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Reliability and Security Characteristic Analysis on Complicated Equipment Operation

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Abstract

To aim at some large-size complex equipments widely used in industrial enterprises, this paper analyzed the operation maintenance and management feature, and established the full life cycle model with four states, and then performed the reliability and security analysis on it. During modeling we not only considered all the possibly emerging states of the equipment such as working, storage, preventive maintenance, and corrective maintenance, but also still conducted the detailed analysis on the transfer relations among these states. Based on it, we constructed the non-Markov process model of the equipment, where the checking and repairing time were considered to follow the arbitrary distribution during preventive maintenance and corrective maintenance, and the failure time distribution was thought to be exponential. Hence, by the use of supplementary variables, we converted the non-Markov model into the Markov process model, and calculated the relative reliability and security indexes. Compared with classical two-state and three-state models, the results show the established model is accurate and effective, and possesses a broad adaptability.

Keywords: *Complicated equipments, preventive maintenance, corrective maintenance, reliability, security, Markov mode*

1. Introduction

Modern industrial enterprises and test centers usually possesses the large-size complicated production and processing equipments. These devices are noted for their complicated structure and advanced processing technology, and the majority of them belong to imported equipments. The working feature of such devices is in storage state at ordinary time, when a demand occurs it then adds electricity to work, and after the completion of the work it is in the block up state again. Due to its expensive cost and outstanding performance, industrial enterprise has conducted the detailed preventive maintenance plan in general [1]. Whether the kind of equipments are in working condition or sealed state, it is quite necessary to implement the preventive maintenance routine check on it so as to avoid possible economic losses resulted in due to accidental failure of the equipment[2]. But anyway, the preventive maintenance can not change natural failure rate of the equipment, at most can only maintain or achieve the inherent reliability level of the equipment [3]. Hence, it is inevitable to implement the corrective maintenance on such equipments.

Preventive maintenance refers to doing periodic inspection and test on the state of the equipment, and with an eventual substitution if there is an upcoming failure, which is a kind of active maintenance management strategy, and also known as prior maintenance. Conversely, corrective maintenance refers to using a new part to replace after a damage has

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happened, which is a kind of passive maintenance technology, and also known as the later maintenance. Generally, correction maintenance dimension is big, and cost is higher, and cycle is longer. Obviously, the purpose of preventive maintenance is to reduce the quantity of corrective maintenance so as to avoid the larger economic losses. And therefore, it is applicable for preventive maintenance to the faults whose consequences can threat safety or task completion so as to cause major economic losses. And corrective maintenance is suitable for what kind of failure whose consequence is not important, and low cost, and low maintenance cost [4]. For the large complex equipments, therefore, the preventive maintenance work is particularly important. But as mentioned former, the corrective maintenance is inevitable, and individual parts are particularly costly and scarce. The life cycle model of complex equipments therefore should include four types of working condition such as work, preservation or storage, preventive maintenance, and corrective maintenance. In the process of the evolution of the life cycle, the transitions between the states go on to conduct. We assume that corrective maintenance is perfect, that is, after a completion of correction maintenance the equipment can be repaired as new. We also assume that if there are any defects to be detected out during preventive maintenance, and the preventive maintenance team also can give timely repair. Furthermore, the large-size equipment belongs to complex system, according to the complex system fault process law, the failure rate of which may be defined as a constant [5]. But its check and repair rates can not be defined as a constant, and are the function of time. As the check and repair time of the large-size complex equipments have a closely connection with inspection and repair parts, locations, maintenance personnel, and repair tools related to the advanced degree [6].

For the equipments reliability, the traditional analysis methods expose the several defects. Firstly, one problem is the integrity of state space. In [7] the state space of the equipments only consists of the two kinds of states, which are respectively defined as the normal and the fault, or the three states adding a preventive maintenance condition. It is applicable for the small-size simple devices, but can be unsuitable for the large-size complicated equipments. The reason lies in that the normal state of the complex equipments comprises the two conditions such as the working and the stop-using, and the failure, and as well as the checking rates possess larger difference under the two conditions. In [8] the state space of the equipments is defined as the normal, inventory, and maintenance of the three states, where the preventive maintenance and corrective maintenance share a state, on the basis of it some useful reliability indexes are drawn. But it is hard to draw more and better results due to the state share. Secondly, the transfer rates between the states are defined as a constant, especially, the inspection and maintenance time is defined as a constant. The aim to do so is for convenient analysis and dealing with the problem simpler. But it covers some technical difficulties, so that difficult to practical application. Thirdly, it is lacking to deeply understand for the used time during preventive maintenance when calculating the relevant reliability indexes. According to the traditional definition on the equipment reliability, if the preventive maintenance check does not interrupt the normal work of the equipment, this time should be included in the mean time to failure (MTTF), and not be considered in MTTF otherwise. Although the preventative maintenance check possibly influences the normal function of equipment, but at the time the equipment does not become invalid. That is to say, at this time the equipment is reliable but is not available due to a routine preventative maintenance check work. Hence, whether the preventive maintenance should be considered as the absorption state or not mostly depend on the specific practical situation when calculating the MTTF. Generally speaking, if the first time maintenance time of the equipment is close to the expiry time, and it has a little effect on reliability and MTTF, for instance, the delay-time model in condition based maintenance(CBM) [9, 10]. And otherwise, it may be possible to generate the larger deviation in the calculation of reliability and MTTF. Fourthly, it is thought to be perfect for the preventive maintenance routine inspection, *i.e.*, the preventive maintenance ability is thought to be infinite, and it can check and deal with any problems. As a fact, some concealing function faults are difficult to detect out so as to need to wait for corrective maintenance to handle.

Based on the analysis as above, the paper applies the theory knowledge of stochastic process to establish the life cycle model of this kind of the equipment. The model comprises the equipment operating, storage, preventive maintenance, and corrective maintenance of all the four possible states. Similarly, According to the failure process law of complex system, the failure rate of the equipment is defined as a constant, and the inspection and repair time of the preventive maintenance and corrective maintenance is considered as function of time, thus a non-Markov process model is then established. And then introducing the supplementary variables, the non-Markov process model is converted to Markov process model. On the basis of it, the calculation formula of the relevant reliability indexes is derived out as a theoretical foundation of the analysis and investigation on such equipments.

2. Model Description

In order to set up the life cycle model of such devices, the assumptions are conducted as follows.

Assumption1. The preventive maintenance, or the corrective maintenance, is independently expressed only using a state.

Assumption2. Whether at the operating or storage state, there are no devices the failures of which are no detectable.

