

Stability Analysis for Uncertain Neutral Markovian Jump Systems with Interval Time-varying Delay

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Abstract

The problem of delay-dependent stochastic stability for neutral Markovian jump systems with mixed delays is investigated in this paper. Combined the new constructed Lyapunov functional with the introduced free matrices, and using the analysis technique of matrix inequalities, the delay-dependent stability conditions are obtained. The obtained results are formulated in terms of LMIs, which can be easily checked in practice by Matlab LMI control toolbox. Finally, two numerical examples are given to show the validity and potential of the developed criteria.

Keywords: *Stochastic Stability, Markovian Jump Systems, Neutral Systems, Delays*

1. Introduction

As well known, neutral systems are frequently encountered in various engineering systems, including population ecology, distributed networks containing lossless transmission lines, heat exchangers, and repetitive control. For recent years, a great deal of attention has been drawn to the study of neutral systems, such as [18, 22], and the references therein. In the literature, many good achievements have been made to obtain less conservative delay-dependent conditions. As a matter of fact, an important index of measuring method is the maximum allowable upper bound on the delay. Delay-dependent conditions via Lyapunov functional are often based on a fixed model transformation technique that rewrites the delayed term via integration. Then utilizing the bounding technique to the cross-term, delay-dependent criteria are obtained. According to the classification of [3], there are four basic fixed transformation methods. A good idea is to use the parameter model transformation technique with a new matrix parameter. The parameter model transformation can be classified into two types: one type is that the matrix parameter can be freely chosen in the resulting stability condition based on linear matrix inequalities (LMIs) (see [2, 16, 17]); the other is to transform the matrix parameter into one of matrix variables of the LMIs of the stability condition by some technique (see [14]). However, these methods cannot well deal with the case when the coefficient matrix of the delayed derivative term contains time-varying uncertainties.

On the other hand, systems with Markovian jump parameters have been attracting an increasing attention. This class of systems is very appropriate to model plants whose structure is subject to random abrupt changes due to, for instance, changes of the operating point, abrupt environment disturbance, random component failures, *etc.* A number of control analysis and synthesis problems related to these systems has been analyzed by several authors (see [1, 4, 5, 10-15, 20, 23]). In particular, robust mean square stability of continuous-time

Markov jump systems with uncertain parameters has been studied in [4, 5, 12]. A common feature of the existing methods of robust stability analysis is that they deal with either linear systems, or linear systems with unknown nonlinearities, such as Lipschitz-type and norm-bounded, which are treated as fictitious uncertainties.

In this paper, a class of uncertain neutral Markovian jump systems with mixed delays is taken into account. We first investigate the nominal system and then extend to the uncertain case. Some less conservative delay-dependent stability criteria are presented by introducing new types of Lyapunov functional. Numerical examples are given to show the effectiveness and reduced conservativeness.

The remainder of this paper is organized as follows: **Section 2** contains the problem statement and preliminaries; **Section 3** presents the main results; **Section 4** provides the numerical example to verify the effectiveness of the results; **Section 5** draws a brief conclusion.

1.1. Notations

In this paper, the following notations will be used: \mathbb{R}^n and $\mathbb{R}^{m \times n}$ respectively denote the n -dimensional Euclidean space and the set of all $m \times n$ real matrices. A^T (or x^T) and A^{-1} (or x^{-1}) denote the transpose and the inverse of matrix A or scalar x , $\sigma_{\max}(A)$ and $\sigma_{\min}(A)$ denote the maximal and minimal eigenvalue of a real matrix A , $\|A\|$ denotes the Euclidean norm of matrix A , $|a|$ denotes the absolute value of the scalar a , $E\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector, $P > 0$ stands for a symmetric positive-definite matrix, I is the unit matrix with appropriate dimensions, “*” means the symmetric terms in a symmetric matrix. In addition, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Problem Statement and Preliminaries

