol burst transmission and then, the probabilities of all possible error-events are derived as well-known forms. For BT-ADF relay systems, we derive an outage probability expression over independent and non-identical distributed (INID) Rayleigh fading channels, so that it can show a practical performance. Furthermore, an exact outage probability is approximated to the simplified expression for an arbitrary link SNR. Numerical results obtained from analytical solutions and Monte-Carlo simulations are compared. Then, it is confirmed for BT-ADF relay systems that the numerical result based on [11] and the approximated outage probability in this paper can be the achievable lower bound and the upper bound, respectively.

The remainder of this paper is organized as follows: Section II describes the system model of BT-ADF relay systems. In section III, the derived performance expressions are provided. The numerical and simulation results are presented in Section IV and also concluding remarks are given in Section IV.

2. BT-ADF Relay Networks

Fig.1. shows the block diagram of BT-ADF relay systems with a Source(S), a destination (D), and a relay(R). The number of relays is L . In this paper, it is assumed that S and R relays transmit over orthogonal frequency bands. At first, let us describe the quasi-static Rayleigh fading channel model to derive the analytical approach for BT-ADF relay scheme.

2.1. BT-ADF Relay System Model

Let h_0 , h_{L+r} , and h_r with $r \in \{1,2,\cdots,L\}$ be the channel gains of S-D, S-R, and R-D link channels, respectively. In this paper, wireless channels between any pair of nodes are assumed quasi-static independent and non-identical distributed (INID) Rayleigh fading [9-11]. In means that channel coefficient can be assumed to be a constant during the several symbol times (i.e., during a frame transmission). From here, let us define N_D as the number of modulated data symbols within a burst.

The received signals for S-D, S-R, and R-D links are presented, respectively, as

$$\begin{aligned} y_0[t] &= h_0\sqrt{E_0}s[t] + n_0[t] \\ y_{L+r}[t] &= h_{L+r}\sqrt{E_{L+r}}s[t] + n_{L+r}[t] \\ y_r[t] &= h_r\sqrt{E_r}\hat{s}_r[t] + n_r[t] \end{aligned} \tag{1}$$

where $E_s = E_0 = E_{L+r}$ for $r \in \{1,2,\cdots,L\}$ is the average transmitted symbol energy of the source and $t \in \{1,2,\cdots,N_D\}$ is the time index within a given burst, and each channel is corrupted by complex additive white Gaussian noise (AWGN) term of $n_r[t]$. Without loss of generality, we can assume that $\{n_r[t]\}$ are mutually independent for different r and t with $E[|n_r[t]|] = 0$ and $E[|n_r[t]|^2] = \sigma^2$. The operator $E[\cdot]$ represents statistical expectation. For simplicity, we consider in this paper the first frame transmission. Then, $s[t]\big|_{1 \le t \le N_D}$ is M-ary phase shift keying (MPSK) data symbol. Also, $\{s[t]\}$ are mutually independent for different t with $E[s[t]] = 0$ and $E\left[|s[t]|^2\right] = 1$. In BT-ADF relay systems, the r th relay is only to transmit the regenerated symbol of $\hat{s}_r[t]$ when all symbols within a burst are correctly decoded. It means that the channel condition of the r th S-R link is sufficiently good to allow for successful decoding [5]. In addition, it is assumed that the relay node is capable of perfect forwarding the original message if we can detect an error for each symbol.

2.2. Conditional BER for S-R Link

For the r th S-R link, the decision variable for data symbols can be written as

$$z_r[t]\big|_{1 \le t \le N_D} = h_{L+r}^{*} y_{L+r}[t] / \sigma_{L+r}^{2} \tag{2}$$

and the received instantaneous SNR is expressed as

$$\gamma_{L+r} = \frac{|h_{L+r}|^2}{\sigma^2}. \tag{3}$$

Then, the probability density function (PDF) of γ_{L+r} can be presented for the Rayleigh fading channel as

$$f_{\gamma_{L+r}}(x) = \frac{1}{\bar{\gamma}_{L+r}} \exp\left(-\frac{x}{\bar{\gamma}_{L+r}}\right) \tag{4}$$

where $\bar{\gamma}_{L+r}$ is the average SNR defined as

$$\bar{\gamma}_{L+r} = E[\gamma_{L+r}] = \frac{E\left[h_{L+r}|^2\right]}{\sigma^2}. \tag{5}$$

Furthermore, above derivation can be also applied to S-D and each R-D links so that by replacing $L+r$ with r, we can obtain γ_r, $\bar{\gamma}_r$, and $f_{\gamma_r}(x)$ with $r \in \{0,1,\cdots,L\}$. Consequently, the conditional bit error rate of the r th S-R link can be expressed for BPSK as

$$P_b(\gamma_{L+r}) = Q\left(\sqrt{2\gamma_{L+r}}\right) \tag{6}$$

with $Q\left(\sqrt{2x}\right) = 1/\sqrt{2\pi}\int_{\sqrt{2x}}^{\infty} \exp(-t^2/2)dt$ [12][13].