Assumption3. Both the preventive maintenance and the corrective maintenance, the check and maintenance time of which follow arbitrary distribution, in addition to it any other transfer rates are constant.

Assumption4. Preventive maintenance can not change the natural failure rate of the equipments, *i.e.*, the instantaneous failure rate after the equipment is repaired is same with one before repaired, and only the residual life is owned when it returns to work.

Assumption5. Preventive maintenance is perfect, *i.e.*, if there are faults detected out during preventive maintenance, and then it would be able to get a timely repair.

Assumption6. Corrective maintenance is perfect, *i.e.*, the equipment can restore to the original state after corrective maintenance.

Based on the above assumptions, the life cycle model of the complicated devices can be presented using the state transition diagram in Figure 1.

In Figure 1, the symbol S_i expresses the state of the equipment, where the subscript $i \in [1, 2, 3, 4]$ is applied to respectively express the equipment storage, work, preventive maintenance, and corrective maintenance of four kinds of states; and λ_1 expresses the storage rate of a functioning equipment; and λ_2 expresses the using rate of a stored equipment; and v_1 presents the failure rate of a stored equipment; and v_2 presents the failure rate of a functioning equipment; and u_1 presents the check rate of a stored equipment; and u_2 presents the check rate of a functioning equipment; and ρ_1 expresses the stored probability of the device after a preventive maintenance intervention; and $1\text{-}\rho_1$ expresses the used probability of the device after a preventive maintenance intervention; and ρ_2 expresses the stored probability of the device after a corrective maintenance intervention; and $1\text{-}\rho_2$ expresses the used probability of the device after a corrective maintenance intervention; and $\mu_1(y_1)$ expresses the repair risk rate function of the preventive maintenance, whose average value is $\mu_1^{-1}<\infty$, and life distribution

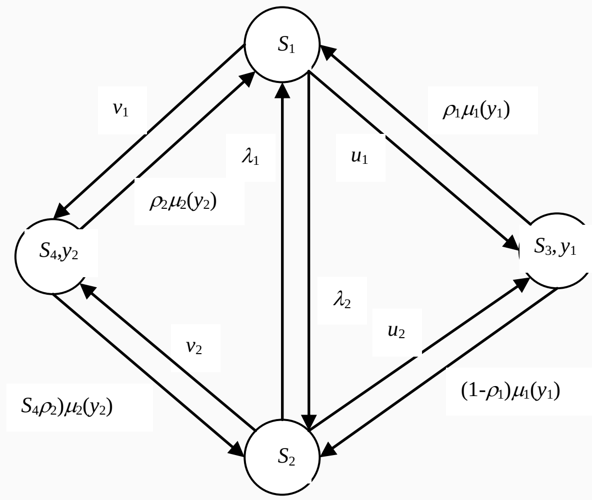

Figure 1. State transition diagram on {*S*(*t*)，*Y*$_1$(*t*)，*Y*$_2$(*t*)}

function is $G_1(t)$, and $g_1(t)$ is the density function; $\mu_2(y_2)$ expresses the repair risk rate function of the corrective maintenance, whose average value is $\mu_2^{-1}<\infty$, and life distribution function is $G_2(t)$, and $g_2(t)$ is the density function. The physical meaning of the transfer rate is to ensure that it is the bounded and the nonnegative.

Since the repair-time follows arbitrary distribution, the complementary variables $Y_1(t)$ and $Y_2(t)$ are introduced to separately express the consumed time of being repaired equipment when it is in S_3 or S_4 at time instant t. Let $\boldsymbol{S}(t)$ be the situated state of the device at time instant t, and then the vector $\{\boldsymbol{S}(t), Y_1(t), Y_2(t)\}$ is a continuous time generalized Markov process, namely, at arbitrary time instant t, as the specific numerical values of $\boldsymbol{S}(t)$, $Y_1(t)$, $Y_2(t)$ are given, and the operation law after the time instant t of the process $\{S(t), Y_1(t), Y_2(t), t \geq 0\}$ has nothing to do with the history before t.

Definition 1. The availability degree $A(t)$ of the equipment is defined as the probability that it can function normally at the time instant t. According to the definition, the equipment can be thought to possess the availability if it is being in S_1 or S_2 in at the time instant t, and otherwise we can say it is unavailable at t if it is not being in S_1 or S_2. And then, from Figure 1, we can have

$$A(t) = x_1(t) + x_2(t) \tag{1}$$

Definition 2 [11]**.** The safety availability degree $SA(t)$ of the equipment is defined as the probability that it is being in safe states at the time instant t. Clearly, $SA(t)$ is difference with $A(t)$. At the preventive maintenance state the equipment is unavailable, but it is safe. The security of the equipment is therefore defined as

$$SA(t) = x_1(t) + x_2(t) + x_3(t) = 1 - x_4(t) \tag{2}$$

Clearly, the probability that the device is being in dangerous unavailable state at the time instant t can be presented below.

$$\overline{SA}(t) = x_4(t) \tag{3}$$

Definition 3. The reliability degree of the equipment is defined as the probability that it can work normally within the scope of the time from zero to t. It does not allow failure, and so does not maintenance. According to the definition, the reliability degree $R(t)$ should have nothing to do with traversing what states before the device becomes the failure for the first time, the key is to see how to define the failure state. If the corrective maintenance is defined as equipment failure state, and all the once traversed states would be reliable before it enters into S_4 for the first time. Likewise, if the preventive maintenance is defined as equipment failure state, and all the once traversed states would be reliable before it enters into S_3 for the first time. It is noted that all the once traversed states here do not include S_4, because in general, the equipment enters into the preventive maintenance state earlier than the preventive maintenance state. And the three types of the definitions on reliability degrees here are then generated as follows.

(1) If the preventive maintenance state is defined as equipment failure, and then reliability degree can be denoted by

$$R_1(t) = x_1(t) + x_2(t) \tag{4}$$

At this moment, the reliability degree is same with the availability degree before the first time failure of the device.

(2) If the corrective maintenance state is defined as equipment failure, and then reliability degree can be denoted by

$$R_2(t) = x_1(t) + x_2(t) + x_3(t) \tag{5}$$

The definition views the preventive maintenance as the normal state of the equipment, *i.e.*, it thinks that the preventive maintenance only is a routine check on the device, there is a difference in nature compared with the failure or fault of the equipment. In many simple preventive maintenance works, the time of the inspection, calibration, and maintenance of the preventive maintenance is shorter compared with the equipment life, and the impact caused by it is not big to fulfill the specified function, and so the preventive maintenance can usually be regarded as normal state, and can be neglected in the calculation.