Given a probability space $\{\Omega, \mathcal{F}, P\}$ where Ω is the sample space, \mathcal{F} is the algebra of events and P is the probability measure defined on \mathcal{F} . $\{r_t, t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in a finite set $S = \{1, 2, 3, \dots, N\}$, with the mode transition probability matrix

$$P(r_{t+\Delta t} = j | r_t = i) = \begin{cases} \pi_{ij} \Delta t + o(\Delta t) & i \neq j \\ 1 + \pi_{ii} \Delta t + o(\Delta t) & i = j \end{cases}$$

where $\Delta t > 0$, $\lim_{\Delta t \rightarrow 0} o(\Delta t)/\Delta t = 0$ and $\pi_{ij} \geq 0$ ($i, j \in S, i \neq j$) denotes the transition rate from mode i to j . For any state or mode $i \in S$, we have

$$\pi_{ii} = - \sum_{j=1, j \neq i}^N \pi_{ij}.$$

In this paper, we consider the following uncertain neutral system with Markovian jump parameters and interval time-varying delay as follows:

$$\begin{aligned} \dot{x}(t) - (C(r_t) + \Delta C(r_t)) \dot{x}(t - \tau) &= (A(r_t) + \Delta A(r_t)) x(t) \\ &+ (B(r_t) + \Delta B(r_t)) x(t - d(t)) \end{aligned} \quad (1)$$

$$x(s) = \varphi(s), \quad r_s = r_0 \quad s \in [-\rho, 0] \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the system state and τ is a constant neutral delay. The initial condition $\varphi(s)$ is a continuously differentiable vector-valued function. The continuous norm of $\varphi(s)$ is defined as

$$\|\varphi\|_c = \max_{s \in [-\rho, 0]} |\varphi(s)|, \quad \rho = \max\{\tau, d_2\}.$$

It is assumed that the time-varying delay $d(t)$ satisfies:

$$0 < d_1 \leq d(t) \leq d_2, \quad \dot{d}(t) \leq \mu \quad (3)$$

where d_1, d_2 and $\mu > 0$ are constant real values. $A(r_i) \in \mathbb{R}^{n \times n}, B(r_i) \in \mathbb{R}^{n \times n}, C(r_i) \in \mathbb{R}^{n \times n}$ are constant matrices, while $\Delta A(r_i) \in \mathbb{R}^{n \times n}, \Delta B(r_i) \in \mathbb{R}^{n \times n}, \Delta C(r_i) \in \mathbb{R}^{n \times n}$ are uncertainties. For simplicity in the sequel, we denote $C(r_i)$ by $C_i, \Delta C(r_i)$ by $\Delta C_i, C_i(t) = C_i + \Delta C_i(t)$, and so on for $r_i = i \in S$. Then in this paper the admissible parametric uncertainties are assumed to satisfy the following condition:

$$[\Delta A_i(t) \quad \Delta B_i(t) \quad \Delta C_i(t)] = H_i F_i(t) [E_{Ai} \quad E_{Bi} \quad E_{Ci}] \quad (4)$$

where $H_i, E_{Ai}, E_{Bi}, E_{Ci}$ are known constant matrices with appropriate dimensions and $F_i(t)$ is an unknown and time-varying matrix satisfying:

$$F_i^T(t) F_i(t) \leq I, \quad \forall t \quad (5)$$

Throughout this paper, in order to guarantee the stability of neutral operator we assume that the parametric matrix $C_i(t)$ is Schur stable. Particularly, when we consider $F_i(t) = 0$, we get the nominal systems which can be described

$$\dot{x}(t) - C_i \dot{x}(t - \tau) = A_i x(t) + B_i x(t - d(t)) \quad (6)$$

Definition 2.1 (Boukas et al.[4]) The systems which are described by (1) and (2) are said to be stochastically stable if there exist a positive constant γ such that

$$E \left\{ \int_0^\infty \|x(r_i, t)\|^2 dt \mid \varphi(s), s \in [-2d, 0], r_0 \right\} < \gamma \quad (7)$$

Before ending this section, let us provide the following results which will be required to derive our main results in this paper. They are stated in the lemmas given below.

