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Quad-Rotor Airship Modeling and Simulation Based on Backstepping Control

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Abstract

Firstly, a novel structure of indoor quad-rotor airship is proposed to overcome the traditional indoor airship shortcomings such as poor balance and lack of buoyancy in this paper. The kinetic model is established according to the design structure and flight characteristics by the principle of Newtonian mechanics and the laws of physics. The force analysis on the quad-rotor airship, including gravity, buoyancy, aerodynamic, wind resistance, propeller force, etc. Secondly, binding Kinetics model, the outer loop position controller and the inner loop attitude controller are designed based on the PID control algorithm. The outer loop position controller includes instantaneous position and velocity, while the inner loop attitude controller includes body angular velocity, Euler angular velocity and Euler angles. The simulation platform is designed by using Matlab/Simulink. The simulation results demonstrate the correctness of the dynamic model and the effectiveness of the control method. Finally, according to the PID's effect on time-varying system and nonlinear system is not very well perfect, we adopted Lyapunov stability theorem and backstepping algorithm to design the controller and divides the whole system into two return circuits and each circuit are divided into three second-order systems which reduce order number of controller effectively. And the control algorithm achieved good results.

Keywords: *Quad-Rotor, airship, adaptive control, PID*

1. Introduction

Indoor airship is a kind of aerostats which is different from the high-altitude large airship. It is featured by small size, distinctive appearance, novel structure, low altitude and slow flight. Although there are many advantages with it, it also has some disadvantages such as poor balance, low stability and lacking of buoyancy. Quad-rotor airship(as shown in Figure 1) ,draw lessons from the structure of quad-rotor helicopter, eliminating the tail rudder, four rotors as direct source of power satisfied the airship rise, fall, inverted flight, as well as forward flight. Because of those structures, Quad-rotor airship is more compact, with greater lift force, flexible control, and it also has other advantages such as strong maneuverability, safety and reliability, flight stability, vertical take-off and landing, *etc.* [1].

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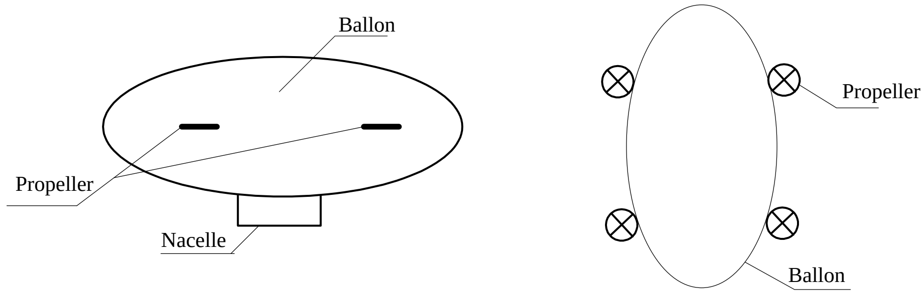

Figure 1. Schematic diagram of the Quad-Rotor airship

In recent years, as high altitude large airship has special strategic significance and wide application prospects [2], more and more countries have focused on the study. Thus Dynamic model was established, and the relevant control system was designed, too, but there is still few research data about indoor shaped airship. In the model data of blimp, the structure and control system are similar to the high altitude airship. The structure of the quad-rotor airship is different from the traditional airship but similar to the quad-rotor helicopter.

The paper takes the quad-rotor airship as the research object, according to the design framework and flight characteristics, drawing lessons from the modeling method of quad-rotor helicopter, Mechanism model is designed based on the Newtonian mechanics principle and the laws of physics. The outer loop position controller and the inner loop attitude controller are designed based on the PID control algorithm. According to the dynamic model of quad-rotor airship, a simulation platform is designed by using Matlab/Simulink. The simulation results demonstrated the correctness of dynamic model and the effectiveness of the control method. Finally, the PID's effect on time-varying system and nonlinear system is not very well, for which we designed a fuzzy adaptive PID controller, combining the fuzzy control theory with the adaptive control and PID control. And the controller achieved good results.

2. Quad-Rotor Airship Coordinate System Selection and Force Analysis

2.1. Selection of coordinate system

The geodetic coordinate system (G--frame) --also called inertial coordinate system--is used to determine the Quad-Rotor airship' relative position to the ground. We need to use the body coordinate system (B--frame) (as shown in Figure 2) to indicate Determination attitude angle of Quad-Rotor airship. The choice of coordinate system abiding by the Right-Hand Rule [3] is defined as follows:

The airship position information: $P=[X \quad Y \quad Z]^T$

The attitude angle vector: $A=[\phi \quad \theta \quad \psi]^T$

The instantaneous center velocity vector of the airship: $V=[v_x \quad v_y \quad v_z]^T$

The Quad-rotor airship's instantaneous velocity in B--frame : $\tilde{V}=[u \quad v \quad w]^T$

The angular velocity vector: $\Omega=[p \quad q \quad r]^T$

The body inertia matrix: $J=[I_x \quad I_y \quad I_z]^T$

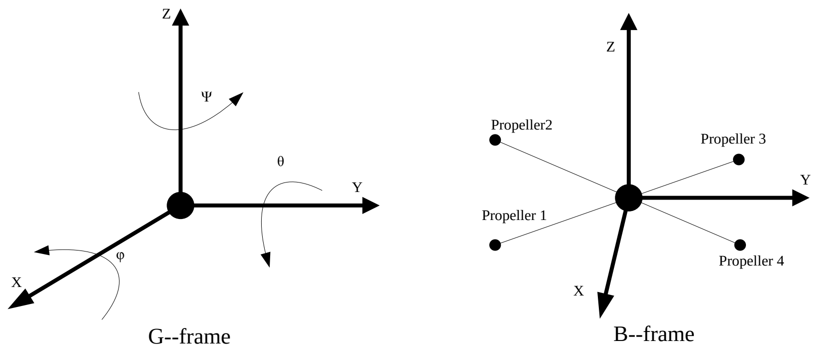

Figure 2. The G--frame and the B--frame

The following are the converted matrix T about the body coordinate system and the ground coordinate system:

$$T=\begin{pmatrix} \cos\theta\cos\psi & \cos\psi\sin\phi\sin\theta-\sin\psi\cos\phi & \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi \\ \sin\theta\sin\psi & \sin\theta\sin\phi\sin\psi+\cos\phi\cos\psi & \sin\theta\sin\psi\cos\phi-\cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{pmatrix} \quad (2.1)$$