3. Average Outage Probability for BT-ADF Relay Systems

In BT-ADF relay systems, the r th relay participates in transmitting the regenerated symbol of $\hat{s}_r[t]$ only when all symbols within a burst are correctly decoded. Then,

$\hat{s}_r[t]$ can be two values, which are $\hat{s}_r[t] = 0$ with the probability of $1 - \left[1 - P_b(\gamma_{L+r})\right]^{N_D}$ and $\hat{s}_r[t] = s[t]$ with the probability of $\left[1 - P_b(\gamma_{L+r})\right]^{N_D}$.

3.1. Error-Event of Relay Nodes and its Probability

In order to generally derive the analytical method based on error-events at relays, let us define the p th error-event vector E^p as [11]

$$E^p = \left[e_1^p \cdots e_r^p \cdots e_L^p\right] \tag{7}$$

with $p \in \left\{1, 2, \cdots, 2^L\right\}$ and the total number of error-events is 2^L. Generally, we can define that E^1 is all-zero vector, E^{2^R} is all-one vector, and so on. Note that for the p th error-event, $e_r^p = 0$ means the correct burst detection at the r th relay and $\hat{s}_r[t]\big|_{1 \le t \le N_D} = s[t]$ with the probability of

$$P_C^{N_D}(\gamma_{L+r}) = \left[1 - P_b(\gamma_{L+r})\right]^{N_D} = \sum_{k=0}^{N_D} \binom{N_D}{k} (-1)^k P_b^k(\gamma_{L+r}). \tag{8}$$

In addition, the average burst correction probability can be written as

$$P_C^{N_D}(\overline{\gamma}_{L+r}) = E\left[P_C^{N_D}(\gamma_{L+r})\right] = \sum_{k=0}^{N_D} \binom{N_D}{k} (-1)^k E\left[P_b^k(\gamma_{L+r})\right] \tag{9}$$

with

$$\begin{aligned} E\left[P_b^k(\gamma_{L+r})\right] &= \int_0^\infty P_b^k(\gamma)\, f_{\gamma_{L+r}}(\gamma)\, d\gamma \\ &= \int_0^\infty Q^k\left(\sqrt{2\gamma}\right) f_{\gamma_{L+r}}(\gamma)\, d\gamma. \end{aligned} \tag{10}$$

Also, $e_r^p = 1$ leads to $\hat{s}_r[t]\big|_{1 \le t \le N_D} = 0$ with the probability of $1 - P_C^{N_D}(\overline{\gamma}_{L+r})$. Furthermore, the probability of the p th error-event at BT-ADF relay systems can be presented as

$$Pr^p = \prod_{r=1}^{L} \left[P_C^{N_D}(\overline{\gamma}_{L+r})\right]^{\overline{e_r^p}} \left[1 - P_C^{N_D}(\overline{\gamma}_{L+r})\right]^{e_r^p} \tag{11}$$

with $\overline{e_r^p} = (e_r^p + 1) \bmod 2$ [11]. The evaluation of (11) can be carried out by using the '*integral*($\cdot$)' function of MATLAB. Note that for $k \in \{1, 2, 3, 4\}$ the closed-form derivation of (11) is presented in [13]. Furthermore, when we apply the upper-bound of the Q-function shown in [12] as

$$P_b(\gamma_{L+r}) = Q\left(\sqrt{2\gamma_{L+r}}\right) < e^{-\gamma_{L+r}} \tag{12}$$

into (10), we can obtain the approximated version of (10) as

$$E\left[P_b^k(\gamma_{L+r})\right] < \int_0^\infty e^{-k\gamma_{L+r}} f_{\gamma_{L+r}}(\gamma)\, d\gamma = \frac{1}{1 + k\overline{\gamma}_{L+r}} = E\left[P_b^k(\gamma_{L+r})\right]_{UB} \tag{13}$$

and from $E\left[P_b^k(\gamma_{L+r})\right]_{UB}$, the results of both (9) and (10) can be simplified.