Definition 2.2 (Mao [21]) In the Euclidean space $\{\mathbb{R}^n \times M \times \mathbb{R}^+\}$, we introduce the stochastic Lyapunov–Krasovskii function of system (1) as $V(x(t), r_i = i, t > 0) = V(x(t), i)$, the infinitesimal generator satisfying

$$\begin{aligned} \mathcal{L}V(x(t), i) &= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \left[E \left\{ V(x(t + \Delta t), r_{t+\Delta t}, t + \Delta t) \mid x(t) = x, r_t = i \right\} - V(x(t), i, t) \right] \\ &= \frac{\partial}{\partial t} V(x(t), i, t) + \frac{\partial}{\partial x} V(x(t), i, t) \dot{x}(t, i) + \sum_{j=1}^N \pi_{ij} V(x(t), j, t) \end{aligned}$$

Lemma 2.1 (Gu [8]) For any constant matrix $H \in \mathbb{R}^{n \times n}$, $H > 0$ and scalar $\gamma > 0$, vector function

$$\omega: [0, \gamma] \rightarrow \mathbb{R}^n$$

such that the integrations $\int_0^\gamma \omega^T(s)H\omega(s)ds$ and $\int_0^\gamma \omega^T(s)ds$ are well defined, then the following inequality holds:

$$-\int_0^\gamma \omega^T(s)H\omega(s)ds \leq -\frac{1}{\gamma} \left[\int_0^\gamma \omega^T(s)ds \right] H \left[\int_0^\gamma \omega(s)ds \right] \quad (8)$$

Lemma 2.2 (Xie [9]) For given matrices $Q = Q^T$, M and N with appropriate dimensions, then

$$Q + MF(t)N + N^T F^T(t)M^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq I$, if and only if there exists a scalar $\delta > 0$, such that

$$Q + \delta^{-1}MM^T + \delta N^T N < 0 \quad (9)$$

Lemma 2.3 (Boyd [19] Schur complement) Given constant matrices Ω_1 , Ω_2 and Ω_3 , where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3^T \\ * & \Omega_1 \end{bmatrix} < 0$$

3. Main Results

In this section, we firstly consider the nominal systems described by (6) and extend to the uncertain case. The following theorems present sufficient conditions to guarantee the stochastic stability for the neutral system with Markovian jump parameters and time-varying delay.

3.1. Stochastic stability for nominal systems

Theorem 3.1 Given scalars τ , μ , d_1 , d_2 and $d_{12} = d_2 - d_1$, the systems described by (6) and (2) are stochastically stable if there exist symmetric positive-definite matrices

$$P_i > 0, Q_j > 0, R_k > 0, Z_l > 0$$

with appropriate dimensions for $i \in S$, $j = 1, 2, 3$, $k = 1, 2, 3, 4, 5$, $l = 1, 2, 3, 4$ such that the following LMIs hold:

$$\begin{aligned} \Theta_1 = & \Sigma_i + \Gamma^T M \Gamma - (e_1 - e_{10}) Q_3 (e_1^T - e_{10}^T) - (e_1 - e_3) Z_1 (e_1^T - e_3^T) \\ & - 2(e_2 - e_4) Z_2 (e_2^T - e_4^T) - (e_3 - e_2) Z_2 (e_3^T - e_2^T) \\ & - 2e_9 Z_4 e_9^T - e_8 Z_4 e_8^T < 0 \end{aligned} \quad (10)$$

$$\begin{aligned} \Theta_2 = & \Sigma_i + \Gamma^T M \Gamma - (e_1 - e_{10}) Q_3 (e_1^T - e_{10}^T) - (e_1 - e_3) Z_1 (e_1^T - e_3^T) \\ & - (e_2 - e_4) Z_2 (e_2^T - e_4^T) - 2(e_3 - e_2) Z_2 (e_3^T - e_2^T) \\ & - e_9 Z_4 e_9^T - 2e_8 Z_4 e_8^T < 0 \end{aligned} \quad (11)$$