(3) The safety reliability degree of the device is defined as the probability that the equipment is being in safety states in range of the time from zero to t before it enters into the dangerous states for the first time. In accordance with the definition, the reliability degree of the device can be defined by

$$SR(t) = R_2(t) \tag{6}$$

Definition 4. To aim at the three kinds of the reliability degrees in definition 3, accordingly, three kinds of $\mathrm{MTTFF_S}$ are defined by

$$\mathrm{MTTFF}_1 = \int_0^\infty R_1(t)\mathrm{d}t \tag{7}$$

$$\mathrm{MTTFF}_2 = \int_0^\infty R_2(t)\mathrm{d}t \tag{8}$$

$$\mathrm{MTTFD} = \int_0^\infty SR(t)\mathrm{d}t \tag{9}$$

According to reliability mathematics theory [12, 13], we have

$$\mu(t)\Delta t = p\{t < Y \le t+\Delta t \mid Y > t\} = \frac{g(t)\Delta t}{1-G(t)} \tag{10}$$

where Y expresses the check or repair time. From (10), we can get

$$\int_0^\infty \mu(t)\mathrm{d}t = \int_0^\infty \frac{1}{1-G(t)}\mathrm{d}G(t) \tag{11}$$

Through simple transformation, and then

$$G(y) = 1-\exp[-\int_0^y \mu(\tau)d\tau] \tag{12}$$

$$g(y) = \mu(y)[1-G(y)] = \mu(y)\exp[-\int_0^y \mu(\tau)d\tau] \tag{13}$$

After Laplace(L) transform, we can have

$$g(s) = \int_0^\infty g(y)\exp(-sy)\mathrm{d}y \tag{14}$$

For $\forall\ t \ge 0$, $y_1 \ge 0$, $y_2 \ge 0$, we define

$$x_1(t) = p\{S(t)=1\} \tag{15}$$

$$x_2(t) = p\{S(t)=2\} \tag{16}$$

$$x_3(t, y_1)dy_1 = p\{S(t)=3, y_1 < Y_1(t) \le y_1+dy_1\} \tag{17}$$

$$x_4(t, y_2)dy_2 = p\{S(t)=4, y_2 < Y_2(t) \le y_2+dy_2\} \tag{18}$$

From Figure 1, let $\lambda_3=u_1+v_1$ and $\lambda_4=u_2+v_2$, we then can write out Kolmogorov partial calculous equation group below.

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}=-(\lambda_2+\lambda_3)x_1(t)+\lambda_1 x_2(t)+\rho_1\int_0^\infty \mu_1(y_1)x_3(t,y_1)\mathrm{d}y_1+\rho_2\int_0^\infty \mu_2(y_2)x_4(t,y_2)\mathrm{d}y_2 \tag{19}$$

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t}=\lambda_2 x_1(t)-(\lambda_1+\lambda_4)x_2(t)+(1-\rho_1)\int_0^\infty \mu_1(y_1)x_3(t,y_1)\mathrm{d}y_1+(1-\rho_2)\int_0^\infty \mu_2(y_2)x_4(t,y_2)\mathrm{d}y_2 \tag{20}$$

$$\frac{\partial x_3(t,y_1)}{\partial t}+\frac{\partial x_3(t,y_1)}{\partial y_1}=-\mu_1(y_1)x_3(t,y_1) \tag{21}$$

$$\frac{\partial x_4(t,y_2)}{\partial t}+\frac{\partial x_4(t,y_2)}{\partial y_2}=-\mu_2(y_2)x_4(t,y_2) \tag{22}$$

The boundary conditions satisfy

$$x_3(t,0)=u_1x_1(t)+u_2x_2(t) \tag{23}$$

$$x_4(t,0)=v_1x_1(t)+v_2x_2(t) \tag{24}$$

The Initial conditions satisfy

$$x_1(t)|_{t=0}=x_1(0),\qquad x_2(t)|_{t=0}=x_2(0),\qquad x_1(0)+x_2(0)=1,$$
$$x_3(t,y_1)|_{t=0}=0,\qquad x_4(t,y_2)|_{t=0}=0. \tag{25}$$

Conducting L transform from (19) to (25), and after arrangement we can obtain

$$sx_1(s)=-[\lambda_2+\lambda_3-\rho_1u_1g_1(s)-\rho_2v_1g_2(s)]x_1(s)+[\lambda_1+\rho_1u_2g_1(s)+\rho_2v_2g_2(s)]x_2(s)+x_1(0) \tag{26}$$

$$\begin{aligned}sx_2(s)=&[\lambda_2+(1-\rho_1)u_1g_1(s)+(1-\rho_2)v_1g_2(s)]x_1(s)-\\&[\lambda_1+\lambda_4-(1-\rho_1)u_2g_1(s)-(1-\rho_2)v_2g_2(s)]x_2(s)+x_2(0)\end{aligned} \tag{27}$$

$$x_3(s,y_1)=x_3(s,0)e^{-sy_1}\bar{G}_1(y_1) \tag{28}$$

$$x_4(s,y_2)=x_4(s,0)e^{-sy_2}\bar{G}_2(y_2) \tag{29}$$

$$x_3(s,0)=u_1x_1(s)+u_2x_2(s) \tag{30}$$

$$x_4(s,0)=v_1x_1(s)+v_2x_2(s) \tag{31}$$

Arranging (26) and (27) in matrix form, and so we have

$$sX(s) = AX(s) + X(0) \tag{32}$$

where

$$X(s) = \begin{vmatrix} x_1(s) \\ x_2(s) \end{vmatrix}; \qquad X(0) = \begin{vmatrix} x_1(0) \\ x_2(0) \end{vmatrix}; \qquad A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$a_{11} = -[\lambda_2 + \lambda_3 - \rho_1 u_1 g_1(s) - \rho_2 v_1 g_2(s)]$$

$$a_{12} = \lambda_1 + \rho_1 u_2 g_1(s) + \rho_2 v_2 g_2(s)$$

$$a_{21} = \lambda_2 + (1-\rho_1)u_1 g_1(s) + (1-\rho_2)v_1 g_2(s)$$

$$a_{22} = -[\lambda_1 + \lambda_4 - (1-\rho_1)u_2 g_1(s) - (1-\rho_2)v_2 g_2(s)]$$

