

Resource Allocation of Large Scale Engineering Based on Optimal Control Theory

Qiao Jianfeng

*Economy and Management school, Beijing University of Posts and
Telecommunications
10th Xitucheng Road of Haidian District, Beijing China 100876
jianfeng_qiao@126.com*

Abstract

It is the common issue for all of countries that lots of safety risk accidents happen in large scale engineering projects. To save cost, generally, no enough resource is booked, or not very qualified resource is selected, which is one of the import safety risk factors in large scale engineering projects. But less scientific strategies are available to balance between safety risk values and safety risk investment. In the large scale engineer safety risk management filed, currently most of methods are qualitative, so it is a hot and difficult topic that constructs a quantitative model for practical problems, especially based on optimal control theory. In this paper, we will take both advantages of maximum principle and dynamic programming to solve the large scale engineering project safety risk issues caused by resource numbers and resource quality. Firstly, by maximum principle, establishes safety risk model considering resource numbers, and give iterative steps of the gradient descent method for numerical solution; Secondly, by dynamic programming, establishes safety risk economic model which takes into account the quality of resource; In the end, both the models are validated in one practical equipment case by combining the advantages of each algorithms .

Keywords: *Large scale engineering, Safety risk, Resource allocation, Optimal control, Maximum principle, Dynamic programming*

1. Introduction

Large scale engineering refers to the complex multi discipline engineering (both design and construction) which is encountered at the top-end of the construction industry's spectrum of activities [1]. And large scale engineering includes complex buildings, process plant, infrastructure, significant civil engineering work and other major construction works. A large scale engineering project has the following attributes:

- Investment is very large required high capital cost
- The construction period is relatively long under urgency
- Internal structures are very complex involved many organizations
- Technology is difficult and need multi-disciplinary knowledge
- System integrations are very complex requiring with high quality
- The construction environment is very complex with uncertainty

Large scale engineering is characterized by a great number of functions and parts, and plays a very important role in facilitating, enhancing, or extending our life on the earth.

However, due to its obvious attributes, often leads to safety risks of major disasters accidents, resulting in huge economic losses to the state property, ecosystem destruction and a large number of personnel casualties, even seriously harm the national public safety.

1.1. Safety Risk Management Process

So, in a way, whether a large scale engineer project is success or not, depending on safety risk management. Any mistakes of safety risk have the potential to waste large sums of money and likely more seriously human lives and environmental damage. “Any safety aspect related to people, property or the environment”. Generally safety risk management is including four processes [2], see Figure 1:



Figure 1. Safety Risk Management Process

- Identify hazards. Hazard is anything (*e.g.*, condition, situation, practice, behavior) that has the potential to cause harm, including injury, disease, death, environmental, property and equipment damage [3]. A hazard can be a thing or a situation.
- Assess risks. Risk is the likelihood, or possibility, that harm (injury, illness, death, damage, *etc.*) may occur from exposure to a hazard.
- Control risks. Take actions to eliminate health and safety risks to reach acceptance level from practice. Where risks can be eliminated, then implementation of control measures is required, to minimize risks.
- Review measures. This involves ongoing monitoring of the hazards identified, risks assessed and risk control processes and reviewing them to make sure they are working effectively.

Safety risk management should be incorporated into design processes from the initial stages and that more scientific and objective approaches are required in order to help prevent major accidents, to demonstrate safety and to describe the safe operational requirements. In 1979, UK, C.B. Chapman, one of first researchers began research in large engineering project risk analysis, provided a systematic approach to the planning and financial evaluation of large engineering projects involving significant risk [4]. And until 2000, there were still lots of risk management related papers written by him.

In 1995, Netherlands, Christian Preyssl working in European Space Agency, gave a hazard analysis and probabilistic risk assessment to minimize the safety risk of space flight projects [5]. Expert judgment was used in a structured way and risk assessment results were used to prioritize and optimize risk reduction efforts and to support risk acceptance evaluation.

In 2003, UK, Liu Jun proposed a framework for modeling the safety of an engineering system with various types of uncertainties using a fuzzy rule based evidential reasoning approach; a fuzzy rule based designed on the basis of a belief structure was used to capture uncertainty and non-linear relationships between these parameters and safety level [6].