3.2. Received SNR and Its PDF & CDF

At the destination node, a maximal ratio combing (MRC) scheme can be applied in order to combine signals from S-D and R-D links. For MRC, the noise variance normalization process is necessary in order to fully obtain the diversity. Then, the decision variable for the p th error-event can be obtained as

$$z_{tot}^{p}[t]\Big|_{1\le t\le N_D} = \sum_{r=0}^{L} h_r^{*}\overline{e_r^{p}}\, y_r[t]/\sigma^2 . \tag{14}$$

Note that the combined instantaneous SNR can be written as $\gamma_C^p = \gamma_0 + \sum_{r=1}^{L}\overline{e_r^p}\,\gamma_r$. It means that when there is a burst error detection at the r th relay node for the p th event vector (i.e., $e_r^p = 1$), no-transmission gives $\overline{e_r^p}\,\gamma_r = 0$. Therefore, in BT-ADF relay systems, $\overline{e_r^p}$ can be regarded as the transmission indicator for the r th relay of the p th error-event. Consequently, the PDF of γ_C^p can be presented as

$$f_{\gamma_C^p}(x) = \sum_{r=0}^{L}\frac{\pi_r^p}{\overline{\gamma}_r}\exp\left(\frac{-\overline{e_r^p}\,x}{\overline{\gamma}_r}\right) \tag{15}$$

with $\pi_r^p = \prod_{i=0,i\ne r}^{L}\frac{\overline{e_r^p}\,\overline{\gamma}_r}{\overline{e_r^p}\,\overline{\gamma}_r - \overline{e_i^p}\,\overline{\gamma}_i}$ [12]. Note that for S-D link terms in (15), we can define $\overline{e_0^p} = 1$ so as to $\overline{e_0^p}\,\overline{\gamma}_0 = \overline{\gamma}_0$. Also, the cumulative distribution function (CDF) of γ_C^p can be written as

$$F_{\gamma_C^p}(x) = \int_0^x f_{\gamma_C^p}(\gamma)\,d\gamma . \tag{16}$$

3.3. Average Outage Probability

When the outage probability of P_{out}^p for the p th error-event is defined as the probability that the channel mutual information (I) falls below the required transmitting rate T_R, it is expressed as $I = \frac{1}{L+1}\log_2\left(1+\gamma_C^p\right) \le T_R$. The ratio of $\frac{1}{L+1}$ is caused by the fact that we need $L+1$ orthogonal channels for BT-ADF relay systems. Therefore, for the p th error-event, P_{out}^p can be written as

$$P_{out}^p = Pr\left(\frac{1}{2}\log_2\left(1+\gamma_C^p\right) \le T_R\right) = F_{\gamma_C^p}\left(2^{(L+1)T_R} - 1\right). \tag{17}$$

Consequently, when we consider all the possible error-events, the average outage probability can be shown as

$$P_{out} = \sum_{p=1}^{2^L} Pr^p\left(\{\overline{\gamma}_{L+r}\}_{r=1}^{L}\right) P_{out}^p . \tag{18}$$

where $Pr^p\left(\{\overline{\gamma}_{L+r}\}_{r=1}^{L}\right)$ of (18) can be considered as the probability of occurrence for the p th error-event's outage.

4. Numerical and Simulation Result

In this section, we show numerical results of average outage probability and verify its accuracy by comparing simulation results. For simplicity, it is assumed that $E_r = E_s / L$ for $r = 1, \cdots, L$. To capture the effect of path-loss on the outage performance, $\alpha_r \left(= r / (L + 1)\right)$ is defined as the relative distance between source and the r th relay when the distance between source and destination is 1. Then, we introduce the channel model that $E\left[|h_{L+r}|^2\right] = E\left[|h_0|^2\right] / \alpha_r^{\mu}$ and $E\left[|h_r|^2\right] = E\left[|h_0|^2\right] / (1 - \alpha_r)^{\mu}$ with the path-loss factor μ. From here, we use $\mu = 3.76$ which is the parameter of outdoor hot-zone model [Table A.2.1.1.2-3] in [14] and SNR is defined as

$$\mathrm{SNR} = \frac{E\left[\left|h_0\right|^2\right] E_s}{\sigma^2}.$$

From here, '*Exact Analysis*' means numerical results obtained from (18) with the evaluation of (10) by using the '*integral*$(\cdot)$' function of MATLAB and '*App. Analysis*' indicates the approximated performance bound which are obtained from (18) with $E\left[P_b^k(\gamma_{L+r})\right]_{UB}$ in (13).