2.2 The force analysis of the airship

Before analyzing the force on the quad-rotor airship, we generally make the following assumptions [4]:

The body of the quad-rotor airship is rigid, the quality of the airship keeps the same and the elastic effect is ignored;

2）The volume center of the airship coincides with the center of buoyancy;

3) The airship has a symmetry plane XOZ, which is the center of buoyancy and gravity, the product of the inertia $I_{xy}=I_{yz}=0$;

The following is the traditional dynamic model of the Quad-Rotor helicopter [5, 6]:

$$\begin{cases} \dot{P}=v \\ m\dot{v}=F_f-F_d-mge_3 \\ \Omega=T\dot{A} \\ J\dot{\Omega}+\Omega\times J\Omega=M_f-F_d+M_c \end{cases} \quad (2.2)$$

The most significant difference from the Quad-Rotor helicopter is that the Quad-Rotor airship has a pneumatocyst. To Charge lighter-than-air gas such as helium or hydrogen in the pneumatocyst can generate buoyancy. As the volume of the pneumatocyst is relatively large, factors including gravity, buoyancy, fluid inertia force, the wind’s resistance and the force of the propeller which play a major role in the performance of the system must be considered during the process of modeling.

2.2.1. Gravity and buoyancy: In the inertial frame, gravity and buoyancy are always perpendicular to the ground. So the gravity and the buoyancy can be combined into a single force

$$F=\begin{bmatrix}0 & 0 & H\end{bmatrix}^T, H=B-G \tag{2.3}$$

2.2.2. Fluid inertia and additional mass: When the airship is moving in the air, the air must be moved resulting from disturbances. If the movement of the airship changes with time, the motion of the air will also vary with time. The movement of the air is affected by the airship. Therefore, the force, which is generated by the air, acts on the airship in the same strength but at opposite direction. This is the fluid inertia force. The fluid inertia force related to the acceleration of the airship motion, is generally represented by additional mass. Additional quality m_{ij} can be understood as the additional mass, additional inertia and additional net torque in the direction j when the airship is moved by the unit of angular acceleration in the direction of i. The additional mass constantly takes positive value. When the value of the i and j is 1,2,3, represents moving along the x, y, z direction . When the value of i and j is 4,5,6, represents rotation of the x, y, z direction .Moving object in arbitrary shape can have 36 additional mass in total. The m_{ij} only depends on the shape of the object and the coordinate system. We can get the added mass matrix of the airship based on some references [7, 8]:

$$Mass=\begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{66} \end{bmatrix} \tag{2.4}$$

$m_{ij}=0(i\neq j)$. $m_{11}=0.05m_{air}$, $m_{22}=m_{33}=1.5m_{air}$, $m_{44}=0.05I_x$, $m_{55}=1.5I_y$, $m_{66}=1.5I_z$. m_{air} represents the weight of the air displaced by the airship.

By the principles of fluid mechanic, if the object flows in the infinity ideal environment, the turbulent kinetic energy is:

$$T=0.5\sum_{i=1}^{6}\sum_{j=1}^{6}m_{ij}v_iv_j\left(i,j=1,2,\ldots6\right) \tag{2.5}$$

Where, $v_1=u$, $v_2=v$, $v_3=w$, $v_4=p$, $v_5=q$, $v_6=r$ .Therefore, we can rewrite the above equation as：

$$T=0.5\left[m_{11}u^2+m_{22}v^2+m_{33}w^2+m_{44}p^2+m_{55}q^2+m_{66}r^2\right] \tag{2.6}$$

But the kinetic momentum of the fluid, the moment of momentum B_i and the kinetic energy T have the following relationships： $B_i=\dfrac{\partial T}{\partial v_i}\left(i,j=1,2,\ldots6\right)$ in which, $B=\begin{bmatrix}B_1 & B_2 & B_3\end{bmatrix}^T$ is the momentum, $K=\begin{bmatrix}B_4 & B_5 & B_6\end{bmatrix}^T$ is the moment of momentum.

writing out:

$$B_i = m_{ii} v_i \tag{2.7}$$

According to the principle of mechanics, Fluid inertia force F_I and moment M_I acting on the airship floating heart can be expressed as:

$$F_I = -\frac{d\tilde{B}}{dt} = \begin{bmatrix} -\frac{du}{dt} \\ -\frac{dv}{dt} \\ -\frac{dw}{dt} \end{bmatrix} = \begin{bmatrix} -m_{11}\dot{u} + m_{22}vr - m_{33}wq \\ -m_{22}\dot{v} - m_{11}ur + m_{33}wp \\ -m_{33}\dot{w} + m_{11}uq - m_{22}vp \end{bmatrix} \tag{2.8}$$

$$M_I = -\frac{d\tilde{K}}{dt} = -\begin{bmatrix} \frac{dp}{dt} \\ \frac{dq}{dt} \\ \frac{dr}{dt} \end{bmatrix} = \begin{bmatrix} -m_{44}\dot{p} + (m_{22} - m_{33})\ vw + (m_{55} - m_{66})qr \\ -m_{55}\dot{q} + (m_{33} - m_{11})\ uw + (m_{66} - m_{44})\ pr \\ -m_{66}\dot{r} + (m_{11} - m_{22})\ uv + (m_{44} - m_{55})\ pq \end{bmatrix} \tag{2.9}$$