In 2003, Hong Kong, Nguye Duy Long presented problems of large construction projects in Vietnam. Data Analysis revealed that problems could be grouped under five factors: (1) incompetent designers/contractors; (2) poor estimation and change management; (3) social and technological issues; (4) site related issues; (5) improper techniques and tools [7].

In 2007, Thailand, Aksorn pointed out that there were 16 factors having a large influence on the success of safety program implementation including [8]:

- Clear and realistic goals
- Good communication
- Delegation of authority and responsibility
- Sufficient resource allocation
- Management support
- Program evaluation
- Continuing participation of employees
- Personal motivation
- Personal competency
- Teamwork
- Positive group norms
- Personal attitude
- Effective enforcement scheme
- Safety equipment acquisition and maintenance
- Appropriate supervision
- Appropriate safety education and training

In 2008, China, Yusun, identified and assessed safety risk factors inherent in Beijing Olympic Games construction with the involvement of 27 experienced and highly respected experts from government agencies, the construction industry, and academe through brainstorming, work discussions and questionnaire surveys. Based on identified critical safety risks, a risk register was composed and a model was developed in application of the analytic hierarchy process to assess the status of risk on site safety [9].

All in all, based on current researching, we can see that most of researchers have pointed out the safety risk factors of the overview engineering project through experts' analysis, then gave a qualitative model and figured out the safety risk control strategies. They mainly focused on high level study, so they have not gone into detail study about each factors and more focus on hazard identification and risk assessment using qualitative method. But risk

control is also a very import process, and more quantitative methods should be introduced into safety risk management to give detail scientific analysis to each safety risk factors.

1.2. Resource Allocation Issues

“Sufficient resource allocation” is one of important factors pointed by T. Aksorn mentioned in above section. In practice, no enough resource or no qualified resource, which may be caused by saving cost, is one of the biggest safety risks for large scale engineering. In this paper, we want to go in-depth study about resource allocation issues including resource number allocation issue and resource quality allocation issue, in large scale engineering project. All engineering projects are limited by budget, consequently, important practical issues of economic feasibility study are how to quantify uncertainty, how to include it in the formal analysis of an investment proposal and how to manage it. That means resource allocation must consider balance between investment and benefit. The target is to make safety risk to be controlled in an acceptance level.

For resource allocation problem, in 1984, Polish, E. Nowiciki assumed that execution speed was a continuous function of the quantity of resources, and then they solved the optimal resources allocation problem under fixed duration and cost constraints, by using the optimal control theory [10]. In 2008, Russian, Yu Kiselev, by using the Pontryagin maximum algorithm (also called maximum algorithm), solved the optimization problem of one level and binary linear constraint equations in linear production [11].

For safety risk control strategy, there are few papers about resource allocation from quantitative perspective. In this paper, we will learn from the optimal control’s application in other fields including maximum algorithm [12, 13, 14] and dynamic programming [15, 16], to solve resource allocation issue which is one of safety risk factors in large scale engineer.

Through theoretical study, we find that the maximum principle of the optimal control theory is suitable to solve control variables which are continuously different in a certain range; and dynamic programming method is suitable to solve the control variable taking a finite value. So this paper will solve safety risk factor of resource number by maximum principle, and solve safety risk factor of resource quality by dynamic programming. And we will construct the two models respectively based on the real situations in large scale engineering projects and then give exploratory research on both models to validate the effectiveness.

2. The Optimal Control Model of the Maximum Principle

Resource allocation of large scale engineer can be represented as a discrete system, as shown in Figure 2.

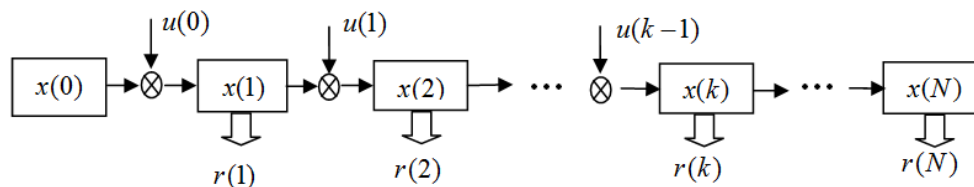


Figure 2. Discrete System Control Model

In which, $x(k)$ represents the number of resources of the system in the k stage, then there is a state vector sequence $x(1), x(2), \dots, x(k), x(N); k = 1, 2, \dots, N$. $x(0)$ represents the

number of resources in the initial state; $x(N)$ represents the number of resources in the end state.