Fig. 2 and Fig. 3 show average outage probability versus SNR with respect to N_D for $L = 2$ and $L = 4$, respectively. We can find that the average outage probability performance decreases in proportion to N_D (number of data symbols within a frame). When N_D increases, the N_D symbols' correct detection probability of (8) decreases, so that each relay's participation probability into the transmission also decreases. Consequently, it leads to the performance degradation, which is only shown as the average SNR loss. Note that even if there is an performance loss according to the increase of N_D, we can still find the diversity gain caused by the increase of L. Also, it is worthwhile to mention that $N_D = 1$ means the case of symbol-by-symbol detection. Therefore, it confirms to previous researches [9-11] and shows an achievable lower outage probability performance.

Fig. 4 shows average outage probability versus SNR with respect to L. We can find that the diversity order linearly increases as the number of relays. Moreover, it is worthwhile to mention that the derived analytical results well match simulation results. As mentioned in Fig. 2 and Fig. 3, it is confirmed that the performance difference between $N_D = 1$ and $N_D = 32$ is in proportion to L. In addition, we can find that the approximated analytical upper bound is tight enough for all SNR values. Consequently, the derived analytical approach can be used as a general tool to verify effects of BT-ADF relay systems on average outage probability over quasi-static Rayleigh fading channels.

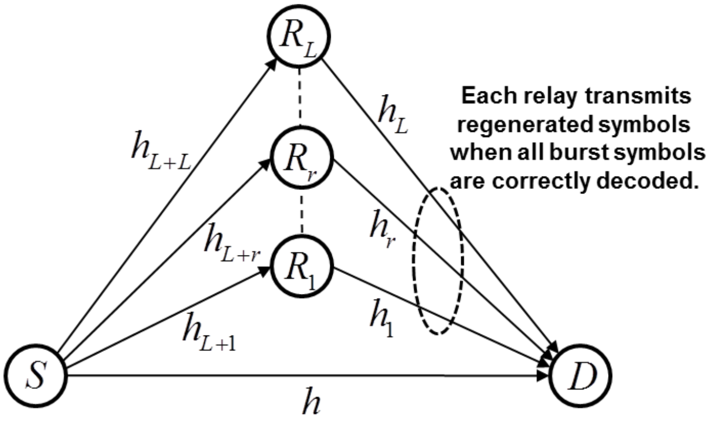

Figure 1 Block Diagram of BT-ADF Relay Systems(source(S), Destination(D), Relay(R))

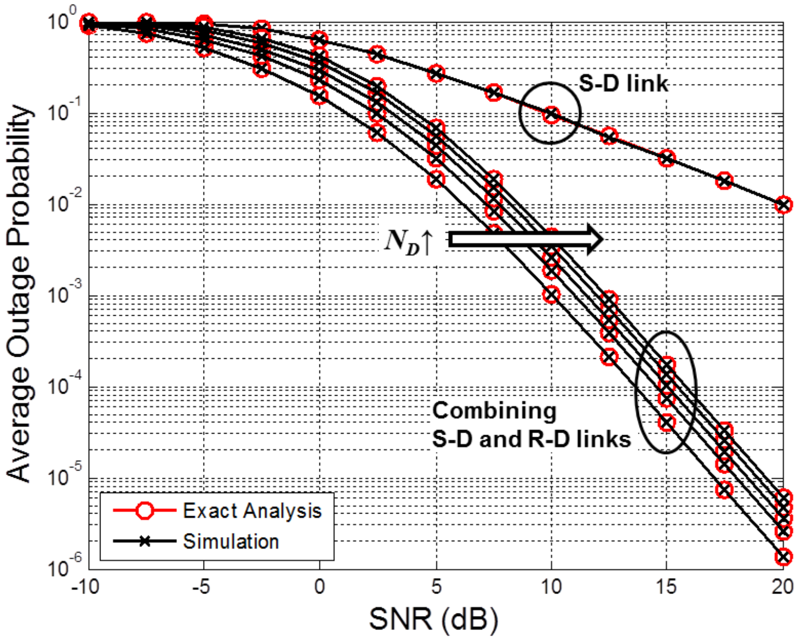

Figure 2 Average Outage Probability Versus SNR for BT-ADF Relay Systems With Respect to Different Number of Burst Symbols

($L = 2$, $N_D = 1, 4, 8, 16, 32$)

