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A New Eccentric Annular Leakage Model for Rod Pump with Couette-Poiseuille Flow

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Abstract

A new eccentric annular leakage model for rod pump with Couette-Poiseuille flow is proposed. This model considers not only the effect of pressure fluctuation yielded by overlying liquid on the plunger, but also the direction of Couette flow leakage, which is consistent to the direction of Poiseuille flow leakage. Furthermore, the consideration of the fluctuation effect is proved to be necessary, and the determination of the direction is justified. An analysis on the relationship between leakage rate, pumping speed and pump depth is developed. The results are consistent with the field observations. And at last, the proposed model is proved to be more accurate when compared with the testing data.

Keywords: *Lifting Device, pump, Leakage, Poiseuille flow leakage, Couette flow leakage*

1. Introduction

Rod pumping is the most widely used artificial lift method in the oil industry. Rod pumps (also called downhole reciprocating pump) are regarded as important parts of the rod pumping system. The leakage from a rod pump can be divided into two categories. One is called valve leakage including both traveling and fixed valve leakage, which is caused due to the long running term, sand production or corrosion. The other one is the annular leakage between the plunger and the inside wall of pump barrel. Annular leakage is the main component of leakage for all the rod pumps, and it is also a vital factor influencing pump delivery system. Therefore, it is an important step to model pump leakage in order to calculate the deliverability and efficiency of the entire rod pumping system.

So far, the petroleum engineers have done a great deal of research on modeling the pump leakage problem with both theoretical and experimental methods. In the early 1930s, Robinson [1] firstly described the equation of pump leakage through experimental method. In the 1940s, Davis [2] created the first theoretical model for pump leakage when he studied the heat transfer in annuli. From then on, Stearns [3], Reekstin [4], and Richard Kyle Chambliss [13, 14] *etc.* put forward some new pump leakage models respectively. In this paper, all these models are referred to as the first kind of pump leakage model, for the reason that most of these models followed the same basic form of Equ. 1.

$$q_v = K\frac{\Delta P D^x \delta^y}{\mu l} \tag{1}$$

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where D is plunger diameter, m. ΔP is the differential pressure of overlying fluid on the plunger and pump intake pressure, Pa. δ is the average width between plunger and inside wall of pump, m. μ is the mixture viscosity, Pa·s. l is the plunger length, m. And K, x, y is some coefficient whose values were differently taken by various theory models or experiments. The above formula is of high accuracy in some certain conditions, especially the model made by Richard Kyle Chambliss in his publication "Developing Plunger Slippage Equation For Rod-drawn Oil Well Pumps"[13] in 2005. The model mentioned in that paper detailed considered the influence of the pump leakage on the factor of the pump clearance, temperature and plunger piston eccentric equivalence. However, it must be pointed out that: any formula which has the form of Equ.1 do not have any parameter that characterizes the pumping rate, which means that all of those models believe pumping rate didn't affect the pump leakage, or at least the influence of the leakage was too small that could be completely neglected. In this paper, the author thinks that view has two questions that can be discussed as follows:

Firstly, take Richard Kyle Chambliss's model as an example. After checking in his experiments, it can be found that his pump intake depth was shallower than 2000m (pump intake depth was about 1170m in his publication, and the rest models are same also), it is doubtful that the formula put forward from that experiment premise can reflect real leakage rate when the pump intake depth is deeper than 2000m, not even to say those wells deep over 4000m at most; Secondly, there are at least two aspects might be noted on the influence of leakage caused by pumping rate in theory: on one hand, the magnitude of pumping rate will affect the change of overburden pressure, which would directly lead to the change of pump leakage; on the other hand, there is a direct relationship between shearing leakage caused by plunger piston's movement relative to pump barrel and plunger piston's movement speed. Up to now, there is no accurate experiment or theory that can prove all those two aspects could be ignored in deep wells.

Besides, the influence of pump depth on leakage didn't reflect in this kind of leakage model. However, there was no evidence could prove that pump depth did or didn't affect the pump loss. At the very least, we can discuss that, the plunger piston's movement changes as long as the pump depth changes. Since the movement of plunger piston caused the differential pressure of up and down of the piston, it will be used to have influence on the pump leakage. So we have necessary to take corresponding research on that factor.