Let $u(k)$ represent the change of supplying resources with respect to $k+1$ stage. This value represents the change of supplying resources between $x(k)$ and $x(k+1)$. If $u(k)$ is positive, it indicates an increase. In the contrary, if it is negative, it indicates a decrease. Then there is the control vector sequence: $u(0), u(1), \dots, u(k), \dots, u(N-1); k=0, 1, 2, \dots, N-1$.

Let $r(k)$ represents the project's demand for resource in the k stage. And the state sequence is as known: $r(1), r(2), \dots, r(k), r(N); k=1, 2, \dots, N$. In general, we can get the number of demand $r(k)$ by project plan.

Based on the above assumptions, follow the below steps to establish resource allocation model using maximum principle of optimal control:

① The system state in each stage is subject to the difference equations:

$$x(k+1) = x(k) + u(k); k = 0, 1, 2, \dots, N-1 \quad (1)$$

The equations show, in the given state $x(0)$, we can get state sequence $x(k)$ by adjusting to the control sequence $u(k)$.

② In the practical engineering problems, $u(k)$ should meet to constraints:

$$x(k) + u(k) \geq 0; k = 0, 1, 2, \dots, N-1 \quad (2)$$

In another word, the number of resources cannot be negative in the practical system.

③ Define system performance based on analysis of safety risk factors. To avoid resource (such as equipment and operating personnel) overwork, take the first indicator: $[x(k) + u(k) - r(k)]^2$. It expresses the supplied resources are consistent with requirement of the project in each stage, then they can ensure the project schedule. However, supplied resources may be less than its demand, so we take the square value to ensure it is a positive value.

From the perspective of project resources fluctuation, take the second indicator: $[x(k) + u(k) - \bar{r}]^2$. It represents that supplied resources should be close to the average requirement of the project in every stage, in order to avoid the negative impact due to huge fluctuations of supplied resources. Especially if there is a substantial huge resource increase in next stage, the resource quality is difficult to guarantee timely, which is easily result to safety risk due to not qualified resource.

$\bar{r} = \sum_{k=1}^N r(k) / N$ represents average requirement of resource in each stage of the project.

In the end, the performance of the system is defined as a function of:

$$J = \phi[x(N), N] + \sum_{k=0}^{k=N-1} \{A[x(k) + u(k) - r(k+1)]^2 + B[x(k) + u(k) - \bar{r}]^2\} / \bar{r}^2 \quad (3)$$

$\phi[x(N), N]$ represents the constraints to the end state. If the system is not constraint to the end state, it can be ignored. A and B are adjustment factor of two performance indicators' weight. \bar{r}^2 is the adjustment factor of square of performance indicators.

④ Introduce Hamiltonian equations

$$H(x, u, \lambda, k) = \{A[x(k) + u(k) - r(k+1)]^2 + B[x(k) + u(k) - \bar{r}]^2\} / \bar{r}^2 + \lambda(k+1)[x(k) + u(k)] \quad (4)$$

⑤ According to the maximum principle of discrete systems, a necessary condition for the minimum value of J , as follows:

$$\lambda(k) = \frac{\partial H(k)}{\partial x(k)} = \frac{2A}{\bar{r}^2} [x(k) + u(k) - r(k)] + \frac{2B}{\bar{r}^2} [x(k) + u(k) - \bar{r}] + \lambda(k+1) \quad (5)$$

$$\frac{\partial H(k)}{\partial u(k)} = \frac{2A}{\bar{r}^2} [x(k) + u(k) - r(k+1)] + \frac{2B}{\bar{r}^2} [x(k) + u(k) - \bar{r}] + \lambda(k+1) \quad (6)$$

Equation (5) and Equation (6) are respectively called as coordination equation and control equation of Hamiltonian canonical equations. The optimal solutions are:

$$H[x^*(k), u^*(k), \lambda^*(k+1), k] = \min_{U \in \Omega} H[x^*(k), u^*(k), \lambda^*(k+1), k] \quad (7)$$

$x^*(k), u^*(k), \lambda^*(k+1)$ let H take the minimum value. $U = [u(0), u(1), \dots, u(N-1)]^T$ belongs to a bounded closed set Ω . $U \in \Omega$ meeting equation(2).

