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The Impact of Imperfect Channel Estimation on Adaptive Power and Rate DS/CDMA Communications

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Abstract. We consider a combined power and rate adaptation scheme in direct-sequence code division multiple access (DS/CDMA) communications over Nakagami fading channels, where the transmission power and the data rate are jointly adapted relative to channel variations. The transmission power proportional to G_i^p is allocated to the data transmission of user i, where G_i is the channel power gain for user i, and p is a real number, and the data rate is simultaneously adapted so that a desired transmission quality is maintained. We analyze the effect of imperfect channel estimation on the average data rate of the joint adaptation scheme.

Keywords: DS/CDMA, adaptive power and rate, estimation error, Nakagami.

1 Introduction

When the transmitter is provided with channel state information (CSI), the transmission schemes can be adapted to this information, enabling more efficient use of the channel. For code-division multiple-access (CDMA) cellular systems, a power adaptation is employed to maintain the received power of each mobile at a desired level [1]. An optimal power adaptation with BPSK signaling with average and peak power constraints was considered in [2], and power adaptation for CDMA systems with successive interference cancellation receivers was examined in [3]. The power adaptation, however, requires a large amount of transmission power to compensate for deep fades. It was shown in [4] that the rate adaptation with a fixed transmission power provides a higher average data rate than the power adaptation with a fixed data rate, when the average transmission power and quality-of-service (QoS) requirements are identical. An optimal rate adaptation scheme with perfect power control was considered in [5] to maximize the throughput performance.

In this paper, we consider a joint power and rate adaptation scheme in uplink DS/CDMA communications over fading channels. The transmission power

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proportional to G_i^p is allocated to the data transmission of user i, where G_i is the channel gain for user i, and p is a real number. The data rate is simultaneously adapted such that a desired QoS can be attained. Power allocation having positive value of p can be interpreted as the same context of *water-filling*, while negative value of p indicates the transmission power is allocated inversely proportional to the channel gain. We note that the proposed joint power and rate adaptation scheme reduces to rate only adaptation when $p = 0$ and power only adaptation when $p = -1$. We analyze the average data rate that the joint adaptation scheme provides, subject to an average transmission power constraint and a maximum transmission power limit. We also discuss the effect of imperfect channel estimation on the average data rate.

The paper is organized as follows. In Section 2, we introduce the system model and analyze the average date rate in general. In Section 3, we consider a joint power and rate adaptation scheme. The effects of channel estimation errors on the performance of the proposed adaptation scheme are described in Section 4. Finally, conclusions are made in Section 5.

2 System Model and Analysis

We consider an uplink DS/CDMA communication system with binary phase-shift-keying (BPSK) signaling. We assume that the channel variation due to fading is slow relative to the bit duration, and the channel is frequency-selective with respect to the spreading bandwidth. We further assume that the channel fading is characterized by the Nakagami-m probability density function (pdf). The Nakagami-m fading model fits experimental data from a variety of fading environments, including urban and indoor multipath propagation [6]. We assume that there are K users in the system, and each non-reference-user signal is misaligned relative to the reference signal by an amount τ_k, $k = 1, 2, \cdots, K$, which is uniformly distributed over a bit interval.

The bit energy-to-equivalent noise spectral density ratio E_b/N_e at the L–finger RAKE receiver output for user i is given by [7]

$$E_b/N_e = \frac{G_i S_i T_i}{\sum_{k=1,k\neq i}^{K} 2G_k S_k T_c/3 + N_0} \tag{1}$$

where

$$G_i \triangleq \sum_{l=1}^{L} G_{i,l}$$

and $G_{i,l}$ is the channel power gain due to multipath fading for user i on the lth path. S_i and T_i are the transmission power and the bit duration for user i, respectively. T_c is the chip time, and N_0 is the one-sided power spectral density of the background noise. We assume that $\{G_{k,l}\}$ are independent random variables with

$$E[G_{k,l}] = \Omega_o \, e^{-\delta(l-1)} \tag{2}$$

where δ reflects the rate at which decay occurs. The pdf of G_i is given by [8]