In the 1980s, Jin. [11] established the annular aperture of cylindrical aperture flow model by using the Navier-Stokes (N-S) equation which neglected the fluid mass. Different from Equ.1, the aperture flow model (see Equ. 2) consists of two parts: the differential pressure flow and shear flow. Zhang *et al.* [12] inducted that form of leakage formula into rod pump leakage calculation; also they did a certain extent of development, and formed the second form of pump leakage calculation model which is different from Equ. 1:

$$q_v = q_{poi}(\Delta P) \pm q_{cou}(v) \qquad (2)$$

Where $q_{poi}(\Delta P)$ is leakage caused by differential pressure of on both ends of plunger piston, namely differential pressure flow leakage (Poiseuille flow leakage), Pa; $q_{cou}(v)$ is leakage caused by plunger piston's movement speed v related to pump barrel, namely shear flow leakage (Couette flow leakage); The advantage of this model is: for v is the function of pumping rate, variation of pumping rate can affect the volume of shear leakage; furthermore, using two separate calculation result adding (or subtracting) would reflect the mechanism difference of the Poiseuille flow leakage and Couette flow leakage more reasonably. So far, some leakage models follow this form [5-7, 12], and the coaxial ring gap loss model mentioned in the literature 11 is the most complete model in the second expressing form of pump leakage calculation model. Although previous researchers have made a lot of improvements and modification on this kind of model, there are two aspects of influence of pumping rate to leakage in the present model that have not been well solved: first, the influence of the piston movement to differential pressure has not been reflected in the model; second, the direction of shear flow leakage is still controversial (literature [5] considers the direction is down, while literature [5-7] think the direction is up). If those two aspects cannot have a good solution, this kind of pump loss model also couldn't reflect the influence of pumping rate and pump depth to pump leakage.

In view of the above problems, this paper inherits the second form of leakage formula and establishes a new pump annular leakage model on the basis of analysis of the overlying liquid pressure fluctuations and the Couette flow direction.

2. Model Development

When the oil well pump works underground, plunger and pump barrel will not be coaxial for the hydraulic jamming force generated by the liquid. That is, plunger would be closer to one side of pump barrel during the up-and-down movement. Therefore, the eccentric model could better simulate the annular leakage than coaxial model. Refer to Chaoming Jin [11], the new eccentric annular leakage model in this paper for rod pump with Couette-Poiseuille flow is derived as follows:

$$q_v(t) = \frac{\pi D \delta^3 (\Delta P_L + P_I(t))}{12 \mu l}\left(1 + \frac{3}{2}\frac{e^2}{\delta^2}\right) + \frac{\pi D \delta v(t)}{2} \tag{3}$$

where D is plunger diameter, m. δ is the average width between plunger and inside wall of pump, m. μ is the mixture viscosity, Pa·s. l is the plunger length, m. e is eccentricity, m, $e \le \delta$. ΔP_L is the difference of static pressure of overlying fluid on the plunger and pump intake pressure, Pa. $P_I(t)$ is the dynamic pressure generated by accelerated motion of the overlying fluid on the plunger , Pa. v(t) is the plunger velocity, m/s.

Eq.3. has two terms on its right hand side. The first one is the leakage caused by the pressure difference across the plunger, which is Poiseuille flow leakage. The second one is the leakage caused by the relative motion between the plunger and pump cylinder, which is Couette flow leakage. The proposed model has two advantages. The first one is the pressure difference ($\Delta P_L + P_I$), in which the $P_I(t)$ is the dynamic pressure generated by the acceleration of the overlying fluid on the plunger, which is ignored in most of the existing leakage models. See Appendix 1 for more details. Besides, the proposed model shows that the directions of shear loss and pressure difference leakage are same, which are all positive. The demonstration is presented in Appendix 2. More details are mentioned in the literature 9. Consequently, the supplementary equations to Eq.3 are listed as follows:

$$\begin{cases} \Delta P_L = \rho g (H_L - H_s) \\ \Delta P_I = \rho c \Delta t \left[a(t+\Delta t) - a(t)\right] \\ v(t) = \dfrac{\partial u(L,t)}{\partial t} = -wO(L)\sin wt + wP(L)\cos wt \\ a(t) = \dfrac{\partial^2 u(L,t)}{\partial t^2} = -w^2 O(L)\cos wt - w^2 P(L)\sin wt \end{cases} \tag{4}$$

Where ρ is the density of mixture near the plunger, kg/m³, t is time, s, c denotes the pressure propagating velocity, m/s, the value range of c is about 1100m/s ~2000m/s.