“Sufficient resource allocation” in here means $x(k) + u(k) \geq r(k)$, which is hardly to meet in practice, due to project manager will make project get maximum profit and save cost from resource allocation. The above resource allocation model will give a good balance between the profit and safety risk.

3. Gradient Descent Calculation Method

In general, the calculation of the maximum algorithm of optimal control is usually divided into two categories: direct and indirect methods. Gradient method is a direct method, used widely [17]. The character of this method is that it will get any a control sequence firstly, which may not meet to the necessary conditions when H gets to the minimal. Then it will use an iterative algorithm to improve U , based on the direction of H gradient decreased, to make it meet to the necessary condition. Calculated as follow steps:

① Firstly, get any control sequence $u^M(k) = u^0(k)$; $k = 0, 1, \dots, k, \dots, N-1$. M is a iterative step. In initial, $M = 0$. u^0 is got from project experience.

② In the M step, calculate the state sequence $x^M(k)$ by order: $k = 1, 2, \dots, k, \dots, N$, based on above step $u^M(k)$ and given initial condition $x(0)$.

③ Calculate gradient vector of Hamilton function:

$$G^M = \left(\frac{\partial H}{\partial U} \right)_M \quad (8)$$

In which, $G^M = [g(0), g(1), \dots, g(N-1)]^T$.

$$\text{And } g(k)_M = \left(\frac{2A}{\bar{r}^2} [x(k) + u(k) - r(k+1)] + \frac{2B}{\bar{r}^2} [x(k) + u(k) - \bar{r}] + \lambda(k+1) \right)_M$$

④ Correct control vector,

$$U^{M+1} = U^M - \alpha G^M \quad (9)$$

α is a step factor, got by projects experience.

$$\text{And } u^{M+1}(k) = u^M(k) - \alpha \left(\frac{2A}{\bar{r}^2} [x(k) + u(k) - r(k+1)] + \frac{2B}{\bar{r}^2} [x(k) + u(k) - \bar{r}] + \lambda(k+1) \right)_M .$$

⑤ Calculate performance index as follow:

$$\left| \frac{J(U^{M+1}) - J(U^M)}{J(U^M)} \right| < \varepsilon \quad (10)$$

$J(U^k) = \sum_{k=0}^{k=N-1} \{A[x(k) + u(k) - r(k+1)]^2 + B[x(k) + u(k) - \bar{r}]^2\} / \bar{r}^2$. ε is the pointed minimum value. If meets the condition, stop the calculation, otherwise go to step (2).

$\frac{\partial H}{\partial U} = 0$ is the necessary conditions to take the minimum of Hamilton function. So $\lambda^k(k) = 0; k = 0, 1, \dots, k, \dots, N-1$. If it is no constraint to the end and transversality condition, $\lambda^k(N) = 0$.

4. Optimal Control Model of Dynamic Programming

In above Section 2 and Section 3, we only consider the optimal control model of the resource number allocation, which make resource allocation not only meet project requirement, but also take safety risks into account. Because quality of resource is varying for each of them, if not qualified resource is used in the large scale engineering project, there will be huge safety risk. Short term tradeoffs between cost saving and safety often exist in the operation such as whether using not very qualified resource.

In 2004, Michelle M. Cowing [18] used dynamic programming to resolve tradeoffs between productivity and safety. In this paper we develop resource quality allocation model to resolve replacement strategy for not very qualified resource. As followings, we will gradually establish optimal control model of dynamic programming.

① The potential risk value.

The risk theory pointed out that the degree of engineering risk depends on two factors: First, the occurrence probability of risk events, known as risk probability p ; Second, the risk loss value g after the incident ^[18]. In this paper we define the potential loss value l of engineering risk is:

$$l = p \times g \quad (11)$$

② Index function and optimal function.