$$P_{G_i}(g) = \left(\frac{m_g}{\Omega_g}\right)^{m_g} \frac{g^{m_g-1}}{\Gamma(m_g)} e^{-\frac{m_g}{\Omega_g} g} \tag{3}$$

where

$$\Omega_g = \Omega_o \sum_{l=1}^{L} e^{-\delta(l-1)} \,, \tag{4}$$

$$m_g = \frac{m(\sum_{l=1}^{L} e^{-\delta(l-1)})^2}{\sum_{l=1}^{L}(e^{-\delta(l-1)})^2} \,, \tag{5}$$

and

$$\Gamma(m) \triangleq \int_0^\infty t^{m-1} e^{-t} dt \,, \qquad m > 0 \,. \tag{6}$$

It follows from (1) that in order to maintain an adequate transmission quality, the data rate $R_i \triangleq 1/T_i$ [bits/sec] and the transmission power S_i of user i should be given by

$$R_i = \frac{1}{(E_b/N_e)_o} \cdot \frac{G_i S_i}{\sum_{\substack{k=1 \\ k\neq i}}^{K} 2G_k S_k/(3R_c) + N_0} \tag{7}$$

where $(E_b/N_e)_o$ is the value required for adequate performance of the modem and decoder, and $R_c \triangleq 1/T_c$ is the chip rate. Typically, $(E_b/N_e)_o$ depends on its implementation, use of error correcting coding, channel impairments such as fading, and error rate requirements. Then, the average data rate $\bar{R}_i$ is given by

$$\bar{R}_i = \frac{E[G_i S_i/S_T]}{(E_b/N_e)_o} \cdot E\left[\frac{1}{I}\right] \tag{8}$$

where

$$I \triangleq \sum_{\substack{k=1 \\ k\neq i}}^{K} 2G_k S_k/(3S_T R_c) + N_0/S_T \tag{9}$$

and S_T is the average transmission power at mobile unit. $E[1/I]$ can be expressed by

$$\begin{aligned} E\left[\frac{1}{I}\right] &= \int_{N_0/S_T}^{\infty} \frac{1}{x} P_I(x) dx \\ &= \frac{3R_c}{4\pi} \int_{N_0/S_T}^{\infty} \frac{1}{x} \int_{-\infty}^{\infty} \varphi^{K-1}(\omega)\, e^{-j\omega 3R_c(x-N_0/S_T)/2}\, d\omega\, dx \end{aligned} \tag{10}$$

where $P_I(x)$ is the pdf of I, and $\varphi(\omega)$ is the characteristic function of $G_k S_k/S_T$. However, since $1/I$ is a *convex* $\cup$ function, a lower bound on the average data rate can be obtained by using the *Jensen's inequality* [9]:

$$\begin{aligned} \bar{R}_i &\geq \frac{E[G_i S_i/S_T]}{(E_b/N_e)_o} \cdot \frac{1}{E[I]} \\ &= \frac{1}{(E_b/N_e)_o} \cdot \frac{1}{2(K-1)/(3R_c) + N_0/(S_T E[G_i S_i/S_T])} \,. \end{aligned} \tag{11}$$

3 Joint Power and Rate Adaptation

We consider adapting the transmission power and the data rate together relative to the channel gain G_i. When the channel gain is below a certain cutoff fade depth γ_0 ($G_i < \gamma_0$), no transmission power is allocated to the data transmission (i.e. the data transmission is suspended); otherwise, the transmission power is allocated in proportion to G_i^p, where p is a real number, and the data rate is simultaneously adapted such that $(E_b/N_e)_o$ can be attained. As p increases, this adaptation makes more transmission power allocated to better channels. We will call this joint adaptation scheme *variable rate and power adaptation*. We assume that transmitters are subject to a maximum transmission power limit of S_{max}. Then, the transmission power of the variable rate and power adaptation is given by

$$S_i/S_T = \begin{cases} 0, & G_i < \gamma_0 \\ \min(c_p G_i^p, \frac{S_{max}}{S_T}), & G_i \geq \gamma_0 \end{cases} \quad (12)$$

for some constant c_p.