where $e_i \{i=1,2,3,\dots,11\}$ are block entry matrices, for instance

$$e_2^T = [0 \ I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\Gamma = [A_i \ B_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ C_i]$$

$$M = Q_2 + \tau^2 Q_3 + R_4 + d_1^2 Z_1 + d_{12}^2 Z_2$$

$$\Sigma_i = \begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Sigma_{i111} \\ * & \Sigma_{i22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Sigma_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Sigma_{i44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Sigma_{i55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Sigma_{i66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{i77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Sigma_{i1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Sigma_{i1111} \end{bmatrix}$$

$$\Sigma_{i11} = A_i^T P_i + P_i A_i + \sum_{j=1}^N \pi_{ij} P_j + Q_1 + R_1 + d_1^2 Z_3 + d_{12}^2 Z_4$$

$$\Sigma_{i12} = P_i B_i, \Sigma_{i111} = P_i C_i$$

$$\Sigma_{i22} = -(1-\mu)R_3, \Sigma_{i33} = -R_1 + R_3 + R_2$$

$$\Sigma_{i44} = -R_2, \Sigma_{i55} = -R_4 + R_5, \Sigma_{i66} = -R_5$$

$$\Sigma_{i77} = -Z_3, \Sigma_{i1010} = -Q_1, \Sigma_{i1111} = -Q_2$$

Proof: Construct a novel legitimate Lyapunov functional candidate as follows:

$$V(x_t, r_t) = V_r(x_t, r_t) + V_\tau(x_t, r_t) + V_d(x_t, r_t) \tag{12}$$

where

$$V_r(x_t, r_t) = x^T(t)P(r_t)x(t) \tag{13}$$

$$V_\tau(x_t, r_t) = \int_{t-\tau}^t x^T(s)Q_1 x(s)ds + \int_{t-\tau}^t \dot{x}^T(s)Q_2 \dot{x}(s)ds + \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s)(\tau Q_3)\dot{x}(s)ds \tag{14}$$

$$V_d(x_t, r_t) = \int_{t-d_1}^t x^T(s)R_1 x(s)ds + \int_{t-d_2}^{t-d_1} x^T(s)R_2 x(s)ds + \int_{t-d(t)}^{t-d_1} x^T(s)R_3 x(s)ds + \int_{t-d_1}^t \dot{x}^T(s)R_4 \dot{x}(s)ds + \int_{t-d_2}^{t-d_1} \dot{x}^T(s)R_5 \dot{x}(s)ds + \int_{-d_1}^0 \int_{t+\theta}^t \dot{x}^T(s)(d_1 Z_1)\dot{x}(s)dsd\theta + \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{x}^T(s)(d_{12} Z_2)\dot{x}(s)dsd\theta + \int_{-d_1}^0 \int_{t+\theta}^t x^T(s)(d_1 Z_3)x(s)dsd\theta + \int_{-d_2}^{-d_1} \int_{t+\theta}^t x^T(s)(d_{12} Z_4)x(s)dsd\theta \tag{15}$$

From **Definition 2.2**, taking \mathcal{L} as its infinitesimal generator, then from (12)-(15) we get the following equalities and inequalities

$$\mathcal{L}V(x_t, r_t) = \mathcal{L}V_r(x_t, r_t) + \mathcal{L}V_\tau(x_t, r_t) + \mathcal{L}V_d(x_t, r_t) \quad (16)$$