According to [14], if $a_{11}a_{22} - a_{21}a_{12} \neq 0$, from (32) we have

$$X(s) = |sI - A|^{-1}\ X(0) \tag{33}$$

where

$$|sI - A| = \begin{vmatrix} s - a_{11} & -a_{12} \\ -a_{21} & s - a_{22} \end{vmatrix}$$

And so, we have

$$X(s) = \frac{1}{\det|sI - A|} \begin{vmatrix} s - a_{22} & a_{12} \\ a_{21} & s - a_{11} \end{vmatrix} X(0) \tag{34}$$

Order

$$\det|sI - A| = s^2 + s[\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + d_1 + d_4] + [\lambda_1\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 + h] + e$$

$$d_1 = -[\rho_1 u_1 g_1(s) + \rho_2 v_1 g_2(s)]$$

$$d_2 = \rho_1 u_2 g_1(s) + \rho_2 v_2 g_2(s)$$

$$d_3 = (1-\rho_1)u_1 g_1(s) + (1-\rho_2)v_1 g_2(s)$$

$$d_4 = -[(1-\rho_1)u_2 g_1(s) + (1-\rho_2)v_2 g_2(s)]$$

$$h = -\{\lambda_1[u_1 g_1(s) + v_1 g_2(s)] + \lambda_2[u_2 g_1(s) + v_2 g_2(s)] - \lambda_3 d_4 - \lambda_4 d_1]\}$$

$$e = (\rho_1 - \rho_2)(u_1 v_2 - u_2 v_1) g_1(s) g_2(s)$$

And then

$$\boldsymbol{X}(s) = \frac{1}{\det|sI - A|} \begin{vmatrix} s + \lambda_1 + \lambda_4 + d_4 & \lambda_1 + d_2 \\ \lambda_2 + d_3 & s + \lambda_2 + \lambda_3 + d_1 \end{vmatrix} X(0) \tag{35}$$

where

$$\begin{aligned} \det|s\boldsymbol{I} - \boldsymbol{A}| = s^2 + s[\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + d_1 + d_4] \\ + [\lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + u_2 v_1 + \rho_1 u_1 v_2 g_2(s) - \rho_2 u_2 v_1 g_2(s)][1 - g_1(s)] \\ + [\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + u_1 v_2 + \rho_1 u_2 v_1 g_1(s) - \rho_2 u_1 v_2 g_1(s)][1 - g_2(s)] \end{aligned}$$

Let D=det|$s\boldsymbol{I}$-$\boldsymbol{A}$|, and then

$$x_1(s) = \frac{[s + \lambda_1 + \lambda_4 - (1-\rho_1) u_2 g_1(s) - (1-\rho_2) v_2 g_2(s)] x_1(0) + [\lambda_1 + \rho_1 u_2 g_1(s) + \rho_2 v_2 g_2(s)] x_2(0)}{D} \tag{36}$$

$$x_2(s) = \frac{[\lambda_2 + (1-\rho_1) u_1 g_1(s) + (1-\rho_2) v_1 g_2(s)] x_1(0) + [s + \lambda_2 + \lambda_3 - \rho_1 u_1 g_1(s) - \rho_2 v_1 g_2(s)] x_2(0)}{D} \tag{37}$$

Substituting (36) and (37) into (30) and (31), and considering (28) and (29), we have

$$x_3(s, y_1) = [u_1 x_1(s) + u_2 x_2(s)] e^{-s y_1} \bar{G}_1(y_1) \tag{38}$$

$$x_4(s, y_2) = [v_1 x_1(s) + v_2 x_2(s)] e^{-s y_2} \bar{G}_2(y_2) \tag{39}$$

Theorem 1. The L transformation of the system instantaneous availability degree can be expressed by

$$A(s) = x_1(s) + x_2(s) = CX(s) \tag{40}$$

where $\boldsymbol{C}$=[1 1], and $\boldsymbol{X}(s)=[x_1(s)\ \ x_2(s)]^T$.

Further, the steady-state availability exists and has nothing to do with the initial states.

$$A = \frac{\lambda_1 + \lambda_2 + \rho_1 u_2 + \rho_2 v_2 + (1-\rho_1) u_1 + (1-\rho_2) v_1}{E} \tag{41}$$

where

$$E = \lim_{s \to 0} \frac{\det|s\boldsymbol{I} - \boldsymbol{A}|}{s} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 - \rho_1 u_1 - \rho_2 v_1 - (1-\rho_1)u_2 - (1-\rho_2)v_2$$
$$+ [\lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + u_2 v_1 + \rho_1 u_1 v_2 - \rho_2 u_2 v_1]\mu_1^{-1}$$
$$+ [\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + u_1 v_2 + \rho_1 u_2 v_1 - \rho_2 u_1 v_2]\mu_2^{-1}$$

$$\mu_1^{-1} = 1\Big/[-\lim_{s\to 0} g_1^{'}(s)] = 1\Big/\int_0^{\infty} y_1 g_1(y_1)\mathrm{d}y_1; \qquad \mu_2^{-1} = 1\Big/[-\lim_{s\to 0} g_2^{'}(s)] = 1\Big/\int_0^{\infty} y_2 g_2(y_2)\mathrm{d}y_2 \qquad (42)$$

Proof. Conducting L transformation on (1) in definition 1, and then

$$A(s) = x_1(s) + x_2(s)$$

Let $\boldsymbol{C}$=[1 1], and $\boldsymbol{X}(s)=[x_1(s)\ \ x_2(s)]^{\mathrm{T}}$ defined by (35). And so, the formula (40) is proved. The system steady-state availability is given by

$$A = \lim_{s\to 0} sA(s) = \lim_{s\to 0}[sx_1(s) + sx_2(s)] =$$
$$\frac{\boldsymbol{C}}{\lim\limits_{s\to 0}\frac{\det|s\boldsymbol{I}-\boldsymbol{A}|}{s}} \begin{vmatrix} \lambda_1 + \lambda_4 - [(1-\rho_1)u_2 + (1-\rho_2)v_2] & \lambda_1 + \rho_1 u_2 + \rho_2 v_2 \\ \lambda_2 + (1-\rho_1)u_1 + (1-\rho_2)v_1 & \lambda_2 + \lambda_3 - [\rho_1 u_1 + \rho_2 v_1] \end{vmatrix} \boldsymbol{X}(0)$$

Noticing that $x_1(0)+x_2(0)=1$, and (42), the formula (41) may be acquired, immediately.