To meet a fixed average transmission power S_T,

$$E[S_i/S_T] = \begin{cases} c_p \int_{\gamma_0}^{\gamma_p} g^p P_{G_i}(g)dg + \frac{S_{max}}{S_T} \int_{\gamma_p}^{\infty} P_{G_i}(g)dg, & p \geq 0 \\ \frac{S_{max}}{S_T} \int_{\gamma_0}^{\max(\gamma_0,\gamma_p)} P_{G_i}(g)dg + c_p \int_{\max(\gamma_0,\gamma_p)}^{\infty} g^p P_{G_i}(g)dg, & p < 0 \end{cases} \quad (13)$$

should be 1, where

$$\gamma_p \triangleq \left(\frac{S_{max}}{c_p S_T} \right)^{\frac{1}{p}}. \quad (14)$$

There is no general closed-form solution for c_p; however, it can be determined using numerical techniques. We use the *Newton method* [10] to find c_p in our work. Once c_p is known, $E[G_i S_i/S_T]$ is given by

$$\begin{aligned} E[G_i S_i/S_T] &= \int_{\gamma_0}^{\infty} \min(c_p g^p, \frac{S_{max}}{S_T}) g P_{G_i}(g)dg \\ &= \begin{cases} c_p \{f_{p+1}(\gamma_0) - f_{p+1}(\gamma_p)\} + \frac{S_{max}}{S_T} f_1(\gamma_p), & p \geq 0 \\ \frac{S_{max}}{S_T} \{f_1(\gamma_0) - f_1(\max(\gamma_0,\gamma_p))\} + c_p f_{p+1}(\max(\gamma_0,\gamma_p)), & p < 0 \,. \end{cases} \end{aligned} \quad (15)$$

A lower bound on $\bar{R}_i$ can be obtained by substituting (15) into (11). Fig. 1 is a plot of the average data rate versus S_T/N_0 for several values of p, which indicates that the lower bound is very close to the exact value.

4 Imperfect Channel Estimation

In this section we consider the effect of imperfect channel estimation on the average data rate of the variable rate and power adaptation scheme. Imperfect

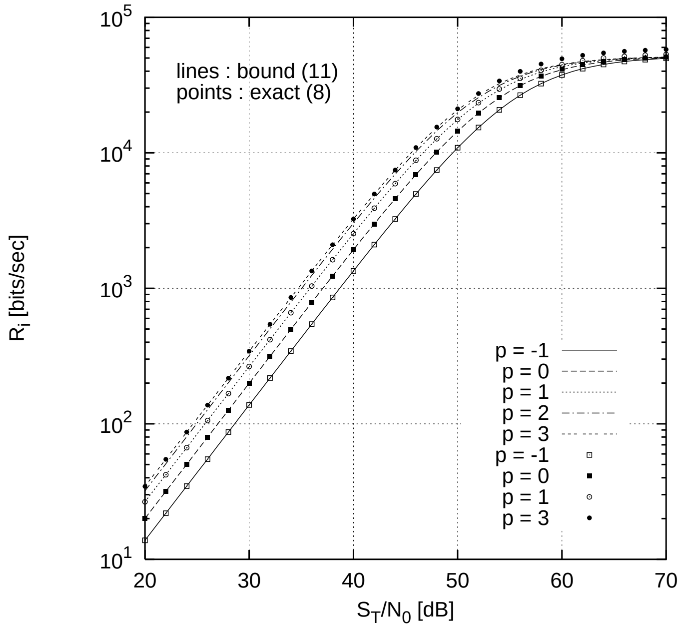

Fig. 1. The average data rate $\bar{R}_i$ versus S_T/N_0; $K = 30$, $\gamma_0 = 0.1$, $S_{max}/S_T = 5$, $(E_b/N_e)_o = 7$[dB], $L = 3$, $m = 1$, $\delta = 0.5$, $\Omega_g = 1$, $R_c = 5M$.

channel estimation will cause the estimated channel fading to differ from the actual fading, inducing thereby a performance degradation.