3. Sensitivity Analysis using the New Eccentric Annular Leakage Model

The parameter sets selected for sensitivity analysis of the new model are listed in Table.1.

Table 1. The parameters for sensitivity analysis

pump depth,m	3813	current production,m³/d	26
reservoir static pressure,MPa	27.9	current PFL,m	1890
tubing ID,mm	62	current stroke	4.96
casing ID,mm	144	current pumping speed,spm	3.23
water cut,%	86.1	current plunger diameter,mm	42.4
GOR (m3/stm3)	147	current pump depth,m	2473
current tubing pressure,MPa	0.43	water specific gravity	1.055
current casing pressure,MPa	0.22	oil specific gravity	0.8515
		gas specific gravity	1.022

Rod String	rod diameter, mm	rod length, m
weighted rod	28	144.8
steel rod	19	1083.9
steel rod	22	726.4
steel rod	25	512.34

3.1. The relationship between leakage rate, pumping speed, and pump grade

Figure 1 shows the leakage rate as a function of pumping speed for different pump grade. The leakage rate here is the average leakage rate for one travel. It can be seen that, the higher speed yields greater leakage rate. Furthermore, when the pump grade is high, the leakage will decrease.

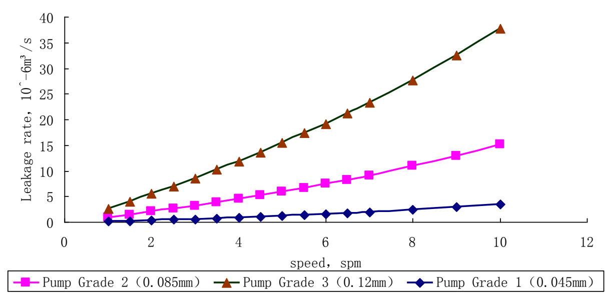

Figure 1. The relationship between the leakage rate and pumping speed for different pump grades

Figure 2 shows the profile of the instantaneous leakage rate in the upstroke. From Figure 2, we can see that the leakage rate is high in the middle of the stroke and low in two ends, which is consistent with the plunger movement.

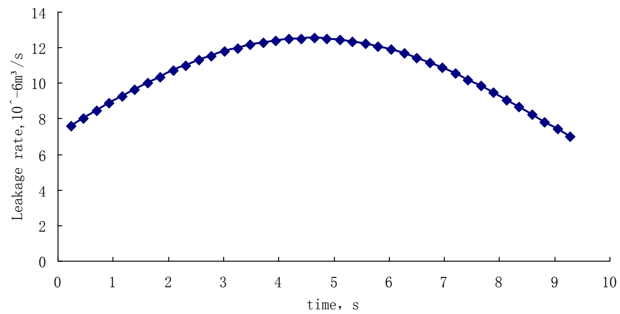

Figure 2. The leakage rate vs time in the upward moving process

From Figure 1 and Figure 2, we can take the conclusion that, higher pumping speed yields greater leakage rate. Accordingly, fast pumping would not reduce the leakage, but intensify the severity of leakage.

This conclusion can prove those popular standpoints in some of China research institutes that increasing the pumping rate can reduce the leakage are wrong. Meanwhile, this conclusion can explain the phenomenon in the field that for those high pump depth wells with low formation energy, increasing pumping speed only cannot increase the production rate in proportion.