Any decision-making process is bound to have a measure of its strategic merits of the scale, a number of indicators, called the index function.

$$V_{k,n} = v_k(x_k, u_k) \oplus v_{k+1}(x_{k+1}, u_{k+1}) \dots \oplus \dots \oplus v_n(x_n, u_n) \quad (12)$$

in which u_k is used to indicate decision variables of the $x(k)$ stage. $v_k(x_k, u_k)$ is the index functions of stage k . The optimal functions of $f_k(x_k)$ can be written as:

$$f_k(x_k) = \max_{u_{k,n}(x_k)} V_{k,n}(x_k; u_{k,n}) \quad (13)$$

③ Dynamic programming equation.

Assume that $e_k(x_k, u_k)$ represents total profit in the phrase of k , in which, state is x_k and control decisions is u_k . Assume that $c_k(x_k, u_k)$ represents system's cost for control measures in k phrase. And assume that $l_k(x_k, u_k)$ represents system's potential risk loss value in the k phrase. Index function based on risk loss is:

$$v_k(x_k, u_k) = e_k(x_k, u_k) - c_k(x_k, u_k) - l_k(x_k, u_k) \quad (14)$$

So the basic equations of dynamic programming are:

$$\begin{cases} f_k(x_k) = \max_{u_k(x_k)} [e_k(x_k, u_k) - c_k(x_k, u_k) - l_k(x_k, u_k) + f_{k+1}(x_{k+1}, u_{k+1})]; k = n, n-1, \dots, 1 \\ \text{End State: } f_{n+1}(x_{n+1}) = 0 \end{cases} \quad (15)$$

The target of above equation is to make the project get the maximum profit during the whole lifecycle of project development considering safety risk. In practice, project manager will make project get the maximum profit, so that may take some risks such as using not very qualified resources. The above resource allocation model will give a good balance between the profit and safety risk. Generally use reverses order to calculate dynamic programming.

5. Resource Allocation Model Based on Optimal Control

For any kind of resource in a large scale engineering project, project manager generally consider two questions: how many resources should be planed and what kinds of resource quality should be selected at the different stages of project development. The decision strategies not only meet project development requirement according to project schedule, but also need consider safety risk. The target is getting maximum profit after take safety risk control strategy.

From above algorithm studies, we can see that the maximum principle of the optimal control theory is suitable to solve control variable with continuously difference in a certain range; and dynamic programming method is suitable to solve the control variable taking a finite value. The former one can be used to resolve resource number allocation issue; and the later one can be resolved resource quality allocation issues such as new equipment replacement decision.

Figure 3 draws a matrix which shows resource number issues can be optimized by maximum principle; and resource quality issues can be resolved by dynamic programming. Combined both algorithms, optimal resource allocation can be totally resolved based on these two solutions.

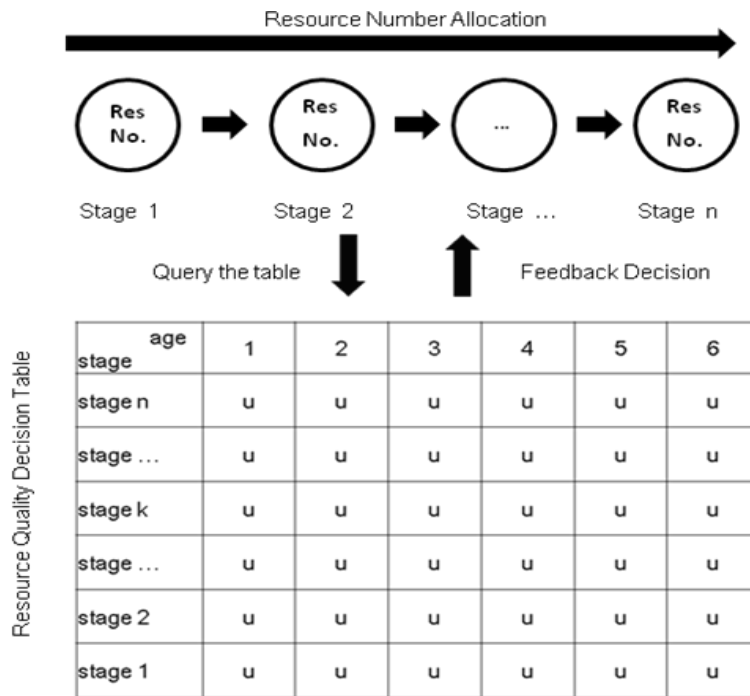


Figure 3. Optimal Resource Allocation Model

At any stage, project managers can make quantitative decision about resource number allocation and resource quality allocations. Optimal resource allocation model of Figure 3 shows that, after resource number is decided, every resource need query the table calculated by dynamic programming, then get the feedback about this resource’s quality decision “u”. “u” is control decision which should be different according to different stages and different ages of the resource.