If we let $\hat{G}_{i,l}$ be the estimate of the channel power gain $G_{i,l}$, then the RAKE receiver for user i forms a decision statistic $\hat{Z}_i$

$$\hat{Z}_i = \sum_{l=1}^{L} \sqrt{\hat{G}_{i,l}} Z_{i,l}. \tag{16}$$

We assume that $\hat{G}_{k,l}$ and $G_{k,l}$ are jointly distributed gamma random variables, and the joint pdf is given by [8]

$$P_{\hat{G}_{k,l},G_{k,l}}(g,\hat{g}) = \frac{1}{(1-\rho)\rho^{(m-1)/2}\Gamma(m)} \left(\frac{m}{\Omega_l}\right)^{m+1} (g\hat{g})^{(m-1)/2} \times I_{m-1}\left(\frac{2m\sqrt{\rho g\hat{g}}}{(1-\rho)\Omega_l}\right) e^{-\frac{m(g+\hat{g})}{(1-\rho)\Omega_l}} \tag{17}$$

where $\rho \in [0,1]$ is the correlation coefficient between the estimated and actual fading. ρ is essentially a function of the employed channel estimation technique. One common way to estimate the channel is to use unmodulated pilot signal. When $\rho = 1$, there is perfect correlation between the estimated and actual fading. As ρ decreases, the correlation diminishes. In the limit as $\rho \to 0$, the fading estimate and its actual value are completely uncorrelated.

The bit energy E_b with the imperfect channel estimate is given by

$$E_b = \left(\sum_{l=1}^{L} \sqrt{\hat{G}_{i,l} G_{i,l}} \right)^2 S_i T_i, \tag{18}$$

and the equivalent noise spectral density $N_e/2$ is given by

$$N_e/2 = \sum_{l=1}^{L} \hat{G}_{i,l} \left[\sum_{\substack{k=1 \\ k \neq i}}^{K} G_k S_k/(3R_c) + N_0/2 \right]. \tag{19}$$

Consequently, the bit energy-to-equivalent noise spectral density ratio E_b/N_e is given by

$$E_b/N_e = \frac{(\sum_{l=1}^{L} \sqrt{\hat{G}_{i,l} G_{i,l}})^2 T_i S_i / S_T}{\sum_{l=1}^{L} \hat{G}_{i,l} \{ \sum_{\substack{k=1 \\ k \neq i}}^{K} 2 G_k S_k/(3 S_T R_c) + N_0/S_T \}}. \tag{20}$$

Notice that when $\hat{G}_{i,l} = G_{i,l}$, (20) reduces to (1). It follows from (12) and (20) that, in order to maintain E_b/N_e at a desired level $(E_b/N_e)_o$, the data rate R_i and power S_i should be related by

$$R_i = \begin{cases} \left(\frac{N_e}{E_b}\right)_o \left[\frac{(\sum_{l=1}^{L} \sqrt{\hat{G}_{i,l} G_{i,l}})^2 \min\left(c_p \hat{G}_i{}^p, \frac{S_{max}}{S_T}\right)}{\sum_{l=1}^{L} \hat{G}_{i,l} \sum_{\substack{k=1 \\ k \neq i}}^{K} 2(1-P_o) E\left[G_k \min\left(c_p \hat{G}_i{}^p, \frac{S_{max}}{S_T}\right)\right] / (3 S_T R_c) + N_0/S_T} \right], & \hat{G}_i \geq \gamma_0 \\ 0, & \hat{G}_i < \gamma_0 \end{cases} \tag{21}$$

where

$$\hat{G}_k \triangleq \sum_{l=1}^{L} \hat{G}_{k,l}. \tag{22}$$

In (21), we approximate the variance of multi-user interference term by its mean value. This approximation makes sense in an environment where the user population is large enough so that the interference is nearly at its average value by the *weak law of large numbers*. Then,

$$E\left[G_k \min\left(c_p \hat{G}_k{}^p, \frac{S_{max}}{S_T}\right)\right] = \underbrace{\int_{G_{k,1}} \int_{\hat{G}_{k,1}} \cdots \int_{G_{k,L}} \int_{\hat{G}_{k,L}}}_{2L} \left(\sum_{l=1}^{L} g_l \right)$$
$$\times \min\left(c_p (\sum_{l=1}^{L} \hat{g}_l)^p, \frac{S_{max}}{S_T}\right) \prod_{l=1}^{L} P_{G_{k,l}, \hat{G}_{k,l}}(g_l, \hat{g}_l)\, dg_1 d\hat{g}_1 \cdots dg_L d\hat{g}_L. \tag{23}$$