3.2. The relationship between leakage rate and pump depth

Figure 3 shows the relationship between the pump depth and pump leakage rate calculated by the new model. We can see the leakage rate increases with the pump depth for fixed pumping speed and stroke. That is because the pressure difference of fluid weight is constant, though the $P_I(t)$ in the Eq.4, the dynamic pressure generated by acceleration of the overlying fluid on the plunger increases with the vibration amplitude of the plunger when the length of rod string is longer. Therefore, the pump discharge coefficient depends mainly on leakage. The deeper pump is located, the less pump discharge coefficient is yielded when the fluid pressure at the pump depth is higher than saturation pressure. Based on this, we can explain why the production rate no longer increases, even decreases sometimes, as the pump is located deeper than a certain depth, below which the pressure is higher than the saturation pressure of oil phase. Of course, maybe we shouldn't doubt the stroke loss is the main reason to explain that, and we just want to present the leakage is also a positive reason for that.

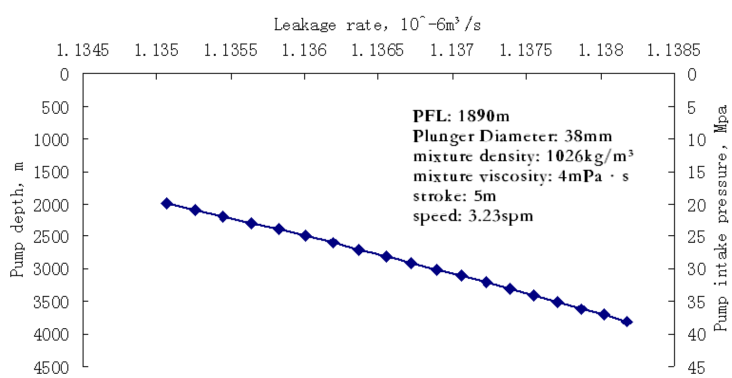

Figure 3. Leakage rate vs pump depth

4. Comparison of the New Leakage Model with the Test Results

Ling Ding [8] has put forward a set of laboratory rod pump lifting simulation test system, which can accurately measure the annular leakage rate without influenced by other factors. Thus, in this paper, a comparison was made between the test result and the calculation with the new leakage model. Also, the difference of results through different previous models and result with the new model is presented. When Ling Ding published her experiment results, there is no specific numbers of pump clearances, so in this paper we took the number according to the most commonly used pump with Grade 1 clearance (0.02mm~0.07mm) and in the practical calculation; we take an average of 0.0045mm as the number of pump clearance. The models used are listed as follows:

Method 1: the model in reference [13]

Method 2: the model in reference [5]

Method 3: the new model.

The results are shown in table 2.

Table 2 shows that: 1) the results from the new proposed model are closest to the actual test results with the average relative error of about 5%. 2) The trend of the leakage rate changing with pumping speed and stroke in the new model is consistent with the one in the test.

We have to admit that compared with this method, the error of method 1 is very small (7.4%), which shows higher accuracy than method 2 (13.9%). In addition, taking the uncertainty of pump clearance value into consideration, the experimental results cannot prove the accuracy of method 2 is higher than method 1 in the overall error. However, the variation of the experimental results of this model does match that of the experimental results when pumping rate varies, which can prove the superiority of this model in some aspects.

Table 2. Comparison of the experimental result of rod pump leakage simulation and the calculated results with different leakage models

stroke m	speed spm	Leakage rate in test, $10^{-6}m^3/s$	Leakage rate in method 1 $10^{-6}m^3/s$	Relative error %	Leakage rate in method 2 $10^{-6}m^3/s$	Relative error %	Leakage rate in new method $10^{-6}m^3/s$	Relative error %
1	3	6.65	7.76	16.7	6.08	8.6	6.87	3.3
1	5	7.4	7.76	4.9	6.21	16.1	7.53	1.7
1	7	7.5	7.76	3.5	6.34	15.5	8.23	9.7
2	3	7.42	7.76	4.6	6.28	15.4	7.84	5.3
Average error				7.4		13.9		5

*6.8MPa, the pressure difference across the plunger which is constant
*A plunger with 44mm diameter is used;
*Pure water is used in the pump barrel.