In fact, the steady-state probabilities being in S_1, S_2, S_3, and S_4 can be worked out as follows.

$$x_1 = \lim_{t\to\infty} x_1(t) = \lim_{s\to 0} sx_1(s) = \frac{\lambda_1 + \rho_1 u_2 + \rho_2 v_2}{E} \qquad (43)$$

$$x_2 = \lim_{t\to\infty} x_2(t) = \lim_{s\to 0} sx_2(s) = \frac{\lambda_2 + (1-\rho_1)u_1 + (1-\rho_2)v_1}{E} \qquad (44)$$

$$x_3 = \lim_{t\to\infty} x_3(t) = \lim_{s\to 0} sx_3(s) = \frac{\lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + \rho_2 u_1 v_2 + (1-\rho_2)u_2 v_1}{\mu_1 E} \qquad (45)$$

$$x_4 = \lim_{t\to\infty} x_4(t) = \lim_{s\to 0} sx_4(s) = \frac{\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + \rho_1 u_2 v_1 + (1-\rho_1)u_1 v_2}{\mu_2 E} \qquad (46)$$

The system steady-state availability can also be obtained by $A=x_1+x_2$, we can get same result with (41) thought substituting (43) and (44) into it.

Theorem 2. The L transformation of the system instantaneous safety availability can be expressed by

$$SA(s) = x_1(s) + x_2(s) + x_3(s) \tag{47}$$

Further, the steady-state safety availability exists and has nothing to do with the initial states.

$$SA = \frac{[\lambda_1 + \lambda_2 + \rho_1 u_2 + \rho_2 v_2 + (1-\rho_1)u_1 + (1-\rho_2)v_1]\mu_1 + \lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + \rho_2 u_1 v_2 + (1-\rho_2)u_2 v_1}{\mu_1 E} \tag{48}$$

The L transformation of the system instantaneous dangerous unavailability can be denoted as

$$\overline{SA}(s) = x_4(s) \tag{49}$$

Further, the steady-state dangerous unavailability exists and can be written as

$$\overline{SA} = \frac{\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + \rho_1 u_2 v_1 + (1-\rho_1)u_1 v_2}{\mu_2 E} \tag{50}$$

Proof. It is known from definition 2 and the former derivation process that the theorem holds obviously. Process (omitting)

Theorem 3. The L transformation of the instantaneous frequency that the system enters into the preventive maintenance state can be presented by

$$m_{\mathrm{fp}}(s) = u_1 x_1(s) + u_2 x_2(s) \tag{51}$$

Further, the steady-state frequency exists and can be written as

$$m_{\mathrm{fp}} = \frac{\lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + \rho_2 u_1 v_2 + (1-\rho_2)u_2 v_1}{E} \tag{52}$$

Likewise, the L transformation of the instantaneous frequency that the system enters into the corrective maintenance state can be presented by

$$m_{\mathrm{fc}}(s) = v_1 x_1(s) + v_2 x_2(s) \tag{53}$$

Further, the steady-state frequency exists and can be written as

$$m_{\mathrm{fc}} = \frac{\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + \rho_1 u_2 v_1 + (1-\rho_1)u_1 v_2}{E} \tag{54}$$

Eventually, the L transformation of the instantaneous joint frequency that the system enters into the preventive and corrective maintenance states can be presented by

$$m_{\mathrm{f}}(s) = \lambda_3 x_1(s) + \lambda_4 x_2(s) \quad (55)$$

Further, the steady-state frequency exists and can be written as

$$m_{\mathrm{f}} = \frac{\lambda_1\lambda_3 + \lambda_2\lambda_4 + \lambda_3\rho_1 u_2 + \lambda_3\rho_2 v_2 + \lambda_4(1-\rho_1)u_1 + \lambda_4(1-\rho_2)v_1}{E} \quad (56)$$

In addition, the following relationship holds between the joint frequency and each independent frequency.

$$m_{\mathrm{f}} = m_{\mathrm{fp}} + m_{\mathrm{fc}} \quad (57)$$

Proof. According to [4], the transfer frequency f_{ij} refers to an expected times of direct transfer from the state i to the state j per unit time, it can be presented by

$$f_{ij}=\lambda_{ij}\, p_i \quad (58)$$

where λ_{ij} expresses the transfer rate from the state i to the state j, and p_i is a probability that the system stay in state i. Seen from (51), the transfer rate λ_{ij} is a condition frequency, the condition is that the system locates in state i at the time instant t.

From (58), the formula (51) is evident. The formula (52) is proved below.

$$m_{\mathrm{fp}} = \lim_{s\to 0} s m_{\mathrm{fp}}(s) = \lim_{s\to 0} s[u_1 x_1(s) + u_2 x_2(s)]$$
$$= \frac{\lambda_1 u_1 + \lambda_2 u_2 + u_1 u_2 + \rho_2 u_1 v_2 + (1-\rho_2) u_2 v_1}{E}$$

Similarly, the formula (53) is evident. The formula (54) is proved below.

$$m_{\mathrm{fc}} = \lim_{s\to 0} s m_{\mathrm{fc}}(s) = \lim_{s\to 0} s[v_1 x_1(s) + v_2 x_2(s)]$$
$$= \frac{\lambda_1 v_1 + \lambda_2 v_2 + v_1 v_2 + \rho_1 u_2 v_1 + (1-\rho_1) u_1 v_2}{E}$$

The formula (55) is evident. The formula (56) is proved below.

$$m_{\mathrm{f}} = \lim_{s\to 0} s m_{\mathrm{f}}(s) = \lim_{s\to 0} s[\lambda_3 x_1(s) + \lambda_4 x_2(s)]$$
$$= \frac{\lambda_1\lambda_3 + \lambda_2\lambda_4 + \lambda_3\rho_1 u_2 + \lambda_3\rho_2 v_2 + \lambda_4(1-\rho_1)u_1 + \lambda_4(1-\rho_2)v_1}{E}$$

Noticing that $\lambda_3=u_1+v_1$, $\lambda_4=u_2+v_2$, the formula (57) can be obtained, promptly.

The reason that the joint frequency is equal to the sum of each independent frequency is that the preventive and corrective maintenance of the two states are transferred from the same states, and there is no state transition between them.

Obviously, the frequency that the system enters into the dangerous state is same with the one that it transfers into the correction maintenance state S_4.