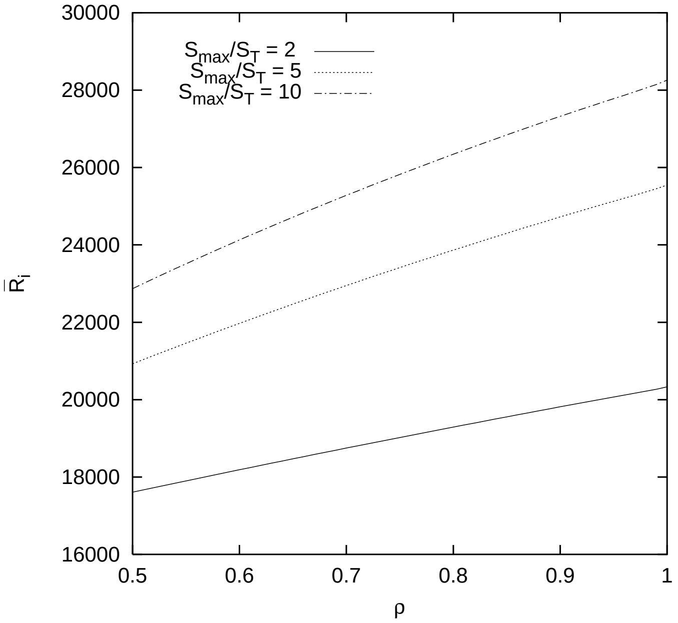

Fig. 2. Average data rate $\bar{R}_i$ versus ρ; $K = 30,\ p = 5,\ L = 1,\ m = 1,\ (E_b/N_e)_o = 7$[dB], $S_T/N_0 = 50$[dB] $\Omega_g = 1,\ R_c = 5M$.

Finally, the average data rate with the imperfect channel estimate is given by

$$\bar{R}_i = \frac{1}{(E_b/N_e)_o \cdot \bar{I}} \underbrace{\int_{G_{k,1}} \int_{\hat{G}_{k,1}} \cdots \int_{G_{k,L}} \int_{\hat{G}_{k,L}}}_{2L} \frac{(\sum_{l=1}^{L} \sqrt{g_l \hat{g}_l})^2}{\sum_{l=1}^{L} \hat{g}_l}$$
$$\times \min\left(c_p(\sum_{l=1}^{L} \hat{g}_l)^p, \frac{S_{max}}{S_T}\right) \prod_{l=1}^{L} P_{G_{k,l},\hat{G}_{k,l}}(g_l, \hat{g}_l)\ dg_1 d\hat{g}_1 \cdots dg_L d\hat{g}_L \quad (24)$$

where

$$\bar{I} \triangleq \sum_{\substack{k=1 \\ k \neq i}}^{K} 2(1 - P_o) E\left[G_k \min\left(c_p \hat{G}_i^p, \frac{S_{max}}{S_T}\right)\right] / (3 S_T R_c) + N_0/S_T. \quad (25)$$

In Fig. 2, we plot the average data rate versus the correlation coefficient ρ for several values of S_{max}/S_T. We note that the imperfect channel estimation degrades the attainable average data rate more significantly as S_{max}/S_T increases. For higher S_{max}/S_T, the dynamic range of transmission powers becomes larger, yielding a mis-allocation of the transmission power even for a small deviation of channel estimation. Fig. 2 indicates that more accurate channel estimation is required for systems with higher peak-to-average power ratios.

5 Conclusion

We considered a joint power and rate adaptation scheme in DS/CDMA communications, where the transmission power and the data rate are simultaneously adapted relative to channel variations. We analyzed the average data rate that meets adequate QoS requirements, subject to an average transmission power constraint and a maximum transmission power limit. We also discussed the effect of channel estimation error on the proposed adaptation scheme.

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