It should be pointed out that this experiment cannot prove the influence of pump depth on pump leakage. It is a pity that in the field we could not find any methods to accurately measure the pump leakage when the pump depth is too deep. Just like the case used in the Chapter 2, it’s a real well coming from Tarim field in China. Although by utilizing this model we can interpret some disciplinary phenomena in the field, we fail to verify this model only using such parameters acquired in the surface as yield, pump depth, length of stroke and pumping rate. it is impossible to certify this pump leakage model by utilizing field data because there are still many factors, such as gas impact, crude oil volume factor and stroke loss, cannot be ignored in deep well. So it is regret of this paper that the influence of pump depth on pump leakage may remain only on theoretical study level for the time being.

5. Conclusions

1) A new eccentric annular leakage model for rod pumps with Couette-Poiseuille flow is proposed, which considers not only the effect of dynamic pressure yielded by the overlying liquid on the plunger, but also the correct direction determination for the Couette flow leakage.

2) The analysis and calculation showed that the dynamic liquid pressure has significant effects on the leakage rate. Ignoring of that will result in underestimation of the leakage rate.

3) A rigorous mathematical model is derived in this paper to prove the Couette flow leakage has the same direction as that for Poiseuille flow leakage.

4) Pumping speed is a non-ignorable factor that has great influence on pump leakage. The pumping speed affects the pump loss through affecting differential pressure around plunger piston and affecting shearing leakage. The faster the pumping speed, the severer the leakage. Those popular standpoints that increasing the pumping rate can reduce the leakage are wrong. This conclusion is consistent with the field observations. And by comparing with the experimental data, the model presented in this paper is proved to be more accurate than the previously published leakage models.

5) The leakage rate increases with the pumping depth. The plunger. The pump depth influences piston's movement and the movement of plunger piston can cause the differential pressure of up and down of the piston, which is how pump depth have effect e on the pump leakage. This conclusion is also consistent with the field observations.

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Appendices

1. Effect of Dynamic Pressure

When plunger is rising up, the differential pressure between upper side and lower side plunger will cause the Poiseuille flow leakage. And the differential pressure can be express as:

$$\Delta P(t) = P_{Lup} + P_I(t) - P_{Ldown} \qquad \text{(a1-1)}$$

Where $\Delta P(t)$ is the difference of the static pressure of overlying fluid on the plunger and pump intake pressure. $P_I(t)$is the dynamic pressure generated by acceleration of the overlying fluid on the plunger moving with the plunger. P_{Lup} is the pressure of liquid column over the plunger; P_{Ldown} is the pressure below the plunger. ΔP_L denotes the difference of static pressure of overlying fluid on the plunger and pump intake pressure, that is $\Delta P_L = P_{Lup} - P_{Ldown}$. And the Eq.(a1-1) can be expressed as follow:

$$\Delta P(t) = \Delta P_L + P_I(t) \qquad \text{(a1-2)}$$

When ignoring the resistance of the fluid as being taken into pump barrel, the variation of pressure difference across the plunger $\Delta P(t)$ equals to the variation of pressure caused by the fluid over the plunger, which is composed of two parts: the weight of the fluid column and the pressure fluctuation when liquid over the plunger moves with the plunger.

This paper uses the following method to calculate the latter part, $P_I(t)$,which is replaced with $P(H_L,t)$ for expressing more clearly.

When plunger accelerates upward, the flow field and pressure field around the plunger and the inside wall of pump will fluctuate. HL is pump depth. Assume the elastic compressibility of liquid can be ignored, Δs is the distance of the pressure wave propagating in Δt after the movement of plunger alters. Accordingly, based on the momentum principle, we can get:

$$[\overline{P}(x_1,t) - \overline{P}(x_2,t)]A\Delta t = \rho A \Delta s[\overline{v}(x_1,t) - \overline{v}(x_2,t)] \qquad \text{(a1-3)}$$

Where $\overline{P}(x,t)$, $\overline{v}(x,t)$ are respectively average pressure and average velocity at x during t~(t+Δt), ρ is the density of mixture near the plunger; $x_1 = H_L$, $x_2 = H_L - \Delta s$.