In this system, the center of buoyancy and gravity are very close, thus we can assume $V \approx \tilde{V}$ [9].So the formula 8 and 9 can be rewritten as following:

$$F_I = -\frac{d\tilde{B}}{dt} = \begin{bmatrix} -\frac{dv_x}{dt} \\ -\frac{dv_y}{dt} \\ -\frac{dv_z}{dt} \end{bmatrix} = \begin{bmatrix} -m_{11}\dot{v}_x + m_{22}v_y r - m_{33}v_z q \\ -m_{22}\dot{v}_y - m_{11}v_x r + m_{33}v_z p \\ -m_{33}\dot{v}_z + m_{11}v_x q - m_{22}v_y p \end{bmatrix} = \begin{bmatrix} K_x \\ K_y \\ K_z \end{bmatrix} \tag{2.10}$$

$$M_I = -\frac{d\tilde{K}}{dt} = -\begin{bmatrix} \frac{dp}{dt} \\ \frac{dq}{dt} \\ \frac{dr}{dt} \end{bmatrix} = \begin{bmatrix} -m_{44}\dot{p} + (m_{22} - m_{33})\ v_y v_z + (m_{55} - m_{66})qr \\ -m_{55}\dot{q} + (m_{33} - m_{11})\ v_x v_z + (m_{66} - m_{44})\ pr \\ -m_{66}\dot{r} + (m_{11} - m_{22})\ v_x v_y + (m_{44} - m_{55})\ pq \end{bmatrix} = \begin{bmatrix} K_l \\ K_m \\ K_n \end{bmatrix} \tag{2.11}$$

2.2.3. The lift force and axial torque provided by the propeller: The lift force provided by the propeller makes the Quad-Rotor airship move. In the body coordinate system, the direction of the lift force remains constant. While in the inertial system, the driving force of lift is decomposed into three directions along the three coordinate axes. So in the ground coordinate system the lift is decomposed into:

$$F_f = R^{-1}\begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix} = \begin{bmatrix} C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\phi S_\theta S_\psi - C_\psi S_\phi \\ C_\phi C_\theta \end{bmatrix} \sum_{i=1}^{4} F_i \quad (2.12)$$

F_i is the lift provided by the i propeller. The expression is： $F_i = b * w_i^2$， b is the lift coefficient.

The propeller provides the torque, including roll, pitch and yaw torque:

$$M_f = \begin{bmatrix} l*(F_4 + F_3 - F_2 - F_1) \\ l*(F_3 + F_2 - F_4 - F_1) \\ d*(F_4 + F_2 - F_3 - F_1) \end{bmatrix} = \begin{bmatrix} l*b*(\omega_4^2 + \omega_3^2 - \omega_2^2 - \omega_1^2) \\ l*b*(\omega_3^2 + \omega_2^2 - \omega_4^2 - \omega_1^2) \\ d*(\omega_4^2 + \omega_2^2 - \omega_3^2 - \omega_1^2) \end{bmatrix} \quad (2.13)$$

While l is the distance from the center of propeller to the center of buoyancy, d is the torque coefficient.

The rotational resistance moment is:

$$M_d = K_f \Omega = \begin{bmatrix} K_{fx} p \\ K_{fy} q \\ K_{fz} r \end{bmatrix} \quad (2.14)$$

$K_f = diag(K_{fx} \quad K_{fy} \quad K_{fz})$ is the rotational resistance coefficient.

Gyro torque：

$$\Omega \times J\Omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_x p \\ I_y q \\ I_z r \end{bmatrix} = \begin{bmatrix} qr(I_z - I_y) \\ pr(I_x - I_z) \\ pq(I_y - I_z) \end{bmatrix} \quad (2.15)$$

2.2.4. Air resistance: The Quad-Rotor airship is more susceptible to the influence of air resistance during flight because of its large surface area. The air resistance is proportional to the translational speed of the airship, which is generally expressed as:

$$F_d = k_d v = diag(k_{dx} \quad k_{dy} \quad k_{dz})V \quad (2.16)$$

$k_d = diag(k_{dx} \quad k_{dy} \quad k_{dz})$ is the air resistance coefficient.

The relationship between Euler angles' rate of change and the body axis angular velocity is:

$$\begin{cases} \dot{\phi} = p + \left(r\cos\phi + q\sin\phi\right)\tan\theta \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = \dfrac{1}{\cos\theta}\left(r\cos\phi + q\sin\phi\right) \end{cases} \quad (2.17)$$

Spatial motion equations of the Quad-Rotor airship:

$$\begin{cases} \ddot{x} = \left(C_\phi C_\psi S_\theta + S_\phi S_\psi\right)\left(\frac{b}{m}\sum_{i=1}^{4}\omega_i^2\right) - \frac{k_{dx}}{m}\dot{x} - \frac{K_x}{m} \\ \ddot{y} = \left(C_\phi S_\theta S_\psi - C_\psi S_\phi\right)\left(\frac{b}{m}\sum_{i=1}^{4}\omega_i^2\right) - \frac{k_{dy}}{m}\dot{y} - \frac{K_y}{m} \\ \ddot{z} = C_\phi C_\theta\left(\frac{b}{m}\sum_{i=1}^{4}\omega_i^2\right) - \frac{k_{dz}}{m} + \frac{H}{m} - \frac{K_z}{m} \\ \dot{p} = qr\left(\frac{I_y - I_z}{I_x}\right) + \frac{l}{I_x}b\left(\omega_4^2 + \omega_3^2 - \omega_2^2 - \omega_1^2\right) - \frac{k_{fx}}{I_x}p - \frac{K_l}{I_x} \\ \dot{q} = pr\left(\frac{I_z - I_x}{I_y}\right) + \frac{l}{I_y}b\left(\omega_3^2 + \omega_2^2 - \omega_4^2 - \omega_1^2\right) - \frac{k_{fy}}{I_y}q - \frac{K_m}{I_y} \\ \dot{r} = pq\left(\frac{I_x - I_y}{I_z}\right) + \frac{d}{I_z}\left(\omega_4^2 + \omega_2^2 - \omega_3^2 - \omega_1^2\right) - \frac{k_{fz}}{I_z r} - \frac{K_n}{I_z} \end{cases} \tag{2.18}$$