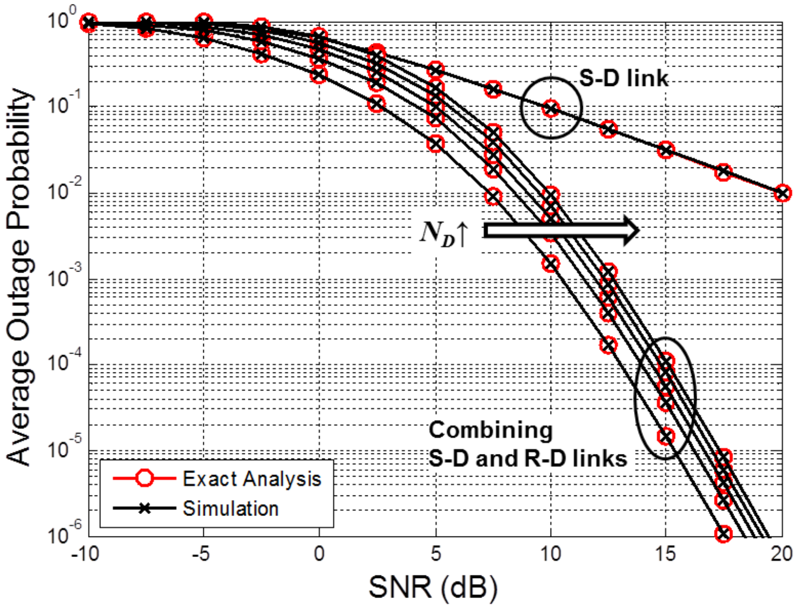

Figure 3 Average Outage Probability Versus SNR for BT-ADF Relay Systems With Respect to Different Number of Burst Symbols ($L=4$, $N_D=1,4,8,16,32$)

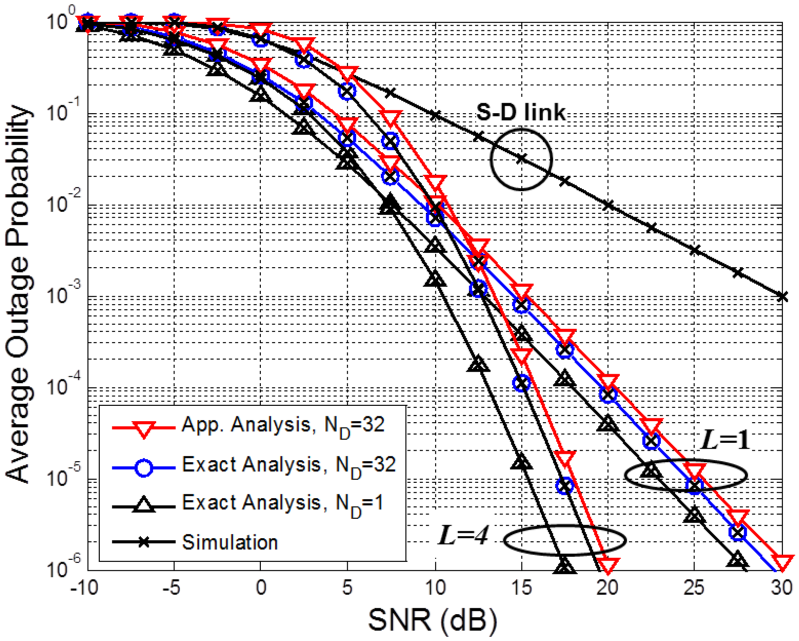

Figure 4 Average Outage Probability Versus SNR for BT-ADF Relay Systems With Respect to Different Number of Relays ($L=1,4$, $N_D=1,32$)

5. Conclusions

In this paper, we have derived the average outage probability for the BT-ADF relay systems over quasi-static INID Rayleigh fading channels. Our proposed analytical approach includes burst transmission, which can be a practical environment. Firstly, the average outage probability expression has been derived as an exact form with numerical integration. Then, the approximated bound is presented as the well-known form and it is confirmed to be an upper outage probability bound by the comparison with simulation results. Therefore, we can conclude that our analytical expressions are very tractable form, and can be used as a tool to verify effects of an erroneous burst detection and transmission at each relay node on the combined SNR, the average outage probability, and cooperative diversity gain.

6. ACKNOWLEDGEMENTS

This research was supported by 2014 Research Grant of Hanseo University.

7. References

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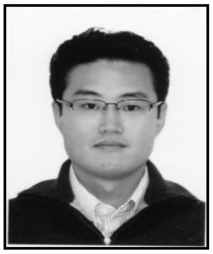

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