Consider the well liquid as elastic liquid, the above equation can be simplified as following:

$$\begin{cases} \overline{P}(x_2,t) \approx \overline{P}(x_1,t-\Delta t) \\ \bar{v}(x_2,t) \approx \bar{v}(x_2,t-\Delta t) \\ \overline{P}(x_1,t) - \overline{P}(x_1,t-\Delta t) \approx P(x_1,t) - P(x_1,t-\Delta t) \\ \bar{v}(x_1,t) - \bar{v}(x_1,t-\Delta t) \approx v(x_1,t) - v(x_1,t-\Delta t) \end{cases} \quad \text{(a1-4)}$$

Where $\overline{P}$(x, t-Δt), $\bar{v}$(x, t-Δt) are respectively average pressure and average velocity at x during (t-Δt)~t; Combine with eqs. a1-3. a1-4,we can get the equation as follows:

$$[P(H_L,t) - P(H_L,t-\Delta t)]A\Delta t = \rho A \Delta s [v(H_L,t) - v(H_L,t-\Delta t)] \quad \text{(a1-5)}$$

That is,

$$[P(H_L,t) - P(H_L,t-\Delta t)] = \frac{\rho \Delta s [v(H_L,t) - v(H_L,t-\Delta t)]}{\Delta t} \quad \text{(a1-6)}$$

Define c=Δs/Δt, which denotes propagating velocity of pressure. Equ. a1-6 can be written as follows:

$$P(H_L,t) - P(H_L,t-\Delta t) = \rho c [v(H_L,t) - v(H_L,t-\Delta t)] \quad \text{(a1-7)}$$

Initial conditions: as t is zero, that is plunger begins to move, both the velocity and the dynamic load are zero, namely:

$$\begin{cases} P(H_L,0) = 0 \\ v(H_L,0) = 0 \end{cases} \quad \text{(a1-8)}$$

Combining with equation (a1-7) and (a1-8), the distribution of dynamic load of liquid over the plunger verses t can be determined with iterative method. If high accuracy is not required, $P(H_L,t)$ could be also calculated by the following equation:

$$P(H_L,t) = \rho c v(H_L,t) \quad \text{(a1-9)}$$

Where $v(H_L,t)$is the velocity of liquid film adhering to the plunger, which equals to the velocity of plunger. It could be calculated with the on dimensional wave equation as follows:

$$v(t) = \frac{\partial u(L,t)}{\partial t} = -wO(L)\sin wt + wP(L)\cos wt \quad \text{(a1-10)}$$

Where O (L), P (L) are two relevant parameters about unit suspended spot movement and casing program. The details for eq. a1-12 are given in the Appendix 3. $P_I(t)$ can be determined by solving Eqs. (a1-7), (a1-8) and (a1-10) together during the plunger moving upward. The results are shown in Fig.A1.1. We can find that the ΔP inferred from Eq. a1-2. is 10% higher with $P_I(t)$ than without that for the given parameters. Therefore, ignoring the pressure of the dynamic load might cause unacceptable errors .

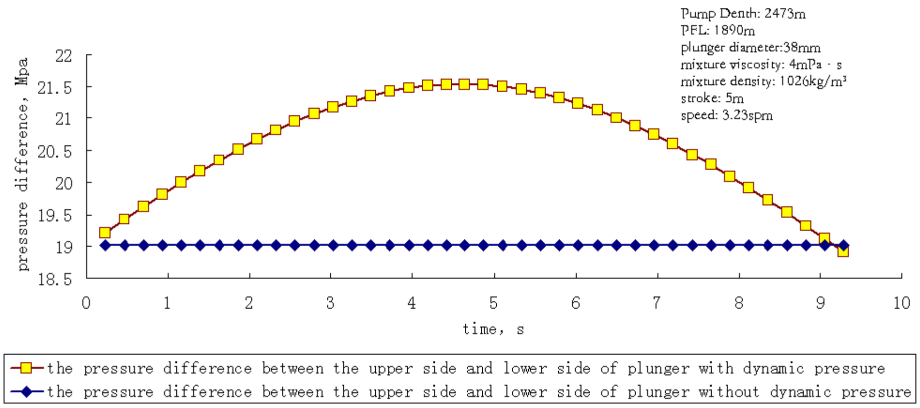

Figure A1.1. The comparison of the pressure difference between the upper side and lower side plunger in the plunger upward moving with dynamic pressure and without dynamic pressure