Control input：

$$\begin{cases} C_1 = \omega_4^2 + \omega_3^2 + \omega_2^2 + \omega_1^2 \\ C_2 = \omega_4^2 + \omega_3^2 - \omega_2^2 - \omega_1^2 \\ C_3 = \omega_3^2 + \omega_2^2 - \omega_4^2 - \omega_1^2 \\ C_4 = \omega_4^2 + \omega_2^2 - \omega_3^2 - \omega_1^2 \end{cases} \tag{2.19}$$

Consequently：

$$\begin{cases} \ddot{x} = \left(C_\phi C_\psi S_\theta + S_\phi S_\psi\right)C_1 - \frac{k_{dx}}{m}\dot{x} - \frac{K_x}{m} \\ \ddot{y} = \left(C_\phi S_\theta S_\psi - C_\psi S_\phi\right)C_1 - \frac{k_{dy}}{m}\dot{y} - \frac{K_y}{m} \\ \ddot{z} = C_\phi C_\theta C_1 - \frac{k_{dz}}{m} + \frac{H}{m} - \frac{K_z}{m} \\ \dot{p} = qr\left(\frac{I_y - I_z}{I_x}\right) + \frac{l}{I_x}bC_2 - \frac{k_{fx}}{I_x}p - \frac{K_l}{I_x} \\ \dot{q} = pr\left(\frac{I_z - I_x}{I_y}\right) + \frac{l}{I_y}bC_3 - \frac{k_{fy}}{I_y}q - \frac{K_m}{I_y} \\ \dot{r} = pq\left(\frac{I_x - I_y}{I_z}\right) + \frac{d}{I_z}C_4 - \frac{k_{fz}}{I_z r} - \frac{K_n}{I_z} \\ p = \dot{\phi} - S_\theta\dot{\psi} \\ q = C_\phi\dot{\theta} + C_\theta S_\phi\dot{\psi} \\ r = -S_\phi\dot{\theta} + C_\phi C_\theta\dot{\psi} \end{cases} \tag{2.20}$$

From formula (20), the Quad-Rotor airship can be summarized as a nonlinear, coupled and under actuated system.

2.3 The Simulation Results

The inputs about the model of Quad-Rotor airship are the propellers' speed, and the outputs contain three position information and three Euler angle information as shown in (2.20).Based on the PID control algorithm, in order to realize the control of the position and the posture, the outer loop position controller and the inner loop attitude controller are designed (as shown in the Figure 3).The outer loop position controller includes position information such as instantaneous position, speed and so on. And the inner loop attitude controller includes the posture information like angular velocity, Euler angular velocity and Euler angle.

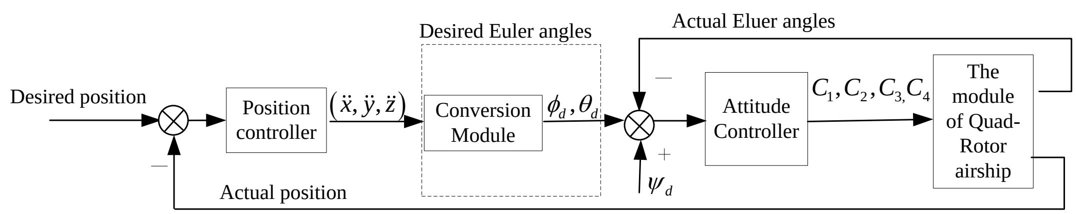

Figure 3. The overall diagram of airship system

Simulation platform:

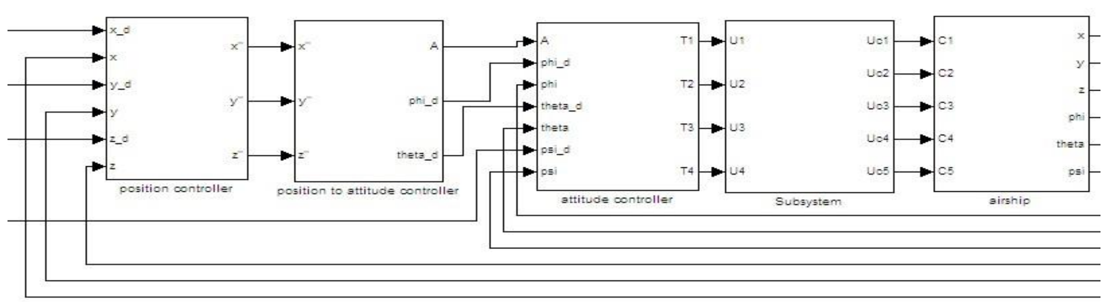

Figure 4. Simulation platform

From left to right are the outer loop position controller, the position attitude conversion module, the inner loop attitude controller, the model of motor, the model of Quad- Rotor airship.