$$\begin{aligned} \mathcal{L}V_r(x_t, r_t) = & 2 \left[x^T(t)A_i^T + x^T(t-d(t))B_i^T + \dot{x}^T(t-\tau)C_i^T \right] P_i x(t) \\ & + \sum_{j=1}^N \pi_{ij} x^T(t) P_j x(t) \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{L}V_\tau(x_t, r_t) = & x^T(t)Q_1 x(t) - x^T(t-\tau)Q_1 x(t-\tau) \\ & + \dot{x}^T(t)Q_2 \dot{x}(t) - \dot{x}^T(t-\tau)Q_2 \dot{x}(t-\tau) \\ & + \dot{x}^T(t) \left(\tau^2 Q_3 \right) \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) \left(\tau Q_3 \right) \dot{x}(s) ds \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{L}V_d(x_t, r_t) = & x^T(t)R_1 x(t) - x^T(t-d_1)R_1 x(t-d_1) \\ & + x^T(t-d_1)R_2 x(t-d_1) - x^T(t-d_2)R_2 x(t-d_2) \\ & + x^T(t-d_1)R_3 x(t-d_1) - (1-\dot{d}(t))x^T(t-d(t))R_3 x(t-d(t)) \\ & + \dot{x}^T(t)R_4 \dot{x}(t) - \dot{x}^T(t-d_1)R_4 \dot{x}(t-d_1) \\ & + \dot{x}^T(t-d_1)R_5 \dot{x}(t-d_1) - \dot{x}^T(t-d_2)R_5 \dot{x}(t-d_2) \\ & + \dot{x}^T(t) \left(d_1^2 Z_1 \right) \dot{x}(t) - \int_{t-d_1}^t \dot{x}^T(s) \left(d_1 Z_1 \right) \dot{x}(s) ds \\ & + \dot{x}^T(t) \left(d_{12}^2 Z_2 \right) \dot{x}(t) - \int_{t-d_2}^{t-d_1} \dot{x}^T(s) \left(d_{12} Z_2 \right) \dot{x}(s) ds \\ & + x^T(t) \left(d_1^2 Z_3 \right) x(t) - \int_{t-d_1}^t x^T(s) \left(d_1 Z_3 \right) x(s) ds \\ & + x^T(t) \left(d_{12}^2 Z_4 \right) x(t) - \int_{t-d_2}^{t-d_1} x^T(s) \left(d_{12} Z_4 \right) x(s) ds \end{aligned} \quad (19)$$

Let us define

$$\begin{aligned} \xi(t) = & \text{col} \left\{ x(t) \ x(t-d(t)) \ x(t-d_1) \ x(t-d_2) \ \dot{x}(t-d_1) \ \dot{x}(t-d_2) \right. \\ & \left. \int_{t-d_1}^t x(s) ds \ \int_{t-d(t)}^{t-d_1} x(s) ds \ \int_{t-d_2}^{t-d(t)} x(s) ds \ x(t-\tau) \ \dot{x}(t-\tau) \right\} \end{aligned}$$

Applying **Lemma 2.1** we have

$$-\int_{t-\tau}^t \dot{x}^T(s) \left(\tau Q_3 \right) \dot{x}(s) ds \leq -\xi^T(t) (e_1 - e_{10}) Q_3 (e_1^T - e_{10}^T) \xi(t) \quad (20)$$

$$-\int_{t-d_1}^t \dot{x}^T(s) \left(d_1 Z_1 \right) \dot{x}(s) ds \leq -\xi^T(t) (e_1 - e_3) Z_1 (e_1^T - e_3^T) \xi(t) \quad (21)$$

$$-\int_{t-d_1}^t x^T(s) \left(d_1 Z_3 \right) x(s) ds \leq - \left[\int_{t-d_1}^t x^T(s) ds \right] Z_3 \left[\int_{t-d_1}^t x(s) ds \right] \quad (22)$$