2. Couette Flow Leakage Model with the Pump Barrel Moving Relatively to the Plunger

The Couette flow rate can be expressed as[11]:

$$q_{vl} = \pm \frac{\pi D \delta v}{2} \tag{a2-1}$$

When the boundary of aperture moves in opposite directions, the shear flow is generated, *i.e.*, the Couette flow. The flow and moving boundary share the same direction[11]. If the aperture simultaneously has Poiseuille flow, when the pressure difference direction and the movement directions are consistent, Eq. a2-1. has positive sign, and negative sign when the directions are opposite. In the rod pump, the plunger external surface and the inside wall of pump barrel consist the annular crack boundary. For the traditional Couette flow leakage rate, Eq. a2-1 takes negative sign, and the plunger moving direction is opposite to the pressure difference across the annular crack [12]. That sounds reasonable, since the plunger is moving from down to up and the direction of pressure difference is downward. However, the direction of the movement velocity v and Couette flow rate q_{v1} in Eq. a2-1 are calculated using the same reference, which is the static boundary. And the leakage rate of pump is only meaningful when it takes the plunger as reference. If we take the plunger as a static boundary, the pump barrel is moving in the same direction as the pressure gradient across the plunger which causes the Poiseuille flow. As a result of this, the Couette flow sign should be positive in Eq. a2-1. The direction determination during this process will be further explained.

S is inner cross sectional area of the pump barrel, S_p is the cross sectional area of plunger, S_e is the annular area, their relationship is as follows:

$$S = S_p + S_e \tag{a2-2}$$

The theoretical production rate qt can be expressed as follows:

$$q_t = S \int_{t_1}^{t_2} v dt \tag{a2-3}$$

Where t_1、t_2 respectively is the time when the plunger starts to move up and stops at the highest point. v is the velocity at any arbitrary time between t_1~t_2.

Ignoring Poiseuille flow leakage, assume that the positive direction is upwards. So the production rate considering leakage q can be expressed as follow:

$$q = S_p \int_{t_1}^{t_2} v dt + S_e \int_{t_1}^{t_2} v_x dt \tag{a2-4}$$

Where v_x is the average velocity in the annulus at any arbitrary time range from t_1 to t_2. The velocity of aperture shear flow at the static boundary is zero, and the velocity of fluid at the moving boundary equals to the velocity of the moving boundary. For the Newtonian fluid, the velocity of Couette flow distributes evenly in the aperture, thus $v_x \approx v/2$, whose direction is same as that of the plunger. The actual leakage rate q_{vl} can be obtained by subtracting leakage q from the theoretical production rate q_t:

$$\begin{aligned} q_{vl} &= q_t - q \\ &= S\int_{t_1}^{t_2} v dt - S_p \int_{t_1}^{t_2} v dt - S_e \int_{t_1}^{t_2} v_x dt \\ &= S_e \int_{t_1}^{t_2} (v - v_x) dt \end{aligned} \tag{a2-5}$$

As $v \geqslant v_x$, $q_{vl} \geqslant 0$ and $q_t \geqslant q$. So we can conclude that the direction of Couette leakage rate is downward and opposite to the direction of plunger, yielding the following equation to calculate the Couette loss:

$$q_{vl} = +\pi D \delta v^{'} \tag{a2-6}$$

Where $v' = v - v_x$, the physical significance is the fluid relative velocity in the crack to plunger, the positive sign representsthe reverse direction of plunger velocity. At the same time, based on the $v_x \approx v/2$, we can get $v' = v/2$, that is equal to v_x, so the Couette flow rate in rod pump should be in the following form:

$$q_{vl} = +\frac{\pi D \delta v}{2} \tag{a2-7}$$

Although the v equals to velocity of plunger, its actual meaning should be the relative velocity to the pump barrel. v can be calculated by equation (a2-8):

$$v(t) = \frac{\partial u(L,t)}{\partial t} = -wO(L)\sin wt + wP(L)\cos wt \tag{a2-8}$$

Equation (a2-8) is the solution of one dimensional wave equation, and the detailed deriving process is given in the Appendix 3.