PID controller structure is simple, achieving good, so we select PID controller. PID control algorithm is done by selecting the appropriate proportional coefficient, integral coefficient, differential coefficient to achieve the desired results. Therefore, the key to get the desired control effect is that coordinated the relationship of the coefficients. In this paper we choice the haalma algorithm [10] to tuning the PID parameters. Calculate the parameters before run in accordance with the parameters. Observing the effect, adjusted the parameters to achieve the satisfactory effect. The parameters as shown in Table 1:

Table 1. The parameters of the controller

Simulation time	80s
x ’s PID	0.6,0.0005,1.5
y ’s PID	1.3,0.002,1.5
z’s PID	3,0.16,5
ϕ ’s PID	0.05,0,3
θ ’s PID	1.5,0.00001,10
ψ ’sPID	0.0162,0.0001,0.02

The expected position information are (8,7,6),the desired yaw angle is 0.5, the simulation time is 80 s. The simulation results are shown as follows:

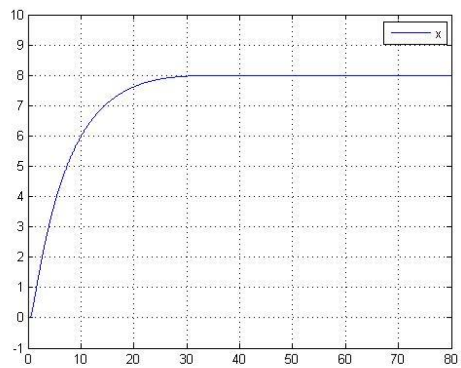

Figure 5. The simulation diagram of X

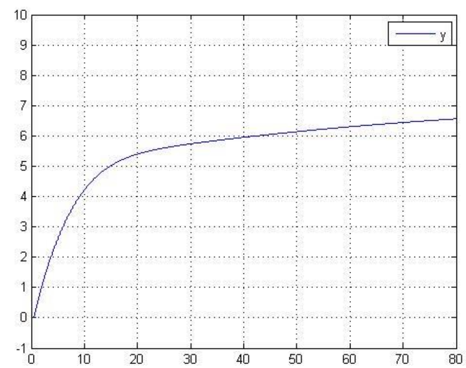

Figure 6. The simulation diagram of Y

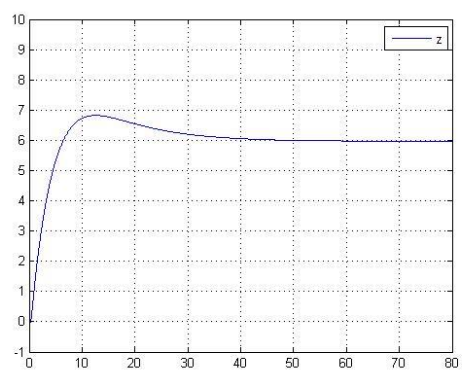

Figure 7. The simulation diagram of Z

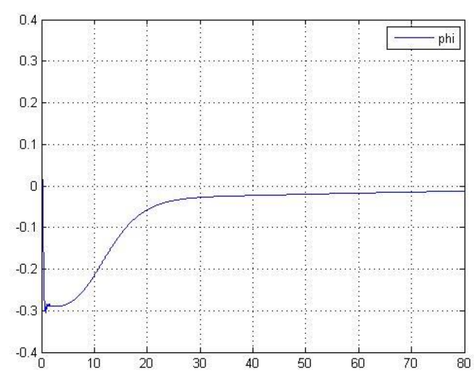

Figure 8. The simulation of roll angle

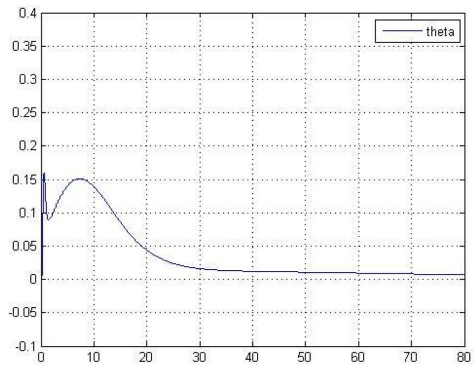

Figure 9. The simulation of pitching angle

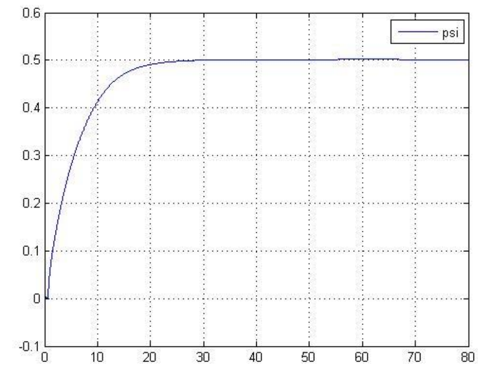

Figure 10. The simulation of yaw angle

From the figure about the simulation we can see X, Z and psi (yaw Angle) achieve the desired value in a relatively short period of time. The effect of Y coordinates is unsatisfactory, but it can tend to the expectation if we extent the period of time. Phi (roll angle) and theta (pitching angle) would be more stable (0) with the extension of time.

In order to more clearly describe the suspended state of the Quad-Rotor airship, we draw the three-dimensional map of its flight trajectory (as shown in the Figure 11), which can see the effectiveness of the control algorithm effect more clearly.