Theorem 4. The L transformation of the instantaneous renewal frequency can be expressed by

$$m_{\mathrm{r}}(s)=[v_1x_1(s)+v_2x_2(s)]g_2(s) \tag{59}$$

Further, the steady-state frequency exists and can be written as

$$m_{\mathrm{r}}=\frac{\lambda_1v_1+\lambda_2v_2+v_1v_2+\rho_1u_2v_1+(1-\rho_1)u_1v_2}{E} \tag{60}$$

Proof. According to assumption 6 and (58), we have

$$m_{\mathrm{r}}(t)=\int_0^{\infty}\mu_2(y_2)x_4(t,y_2)\mathrm{d}y_2$$

After L transformation, we have

$$m_{\mathrm{r}}(s)=\int_0^{\infty}\mu_2(y_2)x_4(s,y_2)\mathrm{d}y_2$$

Substituting (38) into the above formula, and then, the formula (59) is obtained at once. From (59), we have the steady-state probability as follows.

$$m_{\mathrm{r}}=\lim_{s\to 0}sm_{\mathrm{r}}(s)=\frac{\lambda_1v_1+\lambda_2v_2+v_1v_2+\rho_1u_2v_1+(1-\rho_1)u_1v_2}{E}$$

Known from (54) and (60), the transfer frequency that the system enters into the state S_4 from the normal states is fully same with the one that it leaves the state S_4 into any other normal states at steady-state. This embodies the concept of the frequency balance at steady-state. In addition, there is not renewal frequency existing for the preventive maintenance since it is only a check or functional recovery on the equipments.

To resolve the reliability index $R(t)$, we require to determine the absorbing state of the equipment. According to the relationship between the preventive maintenance and corrective maintenance, the preventive maintenance is usually ahead of the corrective maintenance. That is to say, the check rate u of the preventive maintenance is larger than the failure rate v of the corrective maintenance. Hence, if we determine the time that the equipment enters into the state of the preventive maintenance for the first time, the preventive maintenance state S_3 and as well as the corrective maintenance state S_4 must be viewed as the absorbing states at the same time. In another hand, if to determine the time that the equipment enters into the state of the corrective maintenance for the first time, and it is essential that the corrective maintenance state S_4 is seen as the absorbing state. Below we expound the two kinds of cases, respectively.

Firstly, let us determine the time that the equipment enters into the preventive maintenance state S_3 for the first time. At the time, in the fundamental model the connecting arcs from the state S_3 and S_4 leaving to any other normal states must be broken, and thus we would acquire a Markov chain with the absorbing states. Its state transfer diagram is shown in Figure 2.

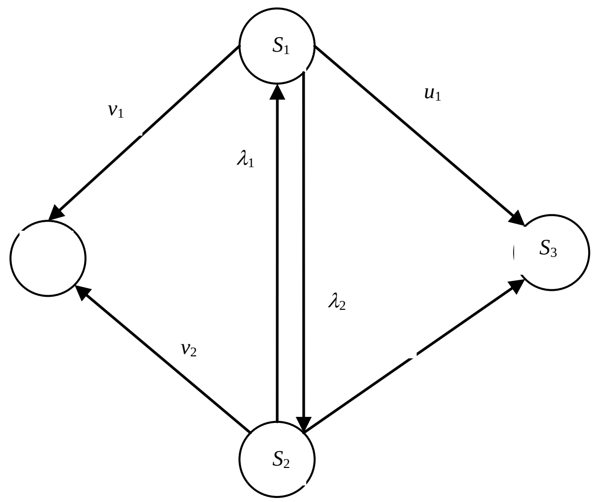

Figure 2. State transition diagram of {*S*(*t*), *t*>0}

According to Figure 2, the equations can be written as follows.

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -(\lambda_2+\lambda_3)x_1(t)+\lambda_1 x_2(t) \tag{61}$$

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = \lambda_2 x_1(t)-(\lambda_1+\lambda_4)x_2(t) \tag{62}$$

The initial conditions satisfy

$$x_1(t)|_{t=0}=x_1(0), \quad x_2(t)|_{t=0}=x_2(0), \quad x_1(0)+x_2(0)=1 \tag{63}$$

After L transformation, we can solve

$$x_1(s)=\frac{(s+\lambda_1+\lambda_4)x_1(0)+\lambda_1 x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4} \tag{64}$$

$$x_2(s)=\frac{\lambda_2 x_1(0)+(s+\lambda_2+\lambda_3)x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4} \tag{65}$$

Theorem 5. The L transformation of the system reliability $R_1(t)$ can be expressed by

$$R_1(s)=\frac{(s+\lambda_1+\lambda_2+\lambda_4)x_1(0)+(s+\lambda_1+\lambda_2+\lambda_3)x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4} \tag{66}$$

Further, the mean time ($MTTFF_1$) that the system enters into the preventive maintenance state S_3 for the first time can be described by

$$\mathrm{MTTFF}_1 = \int_0^{\infty} R_1(t)\mathrm{d}t = \lim_{s \to 0} R_1(s) = \frac{(\lambda_1+\lambda_2+\lambda_4)x_1(0)+(\lambda_1+\lambda_2+\lambda_3)x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4} \quad (67)$$

Proof. According to definition 3, the L transformation of the system reliability is

$$R_1(s) = x_1(s) + x_2(s)$$

Substituting (64) and (65) into the above formula, we easily get (66). Further, from definition 4, substituting (66) into (7), and (67) is acquired, immediately.

Next, we calculate the time that the equipment enters into the corrective maintenance state S_4 for the first time. At the time, the connecting arcs from the state S_4 leaving to any other states must be broken in Figure 1, and thus, a Markov chain with a absorbing state would be obtained, whoses state transfer diagram is shown in Figure 3.

According to Figure 3, the new equations can be written by

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = -(\lambda_2+\lambda_3)x_1(t)+\lambda_1 x_2(t)+\rho_1\int_0^{\infty}\mu_1(y_1)x_3(t,y_1)\mathrm{d}y_1 \quad (68)$$

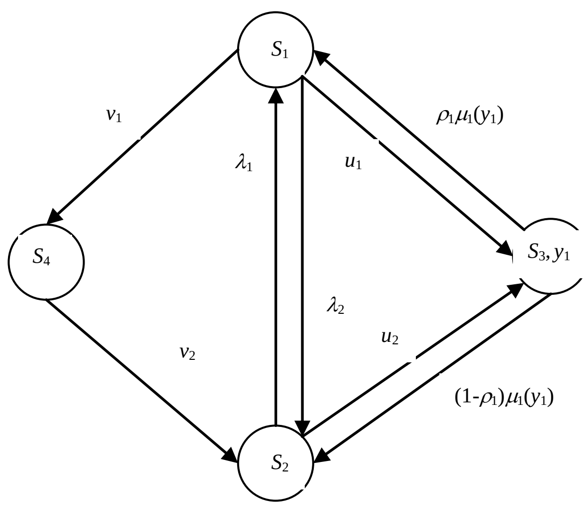

Figure 3. State transition diagram of {$S(t)$, $Y_1(t)$, t>0}

$$\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = \lambda_2 x_1(t)-(\lambda_1+\lambda_4)x_2(t)+(1-\rho_1)\int_0^{\infty}\mu_1(y_1)x_3(t,y_1)\mathrm{d}y_1 \quad (69)$$

$$\frac{\partial x_3(t,y_1)}{\partial t}+\frac{\partial x_3(t,y_1)}{\partial y_1} = -\mu_1(y_1)x_3(t,y_1) \quad (70)$$