3. The Velocity and Acceleration Calculation of the Plunger in the Upward Moving Process

Plunger movement, driven by the pumping units via rod strings, is consist of two parts: the movement driven by the pumping units and the vibration caused by the rod string itself. According to Gibbs [12], we can use one-dimensional wave equation to describe the behavior of the rod string. The movement of the plunger can be considered as the motion of the end of the rod string, which can be specified as follows:

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial t^2} = a^2 \dfrac{\partial^2 u(x,t)}{\partial x^2} - c\dfrac{\partial u(x,t)}{\partial t} \\ u(0,t) = \phi(t) \\ u(x,0) = \psi(x) \\ \dfrac{\partial u(\mathrm{x},0)}{\partial x} = 0 \\ \dfrac{\partial u(L,t)}{\partial x} = \dfrac{W_t}{EA} \end{cases} \quad \text{(a3-1)}$$

Where u(x, t) is the displacement of the x point in the rod string; a is the propagation velocity of pressure wave in sucker rod, $a = \sqrt{E/\rho}$, c is Viscous damping coefficient, whose value can be taken by the equ.a-2 when the sucker rod temporarily are regarded as rigid body. [10]

$$\begin{cases} c = \dfrac{2\pi\mu}{\rho_r f_r}\left[\dfrac{1}{\ln m} + \dfrac{4}{B_2}(B_1+1)^2\right] \\ m = \dfrac{D_t}{D_r} \\ B_1 = \dfrac{m^2-1}{2\ln m} - 1 \\ B_2 = m^4 - 1 - \dfrac{(m^2-1)}{2\ln m} \end{cases} \quad \text{(a3-2)}$$

Ψ(x) is the polished rod load function; and Φ(x) is the polished rod displacement function, which is determined by the geometry of the pumping unit and the torque-speed characteristics of its prime mover [12] . In this paper, we simplified it as harmonic movement:

$$\phi(t) = s(1-\cos wt) \quad \text{(a3-3)}$$

Where s is the stroke of polish rod, w is the crank angular velocity of beam unit, $w = \pi n / 30$; The equation set a3-1 can be solved as follows by combining eqs. a3-2,a3-3:

$$\begin{cases} u(x,t) = \psi(x) + \frac{\gamma_0}{2} + O(x)\cos wt + P(x)\sin wt \\ O(x) = \wp_1 ch\beta x \sin\alpha x + (\wp_1 sh\beta x + \nu_1 ch\beta x)\cos\alpha x \\ P(x) = \wp_1 sh\beta x \cos\alpha x + (\wp_1 ch\beta x + \nu_1 sh\beta x)\sin\alpha x \\ \gamma_0 = 2s \\ \wp_1 = \frac{sMwa\sqrt{1+\sqrt{1+(\frac{c}{w})^2}}}{Ef_r\sqrt{2\left[1+(\frac{c}{w})^2\right]}} \\ \nu_1 = -s \\ \alpha = \frac{w}{a\sqrt{2}}\sqrt{1+\sqrt{1+(\frac{c}{w})^2}} \\ \beta = \frac{w}{a\sqrt{2}}\sqrt{-1+\sqrt{1+(\frac{c}{w})^2}} \end{cases} \quad (a3\text{-}4)$$

Partially differentiate u(x,t) with respect to t:

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = -wO(x)\sin wt + P(x)w\cos wt \\ \frac{\partial^2 u(x,t)}{\partial t^2} = -w^2 O(x)\cos wt - P(x)w^2 \sin wt \end{cases} \quad (a3\text{-}5)$$

Given the length of the rod string as L, the functions of velocity and acceleration of the plunger can be expressed as follow:

$$v(t) = \frac{\partial u(L,t)}{\partial t} = -wO(L)\sin wt + wP(L)\cos wt \quad (a3\text{-}6)$$

$$a = \frac{\partial^2 u(L,t)}{\partial t^2} = -w^2 O(L)\cos wt - P(L)w^2 \sin wt \quad (a3\text{-}7)$$

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