6. Case Study of Resource Allocation Based on Optimal Control

Currently, most of large scale engineering project need modern equipments support during development and operation. These equipments are one of the most import resources in the large scale engineering project, and they are also one of the most import safety risk factors. Such as overwork for long time and expired equipments are still be used to save cost. Meanwhile safety accidents more easily happen. How to balance between profit and safety risk will be resolved in this section.

6.1. Resource Allocation Based on Maximum Principle

Take a certain equipment allocation in a large scale engineering project as an example. The more resource is allocated, the more cost will be spent. The less resource is allocated, the less cost will be spent, but the less resource is allocated, to catch up the project schedule, more overwork is needed and the potential safety risk will be increased. The resource number requirement is different in every month, if there is significant increase in the next month, hardly to allocate qualified resource in short time, which will also cause safety risk. In practice, we need allocate resource based on requirement and also need avoid big fluctuation. Based on above Section 2’s discussion, we can use the maximum principle of the optimal

control theory to optimize the number allocation of the equipment involved in the project. Firstly, we have known the project plan, and quantity requirement for this kind of equipment. Table 1 lists the number of the resource demanded for this project in each month, that mean $r(k)$ is given.

Table.1 Number Requirement of Equipments Monthly

r(k)	r1	r2	r3	r4	r5	r6
Number	11	24	33	7	26	19

Before discussing the issue, make some assumptions, as followings:

① The whole period of project is six months. At the end of sixth month, there is no requirement about how many equipments need to be reserved. In another word, $\phi[x(N), N] = 0$.

② Assume that all equipments have the same quality and performance. Only equipment number is considered which impact safety risk.

③ Assume that there is no equipment when project start. That means $x(0) = 0$.

Based on these assumptions, the issue can be described as: when initial number of equipment is 0, how to allocate the number of the equipment during these six months, which not only meet the project develop requirement, but also consider the project safety risk requirement within limited cost to maximum the project profit. In equation (3), two factors need balanced: one is make real allocation is close to requirement (if real allocation is bigger than requirement, cost will be increased, but safety risk is low; if real allocation is less than requirement, cost will be decreased, but safety risk is high); another is make real allocation is close to the average level to void big fluctuations.

Now step by step, we solve the number allocation of equipment during these six months by applying gradient calculation method of the optimal control theory describe in Section 3. Let $A = 5, B = 3; \alpha = 5$ getting from experience.

First, select any one of the control sequences $u^0(k)$, which will make the first part of the performance index $A[x(k) + u(k) - r(k+1)]^2$ minimize. $r(k)$ can be got from Table 1, and $x(0) = 0$, calculate $u(k)$ and $x(k)$ step by step. According to equation (3) and (8), calculate gradient vector of Hamilton function like Table 2.

Table 2. Each Parameter Values When M=0

N/A	r1	r2	r3	r4	r5	r6
N/A	11	24	33	7	26	19
u0	u1	u2	u3	u4	u5	N/A
11	13	9	-26	19	-7	N/A
x0	x1	x2	x3	x4	x5	x6
0	11	24	33	7	26	19
j0	j1	j2	j3	j4	j5	N/A
0.61	0.12	1.27	1.27	0.27	0.01	N/A
g0	g1	g2	g3	g4	g5	N/A
-0.135	0.06	0.195	-0.195	0.09	-0.015	N/A

Use equation (9) to calculate the control variable in next step. Then go on calculate each parameter values in above table. Take $\varepsilon = 0.01$, If equation (10) is met, stop iteration. After eleven iterations, equation (10) is met; See the Table 3 of each parameter values.