Let $\alpha = (d(t) - d_1)/d_{12}$ and from (3) we obtain

$$\begin{aligned}
 & -\int_{t-d_2}^{t-d_1} \dot{x}^T(s) (d_{12} Z_2) \dot{x}(s) ds \\
 & = -d_{12} \int_{t-d_2}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds - d_{12} \int_{t-d(t)}^{t-d_1} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & = -(d_2 - d(t)) \int_{t-d_2}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \quad - (d(t) - d_1) \int_{t-d_2}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \quad - (d(t) - d_1) \int_{t-d(t)}^{t-d_1} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \quad - (d_2 - d(t)) \int_{t-d(t)}^{t-d_1} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 & \leq -\xi^T(t) (e_2 - e_4) Z_2 (e_2^T - e_4^T) \xi(t) \\
 & \quad - \alpha \xi^T(t) (e_2 - e_4) Z_2 (e_2^T - e_4^T) \xi(t) \\
 & \quad - \xi^T(t) (e_3 - e_2) Z_2 (e_3^T - e_2^T) \xi(t) \\
 & \quad - (1 - \alpha) \xi^T(t) (e_3 - e_2) Z_2 (e_3^T - e_2^T) \xi(t)
 \end{aligned} \tag{23}$$

Similarly,

$$\begin{aligned}
 & -\int_{t-d_2}^{t-d_1} x^T(s) (d_{12} Z_4) x(s) ds \\
 & \leq -\xi^T(t) [e_8 Z_4 e_8^T + e_9 Z_4 e_9^T] \xi(t) \\
 & \quad - \alpha \xi^T(t) (e_9 Z_4 e_9^T) \xi(t) - (1 - \alpha) \xi^T(t) (e_8 Z_4 e_8^T) \xi(t)
 \end{aligned} \tag{24}$$

Considering

$$\dot{x}^T(t) M \dot{x}(t) = \xi^T(t) \Gamma^T M \Gamma \xi(t) \tag{25}$$

We take the above equalities and inequalities (17)-(25) into (16), thus we get the following inequality

$$\begin{aligned}
 \mathcal{L}V(x_t, r_t) & \leq \xi^T(t) \Sigma_i \xi(t) + \xi^T(t) \Gamma^T M \Gamma \xi(t) + \xi^T(t) \\
 & \quad [- (e_1 - e_5) Q_3 (e_1^T - e_5^T) - (e_1 - e_3) Z_1 (e_1^T - e_3^T) \\
 & \quad - (1 + \alpha) (e_2 - e_4) Z_2 (e_2^T - e_4^T) - (2 - \alpha) (e_3 - e_2) Z_2 (e_3^T - e_2^T) \\
 & \quad - (1 + \alpha) e_9 Z_4 e_9^T - (2 - \alpha) e_8 Z_4 e_8^T] \xi(t)
 \end{aligned}$$

So it can be seen that

$$\mathcal{L}V(x_t, r_t) \leq \xi^T(t) [\alpha \Theta_1 + (1 - \alpha) \Theta_2] \xi(t)$$

Since $0 \leq \alpha \leq 1$, $\alpha \Theta_1 + (1 - \alpha) \Theta_2$ is a convex combination of Θ_1 and Θ_2 . Therefore, $\alpha \Theta_1 + (1 - \alpha) \Theta_2 < 0$ is equivalent to (10) and (11). So we choose

$$\beta = \max_{i \in S} \lambda_{\max}(\alpha \Theta_1 + (1 - \alpha) \Theta_2) \text{ and } \beta < 0.$$

Then

$$\mathcal{L}V(x_t, r_t) \leq \beta \|\xi(t)\|^2 \leq \beta \|x(t)\|^2$$

From the Dynkin's Formula

$$E\{V(x_t, r_t)\} - V(x_0, r_0) \leq \beta E\left\{\int_0^t \|x(s)\|^2 ds\right\}$$

Let $t \rightarrow \infty$, then we obtain

$$\lim_{t \rightarrow \infty} E\left\{\int_0^t \|x(s)\|^2 ds\right\} \leq (-\beta)^{-1} V(x_0, r_0).$$

From **Definition 2.1**, we know that the systems described by (6) and (2) are stochastically stable. This completes the proof. ■

3.2. Stochastic stability for the uncertain case

In this subsection, we consider the uncertain case which can be described by (1) and (2). We obtain the following theorem to guarantee the stochastic stability for the uncertain systems based on **Theorem 3.1**.