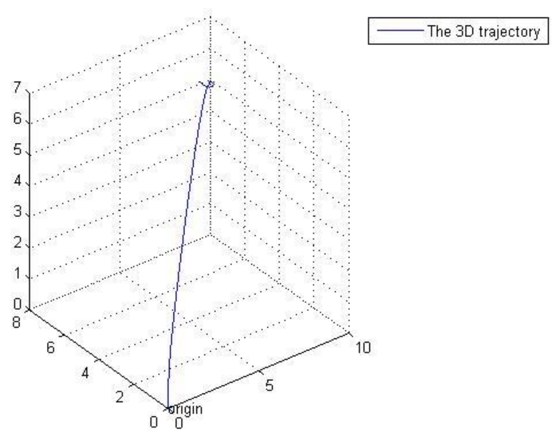

Figure 11. The simulation diagram of 3D trajectory

From the simulation results it can be seen that the dynamic model of Quad-Rotor airship is correct and the control method is feasible.

3. To Design the Control System based on Backstepping Method and the Simulation Results

Backstepping design method [11, 12] is one of the most commonly used methods for nonlinear system controller design, which combines the selection of Lyapunov function with the controller design , and is one of the virus regression design methods. By introducing the virtual control variable and designing the virtual control system gradually during the process from the start of the lowest order differential equations[13], the true control law will be designed at last.

Translate the formula (2.20) of dynamical model established in the second chapter into first-order spatial expression $\dot{X}=f\left(X,C\right)$, in which state variables $X=\left[x_1\ x_2\ x_3 \cdots x_{12}\right]^T$, control variables $C=\left[C_1\ C_2\ C_3\ C_4\right]$. Among them,

$$x_1=\phi \quad x_2=\dot{x}_1=\dot{\phi} \quad x_3=\theta \quad x_4=\dot{x}_3=\dot{\theta} \quad x_5=\psi \quad x_6=\dot{x}_5=\dot{\psi}$$
$$x_7=z \quad x_8=\dot{x}_7=\dot{z} \quad x_9=x \quad x_{10}=\dot{x}_8=\dot{x} \quad x_{11}=y \quad x_{12}=\dot{x}_{11}=\dot{y}$$

$$\dot{X}_1=\begin{cases}x_2\\ a_1x_4x_6-a_2+b_1C_2\end{cases} \quad \dot{X}_4=\begin{cases}x_8\\ \dfrac{H}{m}+1/m\left(\cos x_1\cos x_3\right)C_1\end{cases}$$
$$\dot{X}_2=\begin{cases}x_4\\ a_3x_2x_6-a_4+b_2C_3\end{cases} \quad \dot{X}_5=\begin{cases}x_{10}\\ 1/m\left(u_xC_1\right)\end{cases} \tag{3.1}$$
$$\dot{X}_3=\begin{cases}x_6\\ a_5x_2x_4+b_3C_4-a_6\end{cases} \quad \dot{X}_6=\begin{cases}x_{12}\\ 1/m\left(u_yC_1\right)\end{cases}$$

Table 2. Variable corresponding relational expression

variable	*Relation expression*	*variable*	*Relation expression*
a_1	$\left(I_y-I_z\right)/I_x$	b_1	l/I_x
a_2	$-k_l/I_x$	b_2	l/I_y
a_3	$\left(I_z-I_x\right)/I_y$	b_3	d/I_z
a_4	k_m/I_y	a_5	$\left(I_x-I_y\right)/I_z$
a_6	k_n/I_z		

The whole control system is divided into two control loops: position loop and attitude loop. Each control loop is divided into three second order controllers. Each variable corresponding relationship is as shown in the Table 2.

3.1 The design of attitude control system

With a controller of attitude control loop as a example, the derivation according to the backstepping design method will be as following:

$$\dot{X}_1=\begin{cases}x_2\\ a_1x_4x_6-a_2+b_1C_2\end{cases}=\begin{cases}x_2\\ b_1C_2+f_1\left(x_4,x_6\right)\end{cases} \tag{3.2}$$

where, $\dot{X}_1=\left[\dot{x}_1\ \dot{x}_2\right]^T$, $f_1\left(x_4,x_6\right)=a_1x_4x_6-a_2$

The desired roll angle is $\phi_d=x_{d1}$.The tracking error of attitude angle is: $z_1=x_{d1}-x_1$.According to the Lyapnov theory, the system satisfies $\phi=\phi_d$ at the point. $z_1=0$

Select the positive definite Lyapunov function $V(z_1)$:

$$V(z_1) = \frac{1}{2} z_1^2 \tag{3.3}$$

Along the trajectories $V(z_1)$ of the system, the time derivative is:

$$\dot{V}(z_1) = z_1(\dot{x}_{d1} - x_2) \tag{3.4}$$

Based on the Lyapunov stability theory, $\dot{V}(z_1)$ should be negative semidefinite, $\dot{V}(z_1) < 0$, therefore, lead into a dummy variable x_2^v, regards x_2^v as the virtual control of subsystem of z_1 .set up $x_2^v = \dot{x}_{d1} + \alpha_1 z_1$, $\alpha_1 > 0$ as the adjustable parameter.

The tracking error of velocity is defined as $z_2 = x_2 - x_2^v = x_2 - \dot{x}_{d1} - \alpha_1 z_1$, The structure of two order Lyapunov function is expressed as

$$V(z_1, z_2) = \frac{1}{2}(z_1^2 + z_2^2) \tag{3.5}$$

Then the first order derivative of $V(z_1, z_2)$ for the time derivative is

$$\begin{aligned}\dot{V}(z_1, z_2) &= z_2(a_1 x_4 x_6 \text{-} a_2 + b_1 \mathrm{C}_2) \\ &\quad - z_2\left[\ddot{x}_{d1} - \alpha_1(z_2 + \alpha_1 z_1)\right] - z_1 z_2 - \alpha_1 z_1^2\end{aligned} \tag{3.6}$$

So the control input C_2 can be expressed as the following expressions:

$$C_2 = \frac{1}{b_1}\left[z_1 - a_1 x_4 x_6 - a_2 - \alpha_1(z_2 + \alpha_1 z_1) - \alpha_2 z_2\right] \tag{3.7}$$

if $\alpha_2 > 0$, $\ddot{x}_{d1} = 0$, then $\dot{V}(z_1, z_2) < 0$ .Scilicet,

$$\dot{V}(z_1, z_2) = -\alpha_1 z_1^2 - \alpha_2 z_2^2 \tag{3.8}$$

The steady state of the whole system is not only influenced by the state x_1 and x_2, but also need to explicit state quantity x_4, x_6 ,and the parameters a_1, a_2, b_1 during the control process.