Table 3. Each Parameter Values When M=11

N/A	r1	r2	r3	r4	r5	r6
N/A	11	24	33	7	26	19
u0	u1	u2	u3	u4	u5	N/A
14.09	9.34	4.56	-18.05	15.31	-6.01	N/A
x0	x1	x2	x3	x4	x5	X6
0.00	14.09	23.43	27.99	9.94	25.26	19.25
j0	j1	j2	j3	j4	j5	N/A
0.38	0.09	0.79	0.87	0.21	0.01	N/A
g0	g1	g2	g3	g4	g5	N/A
-0.012	0.037	-0.005	-0.077	0.060	-0.005	N/A

In Table 3, we found that when $r(3) = 33$, $x(3) = 27.99 \approx 28$. That means real number of equipment allocation can be slightly less than the requirement, the benefit is saving cost, but the negative effect is poetical safety risk is increased due to not enough resource. However when $r(4) = 7$, $x(4) = 9.94 \approx 10$. That means, the real resource number allocation can be slightly bigger than the requirement, the benefit is reducing safety risk if increase resource in next month, but the negative effect the cost is increased. These resource allocations are more in line with the actual implementations in practice. For the total demand of resource number, both are 120 which is same before optimizing and after optimizing. In another word, there is no increase in cost after optimizing, even though we consider the safety risk factors. Generally more cost is needed if consider handling safety risk, and it is enough that safety risk is reduced into acceptance level.

6.2 Resource Allocation Based Dynamic Programming

The longer equipment is used, the greater it generates cumulative benefits, but with the equipment's outdated, the use cost and safety risk of the equipment also will increase. So the project will face a decision to keep or update equipments at the stage. Before discussing this issue, make some assumptions, as followings:

- ① The whole period of project is six months, that means $f_7(t) = 0$.
- ② The expire date of equipment is six months, that means $f_k(7) = -\infty$.
- ③ The loss value is 100 KEUR if a safety risk accident happens, and the probability of accident is $p(t)$ which increase monthly if keep using this equipment. Based on equation (11) in section 4, risk values is $l_k(x_k, u_k) = 100 * p(t)$.

Table 4 lists the net profit $v(t)$ at the different stages of the equipment if considering potential safety risk value in the practice.

Table 4. The Equipment Net Profit in Each Stages

	1	2	3	4	5	6
$e(t)$	20	19	18	16	14	11
$c(t)$	2	3	4	6	8	11
$p(t)$	0.0	0.02	0.04	0.06	0.1	0.2
$l(t)$	0.0	2.0	4	6	10	20
$v(t)$	18	14	10	4	-4	-20

Assume that the cost of updating equipments is 30 KEUR:

$$c_k(x_k, u_k) = \begin{cases} 0; u_k = N(\text{Keep}) \\ -30; u_k = P(\text{Updat e}) \end{cases} \quad (16)$$

Based on equation (15) in section 4, write the dynamic programming equation by the known variables in table 4:

$$f_k(t) = \max \begin{cases} e(t) - c(t) - l(t) + f_{k+1}(t+1); u_k = N. \\ 18 - 30 + f_{k+1}(2); u_k = P. \end{cases} \quad (17)$$

Dynamic programming algorithm should be calculated from the end stage. For this case, calculate from $f_6(t)$:

$$f_6(6) = \max\{-20; -12\} = -12(u = P)$$

$$f_6(5) = \max\{-4; -12\} = -4(u = N)$$

...

$$f_6(2) = \max\{14; -12\} = 14(u = N)$$

$$f_6(1) = \max\{18; -12\} = 18(u = N)$$

$$f_5(6) = \max\{-20 - \infty; -12 + 14\} = 2(u = P)$$

...

Table 5. Net Benefit of Dynamic Programming

Stage \ Age	1	2	3	4	5	6
$f_6(t)$	18	14	10	4	-4	-12
$f_5(t)$	32	24	14	2	2	2
$f_4(t)$	42	28	13	12	12	12
$f_3(t)$	46	27	22	16	16	16
$f_2(t)$	45	36	26	20	15	15
$f_1(t)$	54	40	30	24	24	24

After 6*6 steps, $f_1(t)$ can be got. Table 5 is the ultimate results. In the table, shaded tables, means that control decision will select to update with new equipment at this stage. However non-shaded tables, means that control decision will select to keep the old equipment. For example, the project have been taken for four months, assume an equipment has been used for four months , if we consider factors of safety risk, we would get $f_4(4) = 12$ by looking up for Table 5 and need update with new equipment. But if we don't update the equipment, the accumulated net benefit for this equipment is $4 + f_5(5) = 6$. The result shows that updating the equipment will not only increase revenue but also reduce safety risks.