Theorem 3.2 Given scalars $\tau, \mu, \delta_1, \delta_2, d_1, d_2$ and $d_{12} = d_2 - d_1$, the systems described by (1) and (2) are stochastically stable if there exist symmetric positive-definite matrices

$$P_i > 0, Q_j > 0, R_k > 0, Z_l > 0$$

with appropriate dimensions for $i \in S, j = 1, 2, 3, k = 1, 2, 3, 4, 5, l = 1, 2, 3, 4$ such that the following LMIs hold:

$$\begin{bmatrix} \Sigma_{i1} + \frac{1}{\delta_1} \varepsilon_{i2}^T H_i H_i^T \varepsilon_{i2} + \delta_1 \varepsilon_{i1}^T \varepsilon_{i1} & \Gamma^T M + \frac{1}{\delta_1} \varepsilon_{i2}^T H_i H_i^T M \\ M \Gamma + \frac{1}{\delta_1} M H_i H_i^T \varepsilon_{i2} & \frac{1}{\delta_1} M H_i H_i^T M - M \end{bmatrix} < 0 \quad (26)$$

$$\begin{bmatrix} \Sigma_{i2} + \frac{1}{\delta_2} \varepsilon_{i2}^T H_i H_i^T \varepsilon_{i2} + \delta_2 \varepsilon_{i1}^T \varepsilon_{i1} & \Gamma^T M + \frac{1}{\delta_2} \varepsilon_{i2}^T H_i H_i^T M \\ M \Gamma + \frac{1}{\delta_2} M H_i H_i^T \varepsilon_{i2} & \frac{1}{\delta_2} M H_i H_i^T M - M \end{bmatrix} < 0 \quad (27)$$

where

$$\begin{aligned} \Sigma_{i1} = & \Sigma_i - (e_1 - e_{10}) Q_3 (e_1^T - e_{10}^T) - (e_1 - e_3) Z_1 (e_1^T - e_3^T) \\ & - 2(e_2 - e_4) Z_2 (e_2^T - e_4^T) - (e_3 - e_2) Z_2 (e_3^T - e_2^T) \\ & - 2e_9 Z_4 e_9^T - e_8 Z_4 e_8^T \end{aligned}$$

$$\begin{aligned} \Sigma_{i2} = & \Sigma_i - (e_1 - e_{10}) Q_3 (e_1^T - e_{10}^T) - (e_1 - e_3) Z_1 (e_1^T - e_3^T) \\ & - (e_2 - e_4) Z_2 (e_2^T - e_4^T) - 2(e_3 - e_2) Z_2 (e_3^T - e_2^T) \\ & - e_9 Z_4 e_9^T - 2e_8 Z_4 e_8^T \end{aligned}$$

$$\varepsilon_{i1} = [E_{Ai} \ E_{Bi} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ E_{Ci}]$$

$$\varepsilon_{i2} = [P_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

Σ_i, M and Γ have been defined in **Theorem 3.1**.

Proof: On the basis of **Theorem 3.1**, we directly replace A_i, B_i, C_i with $A_i + \Delta A_i(t), B_i + \Delta B_i(t), C_i + \Delta C_i(t)$ and obtain