Similarly we can get the controller of pitch subsystem:

$$C_3 = \frac{1}{b_2}\left[z_3 - a_3 x_2 x_6 - a_4 - \alpha_3(z_4 + \alpha_3 z_3) - \alpha_4 z_4\right] \tag{3.9}$$

The controller of yaw subsystem:

$$C_4 = \frac{1}{b_3}\left[z_5 - a_5 x_2 x_4 - a_6 - \alpha_5(z_6 + \alpha_5 z_5) - \alpha_6 z_6\right] \tag{3.10}$$

The controller of height subsystem:

$$C_1 = \frac{m}{\cos x_1 \cos x_3}\left[z_7 + \frac{H}{m} - \alpha_7(z_8 + \alpha_7 z_7) - \alpha_8 z_8\right] \quad \alpha_7 > 0, \alpha_8 > 0 \tag{3.11}$$

The horizontally positional kinetic equations of the system is

$$\dot{X}_5 = \begin{cases} x_{10} \\ \frac{1}{m} u_x \mathrm{C}_1 \end{cases} \tag{3.12}$$

The tracking error of x and y are defined respectively:

$$e_9 = x_{9d} - x_9 \,,\, e_{11} = y_{11d} - y_9 \,;$$

The tracking error of velocity:

$$e_{10} = \alpha_9 e_9 + \dot{x}_{9d} + \lambda_5 \chi_5 - \dot{x}_9 \,, \quad e_{11} = \alpha_{11} e_{11} + \dot{y}_{11d} + \lambda_6 \chi_6 - \dot{y}_{11}$$

The pitch angle θ and roll angle ϕ are obtained from the yaw angle ψ which is already known, ϕ_d and θ_d are scheduled to enter the attitude loop. Considering the dynamic characteristics of the position error at the x axial is e_9, the virtual control input $x_{10} = \dot{x}_{9d} - \alpha_9 e_9$, selection Lyapunov function

$$\begin{aligned} V(e_9, e_{10}) &= \frac{1}{2}\left(e_9^2 + e_{10}^2\right) \\ \dot{V}(e_9, e_{10}) &= e_9\left(-e_{10} - \alpha_9 e_9\right) + e_{10}\left(\frac{u_x}{m} C_1 - \ddot{x}_{9d} + \alpha_9\left(z_{10} + \alpha_{10} e_9\right)\right) \end{aligned} \tag{3.13}$$

so, $u_x = \frac{m}{U_1}\left[e_9 - \alpha_9\left(e_{10} + \alpha_9 e_9\right) - \alpha_{10} e_{10}\right]$, $u_y = \frac{m}{U_1}\left[e_{11} - \alpha_{11}\left(e_{12} + \alpha_{11} e_{11}\right) - \alpha_{12} e_{12}\right]$,

among them $\alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}$ are positive.

So that, we can get the expression of roll angle ϕ_d and pitch angle θ_d :

$$\begin{aligned} \phi_d &= x_{1d} = \arcsin\left(u_x \sin(x_5) - u_y \cos(x_5)\right) \\ \theta_d &= x_{3d} = \arcsin\left(\frac{u_x - \sin(x_3)\sin(x_5)}{\cos(x_3)\cos(x_5)}\right) \end{aligned} \tag{0.1}$$

To sum up, through the deduction and analysis of the above derivation, the expressions of control input $\left(C_1\, C_2\, C_3\, C_4\right)$ can be obtained .By changing the size of $\left(C_1\, C_2\, C_3\, C_4\right)$ to adjust the performance of the system, so as to achieve the desired flight state.

3.2 The control simulation and the results analysis

In order to verify the effectiveness of the backstepping method controller, complete the design of control system based on the dynamic model in the second chapter, the simulation control system will still uses double loop feedback control. The difference is that outer loop position controls by using the classical PID control algorithm, the inner attitude loop controls by using the backstepping design method. Build the platform (as shown in Figure 12).

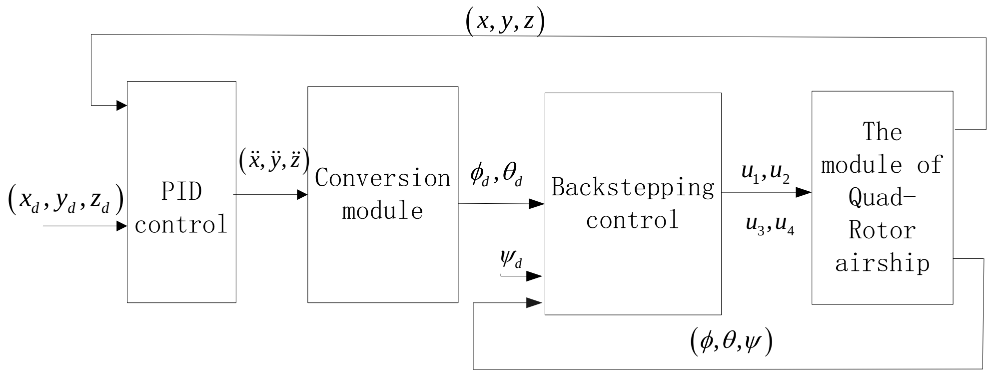

Figure 12. Simulation platform

3.5 The Simulation Results

The expected position information are (8,7,6),the desired yaw angle is 0.5, the simulation time is 80 s. The simulation results are shown as follows:

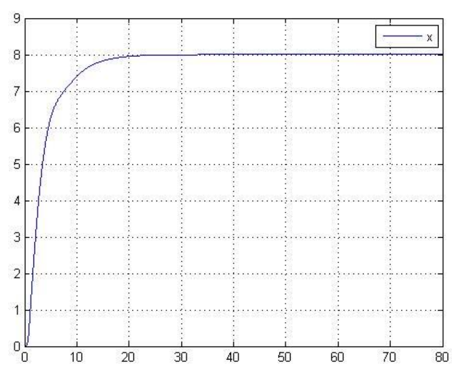

Figure 13. The simulation diagram of X

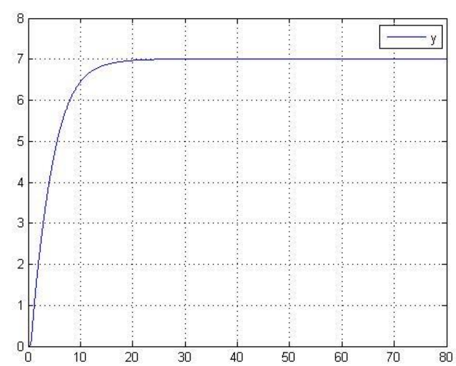

Figure 14.The simulation diagram of Y

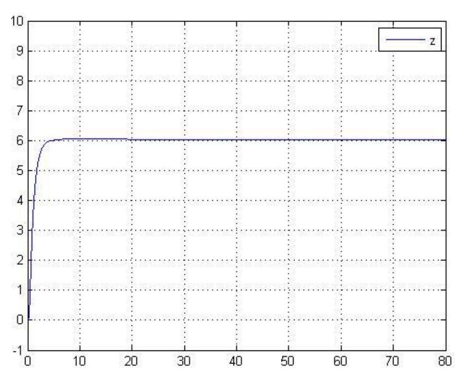

Figure 15. The simulation diagram of Z

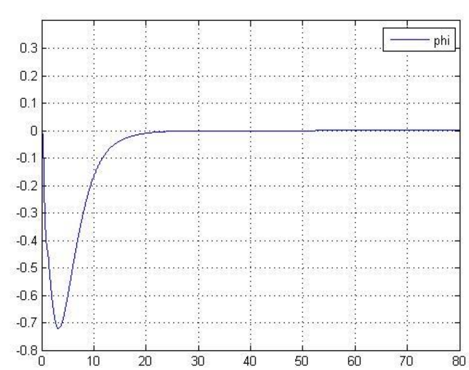

Figure 16. The simulation of roll angle

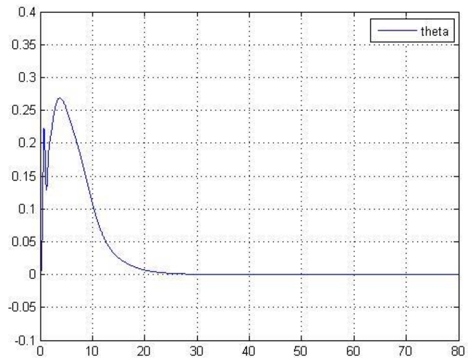

Figure 17. The simulation of pitching angle

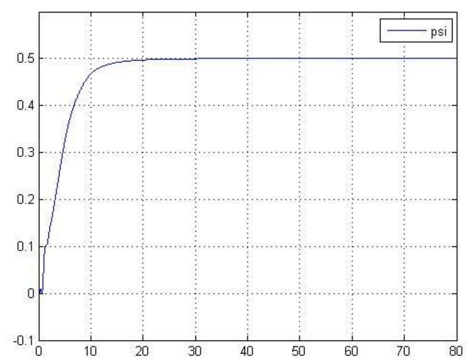

Figure 18. The simulation of yaw angle

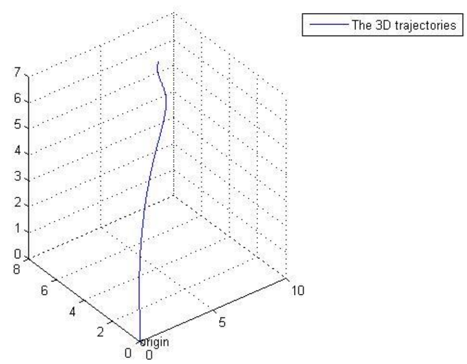

Figure 19. The simulation diagram of 3D trajectory

Through simulation and analysis, the controller designed by using backstepping design method, can reach the desired hovering point quickly, comparing Figure 5 – Figure 11 and Figure 13 – Figure 19 can find, the response time of backstepping design controller is faster than PID controller, the error of backstepping design controller is smaller than PID controller too. In conclusion, the proposed controller can complete the hover task very well, and the effect is better than the PID controller.

4. Conclusion

A novel structure of indoor airship -- Quad-Rotor airship is proposed in this paper. It satisfies the need for indoor such as the required power, the performance of the controller and the smoothness. Dynamic model is established according to the design structure and flight characteristics of Quad-Rotor airship and referencing the dynamic model of Quad-Rotor helicopter. Then the outer loop position controller and the inner loop attitude controller are designed. Combined with the dynamic model of the Quad-Rotor airship, the simulation platform with Matlab/Simulink is designed for the actual system. Based on PID control algorithm and backstepping algorithm, two controllers are designed in this paper, and respectively did corresponding experiments. Finally, the simulation results verify that the dynamics model of the airship is correct and the control system is feasible and valid. The controller designed by using backstepping design method, can reach the desired hovering point quickly, and the effect is better than the PID controller.

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