6.3 Case Summary of Resource Allocation

In summary, resource allocation process can be described in figure 4 for the above case. After resource number is optimized by using the maximum principle, each resource should look into the resource update decision table calculated by dynamic programming, to check whether the resource should be kept or updated. In the dynamic programming table, the decision is easily to get as long as input equipment age t and project stage k. for example in stage 3, 28 equipments need to be booked, and in these equipments, if the equipment is more than 4 months old, it should be replaced with newest one to reduce safety risk and get maximum profit.

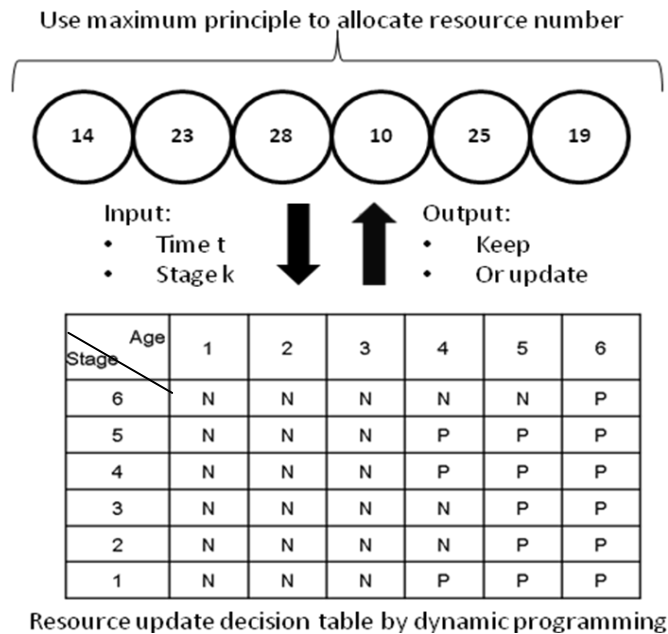


Figure 4. Resource Allocation Case

7. Conclusion

Generally, additional more costs need to be spent for taking safety risk controlling, especially for the large scale engineering. Balancing between cost investment and potential safety risk value is essential objective for the project managers. Large scale numbers of resources need to be involved in the large scale engineering which is one of the import safety risk factors.

The optimal resource allocation model presented in this paper provides a resource allocation framework: firstly, use the maximum principle to resolve resource number allocation issues; secondly, use dynamic programming to resolve resource quality allocation issues. Maximum principle model is constructed with square functions considering two factors: one need meet no significant different with requirement, another need meet no significant fluctuation compared with average value; Dynamic programming model is constructed with net profit function considering three factors: total profit, safety risk control investment and saved potential safety risk value. From practice, maximum principle has been verified as an effective quantitative method to adjust resource numbers; and dynamic programming has been verified as an effective quantitative method for budget tradeoff. Fewer papers take advantages of each optimal control algorithms to resolve one issue. And less quantitative methods have been adopted into resolving safe risk factors. So this paper is a meaningful research to apply the optimal control theory into safety risk management of large-scale engineer.

The safety risk of a large engineering system is affected by many factors such as its design, manufacturing, installation, commissioning, operation and maintenance. Consequently, it may be extremely difficult to construct an accurate and complete mathematical model for the system in order to assess the safety because of inadequate knowledge about the basic failure evens. This leads inevitably to problems of uncertainty in representation. In this paper we select resource allocation safety risk factor to give a detail analysis during the operation, but there are still lots factors disserve in-depth study and more safety risk issue can be resolved by optimal control theory.

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Author



Jianfeng Qiao. He received his M.Sc. in control theory and control engineering in 2003. From 2011 he was PhD student of Beijing University of Posts and Telecommunications at Economy and Management school, major in system integration and system control in large scale engineering.