The boundary conditions satisfy

$$x_3(t,0) = u_1 x_1(t) + u_2 x_2(t) \quad (71)$$

The initial conditions satisfy

$$x_1(t)|_{t=0} = x_1(0),\ \ x_2(t)|_{t=0} = x_2(0),\ \ x_1(0)+x_2(0)=1,\ \ x_3(t,y_1)|_{t=0} = 0 \tag{72}$$

Conducting L transformation from (68) to (72), and then

$$sx_1(s) = -[\lambda_2+\lambda_3-\rho_1 u_1 g_1(s)]x_1(s)+[\lambda_1+\rho_1 u_2 g_1(s)]x_2(s)+x_1(0) \tag{73}$$

$$sx_2(s) = [\lambda_2+(1-\rho_1)u_1 g_1(s)]x_1(s)+[\lambda_1+\lambda_4-(1-\rho_1)u_2 g_1(s)]x_2(s)+x_2(0) \tag{74}$$

$$x_3(s,y_1) = x_3(s,0)e^{-sy_1}\bar{G}_1(y_1) \tag{75}$$

Writing (73) and (74) as the matrix form, and then we have

$$sX(s) = A_1 X(s) + X(0) \tag{76}$$

where

$$\boldsymbol{X}(s) = \begin{vmatrix} x_1(s) \\ x_2(s) \end{vmatrix}; \qquad \boldsymbol{X}(0) = \begin{vmatrix} x_1(0) \\ x_2(0) \end{vmatrix}$$

$$\boldsymbol{A}_1 = \begin{vmatrix} -[\lambda_2+\lambda_3-\rho_1 u_1 g_1(s)] & [\lambda_1+\rho_1 u_2 g_1(s) \\ \lambda_2+(1-\rho_1)u_1 g_1(s) & -[\lambda_1+\lambda_4-(1-\rho_1)u_2 g_1(s)] \end{vmatrix}$$

From (76), we have

$$\boldsymbol{X}(s) = |s\boldsymbol{I}-\boldsymbol{A}_1|^{-1}\ X(0) \tag{77}$$

where

$$\left|s\boldsymbol{I}-\boldsymbol{A}_1\right| = \begin{vmatrix} s+\lambda_2+\lambda_3-\rho_1 u_1 g_1(s) & -[\lambda_1+\rho_1 u_2 g_1(s)] \\ -[\lambda_2+(1-\rho_1)u_1 g_1(s)] & s+\lambda_1+\lambda_4-(1-\rho_1)u_2 g_1(s) \end{vmatrix}$$

Then

$$\begin{aligned}\det|s\boldsymbol{I}-\boldsymbol{A}_1| &= s^2 + s[\lambda_1+\lambda_2+\lambda_3+\lambda_4-(1-\rho_1)u_2 g_1(s)-\rho_1 u_1 g_1(s)] \\ &\quad + \lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4 - [\lambda_1 u_1+\lambda_2 u_2+\rho_1\lambda_4 u_1+(1-\rho_1)\lambda_3 u_2]g_1(s)\end{aligned}$$

And so, we have

$$x_3(s,y_1) = [u_1 x_1(s)+u_2 x_2(s)]e^{-sy_1}\bar{G}_1(y_1) \tag{79}$$

Theorem 6. The L transformation of the system reliability $R_2(t)$ can be expressed by

$$R_2(s) = x_1(s) + x_2(s) + \int_0^\infty x_3(s, y_1)\mathrm{d}y_1 \tag{80}$$

Further, the mean time (MTTFF_2) that the system enters into the corrective maintenance state S_4 for the first time can be described by

$$\mathrm{MTTFF}_2 = \lim_{s\to 0} R_2(s) = \frac{[\lambda_1+\lambda_2+\lambda_4+(1-\rho_1)(u_1-u_2)]x_1(0)+[\lambda_1+\lambda_2+\lambda_3+\rho_1(u_2-u_1)]x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4 - [\lambda_1 u_1+\lambda_2 u_2+\rho_1\lambda_4 u_1+(1-\rho_1)\lambda_3 u_2]}$$
$$+\frac{[\lambda_1 u_1+\lambda_2 u_2+\lambda_4 u_1 x_1(0)+\lambda_3 u_2 x_2(0)]\mu_1^{-1}}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4 - [\lambda_1 u_1+\lambda_2 u_2+\rho_1\lambda_4 u_1+(1-\rho_1)\lambda_3 u_2]} \tag{81}$$

Proof. The formula (80) is clearly. From definition 4, substituting (80) into (8) to quickly obtain (81).

If we ignore the check time of the preventive maintenance, and the formula (81) may written as

$$\mathrm{MTTFF}_2^* = \lim_{s\to 0} R_2^*(s) = \frac{[\lambda_1+\lambda_2+\lambda_4+(1-\rho_1)(u_1-u_2)]x_1(0)+[\lambda_1+\lambda_2+\lambda_3+\rho_1(u_2-u_1)]x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4 - [\lambda_1 u_1+\lambda_2 u_2+\rho_1\lambda_4 u_1+(1-\rho_1)\lambda_3 u_2]} \tag{82}$$

where

$$R_2^*(s) = x_1(s) + x_2(s) \tag{83}$$

Obviously, as the check rate u_1 equals zero, and u_2 equals zero, the formula (82), and (81) are same with (67).

Theorem 7. The L transformation of the system safety reliability $SR(t)$ can be expressed by

$$SR(s)= R_2(s) \tag{84}$$

In addition, the mean time (MTTFD) before the system enters into the dangerous state for the first time can be described by

$$\mathrm{MTTFD}=\mathrm{MTTFF}_2 \tag{85}$$

Proof. From the description in definition 3 and 4, the formula (84) and (85) is obvious.

3. Examples

Example 1. A power enterprise implements the preventive maintenance on the generating unit at ordinary times, and conducts the corrective maintenance as a failure happens, the maintenance parameters of which satisfy: $\rho_1=\rho_2=\rho$, $\mu_1(y_1)= \mu_2(y_2)= \mu$. And then according to (41), (48), (50), (52), (54), (56), (60), (67), (82), and (85), we can have

$$A=\frac{(\lambda_1+\lambda_2)\mu+(1-\rho)\mu\lambda_3+\rho\mu\lambda_4}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$SA=\frac{(\lambda_1+\lambda_2)\mu+(1-\rho)\lambda_3\mu+\rho\lambda_4\mu+\lambda_1u_1+\lambda_2u_2+u_1u_2+\rho u_1v_2+(1-\rho)u_2v_1}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$\overline{SA}=\frac{\lambda_1v_1+\lambda_2v_2+v_1v_2+\rho u_2v_1+(1-\rho)u_1v_2}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$m_{\mathrm{fp}}=\frac{[\lambda_1u_1+\lambda_2u_2+u_1u_2+\rho u_1v_2+(1-\rho)u_2v_1]\mu}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$m_{\mathrm{fc}}=\frac{[\lambda_1v_1+\lambda_2v_2+v_1v_2+\rho u_2v_1+(1-\rho)u_1v_2]\mu}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$m_{\mathrm{f}}=\frac{[\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4]\mu}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$m_{\mathrm{r}}=\frac{[\lambda_1v_1+\lambda_2v_2+v_1v_2+\rho u_2v_1+(1-\rho)u_1v_2]\mu}{[\lambda_1+\lambda_2+(1-\rho)\lambda_3+\rho\lambda_4]\mu+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$\mathrm{MTTFF}_1=\frac{(\lambda_1+\lambda_2+\lambda_4)x_1(0)+(\lambda_1+\lambda_2+\lambda_3)x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4}$$

$$\mathrm{MTTFF}_2^*=\frac{[\lambda_1+\lambda_2+\lambda_4+(1-\rho)(u_1-u_2)]x_1(0)+[\lambda_1+\lambda_2+\lambda_3+\rho(u_2-u_1)]x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4-[\lambda_1u_1+\lambda_2u_2+\rho\lambda_4u_1+(1-\rho)\lambda_3u_2]}$$

$$\mathrm{MTTFD{=}MTTFF}_2=\lim_{s\to0}R_2(s)=\frac{[\lambda_1+\lambda_2+\lambda_4+(1-\rho)(u_1-u_2)]x_1(0)+[\lambda_1+\lambda_2+\lambda_3+\rho(u_2-u_1)]x_2(0)}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4-[\lambda_1u_1+\lambda_2u_2+\rho_1\lambda_4u_1+(1-\rho_1)\lambda_3u_2]}$$
$$+\frac{[\lambda_1u_1+\lambda_2u_2+\lambda_4u_1x_1(0)+\lambda_3u_2x_2(0)]\mu^{-1}}{\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4-[\lambda_1u_1+\lambda_2u_2+\rho\lambda_4u_1+(1-\rho)\lambda_3u_2]}$$

Let $\rho\mu=\mu_1$, and $(1-\rho)\mu=\mu_2$, and substituting them into the above formula, and then we can get the same results in [8]. As a fact, the state S_3 and S_4 may be combined into a state under the condition of the above parameters given. As the conditions of the states merger can not be satisfied, the results mentioned should be applied in this paper.

Example 2. A power enterprise implements the preventive maintenance on the generator at ordinary times, and conducts the corrective maintenance as a failure happens, but the generating unit has been used and never terminated, which means that the normal state of the generator is only a working state and does not have the storage state. And so the state S_1 may be eliminated from Figure 1, and the generator becomes the device of the three states, *i.e.*, the working state, and the preventive state, and as well as the corrective state. The relative reliability indexes of such devices generally may be obtained by redrawing the state transfer diagram and rewriting the state transfer equations. But using the results presented in this paper, we can also export the steady-state reliability indexes of the equipments with the three states. In another hand, ignoring the state S_1 means $\rho_1=\rho_2=0$, $u_1= 0$, $\lambda_1=\lambda_2=0$ in Figure 1. But it should be noted that at the time the failure rate v_1 should be reserved, it is the natural attribute of the equipment. In essence, it can be divided out during the formula reduction. In addition, we also note that the initial conditions should be changed as $x_1(0)=0$ and $x_2(0)=1$. And so, from (41), (48), (50), (52), (54), (56), (60), (67), (82), and (85), we then have

$$A=\frac{\mu_2}{\mu_2+v_2+u_2\mu_2\mu_1^{-1}}$$

$$SA=\frac{(\mu_1+u_2)\mu_2}{(\mu_1+u_2)\mu_2+v_2\mu_1}$$

$$\overline{SA}=\frac{\mu_1 v_2}{\mu_1\mu_2+u_2\mu_2+v_2\mu_1}$$

$$m_{\mathrm{fp}}=\frac{u_2\mu_2}{\mu_2+u_2\mu_2\mu_1^{-1}+v_2}$$

$$m_{\mathrm{fc}}=\frac{v_2\mu_2}{\mu_2+u_2\mu_2\mu_1^{-1}+v_2}$$

$$m_{\mathrm{f}}=\frac{\lambda_4\mu_2}{\mu_2+u_2\mu_2\mu_1^{-1}+v_2}$$

$$m_{\mathrm{r}}=\frac{v_2\mu_2}{\mu_2+u_2\mu_2\mu_1^{-1}+v_2}$$

$$\mathrm{MTTFF}_1=\frac{1}{\lambda_4}$$

$$\mathrm{MTTFF}_2^*=\frac{1}{v_2}$$

$$\mathrm{MTTFD}=\mathrm{MTTFF}_2=\frac{u_2+\mu_1}{v_2\mu_1}$$

The results are fully consistent with ones in [7].

4. Conclusion

The life cycle model presented during the complicated equipment operation in this paper may be better used to evaluate the calculation formula of the life parameters of such model so as to be able to provide the theory basis for complex systems analysis. It not only can be used to evaluate the reliability and security of the large-size complex equipment, but also to guide its reserve plan, and as well as the evaluation of the preventive maintenance project. Compared with the traditional models with the two-state or three-state, applying the four-state model to evaluate the reliability and security of the equipments would be more accurate, and objective, and available. And so the model proposed in the paper would be more flexible and ubiquitous, and possesses a broadly application prosperity.

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