$$\Theta_1(t) = \Sigma_{i1}(t) + \Gamma^T(t)M\Gamma(t) < 0 \quad (28)$$

$$\Theta_2(t) = \Sigma_{i2}(t) + \Gamma^T(t)M\Gamma(t) < 0 \quad (29)$$

where

$$\begin{aligned} \Sigma_{i1}(t) &= \Sigma_i(t) - (e_1 - e_{10})Q_3(e_1^T - e_{10}^T) - (e_1 - e_3)Z_1(e_1^T - e_3^T) \\ &\quad - 2(e_2 - e_4)Z_2(e_2^T - e_4^T) - (e_3 - e_2)Z_2(e_3^T - e_2^T) \\ &\quad - 2e_9Z_4e_9^T - e_8Z_4e_8^T \\ \Sigma_{i2}(t) &= \Sigma_i(t) - (e_1 - e_{10})Q_3(e_1^T - e_{10}^T) - (e_1 - e_3)Z_1(e_1^T - e_3^T) \\ &\quad - (e_2 - e_4)Z_2(e_2^T - e_4^T) - 2(e_3 - e_2)Z_2(e_3^T - e_2^T) \\ &\quad - e_9Z_4e_9^T - 2e_8Z_4e_8^T \end{aligned}$$

Considering (28), by **Lemma 2.3** we have

$$\begin{bmatrix} \Sigma_{i1}(t) & \Gamma^T(t)M \\ M\Gamma(t) & -M \end{bmatrix} < 0$$

Since the uncertainty described by (4) and (5), we obtain

$$\begin{bmatrix} \Sigma_{i1} & \Gamma^T M \\ M\Gamma & -M \end{bmatrix} + \begin{bmatrix} \varepsilon_{i2}^T H_i F_i(t) \varepsilon_{i1} + \varepsilon_{i1}^T F_i^T(t) H_i^T \varepsilon_{i2} & \varepsilon_{i1}^T F_i^T(t) H_i^T M \\ MH_i F_i(t) \varepsilon_{i1} & 0 \end{bmatrix} < 0$$

That is

$$\begin{bmatrix} \Sigma_{i1} & \Gamma^T M \\ M\Gamma & -M \end{bmatrix} + \begin{bmatrix} \varepsilon_{i2}^T & 0 \\ M & 0 \end{bmatrix} H_i F_i(t) \varepsilon_{i1} + \varepsilon_{i1}^T F_i^T(t) H_i^T \begin{bmatrix} \varepsilon_{i2} & M \\ 0 & 0 \end{bmatrix} < 0$$

By **Lemma 2.2** we get

$$\begin{bmatrix} \Sigma_{i1} & \Gamma^T M \\ M\Gamma & -M \end{bmatrix} + \frac{1}{\delta_1} \begin{bmatrix} \varepsilon_{i2}^T & 0 \\ M & 0 \end{bmatrix} H_i H_i^T \begin{bmatrix} \varepsilon_{i2} & M \\ 0 & 0 \end{bmatrix} + \delta_1 \begin{bmatrix} \varepsilon_{i1}^T \varepsilon_{i1} & 0 \\ 0 & 0 \end{bmatrix} < 0$$

Therefore, we obtain the linear matrix inequality (26). Similarly, we can get LMI (27). Finally, following from the latter proof of **Theorem 3.1**, we know that the uncertain systems described by (1) and (2) are stochastically stable. This completes the proof. ■

4. Numerical examples

In this section, two numerical examples are given to show the effectiveness and the improvement of the proposed method over some previous ones.

Example 1: Consider the nominal neutral Markovian jump system with $N=2$ and the following parameters are used:

$$\dot{x}(t) - C_i \dot{x}(t-0.1) = A_i x(t) + B_i x(t-d(t))$$

where

$$\begin{aligned} [\pi_{ij}] &= \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}, \quad i, j \in S = \{1, 2\} \\ A_1 &= \begin{bmatrix} 0.4 & 0.5 \\ 1.0 & -1.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.0 & 0.1 \\ 0 & -2.0 \end{bmatrix} \\ B_1 &= \begin{bmatrix} -0.6 & 0.5 \\ 0.1 & -0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1.0 & -0.2 \\ 0.2 & -0.5 \end{bmatrix} \\ C_1 &= \begin{bmatrix} -0.3 & 0.1 \\ 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.5 & -0.2 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

We assume that

$$d_1 = 0.2, \quad d_2 = 0.6, \quad \mu = 0.5$$

By **Theorem 3.1**, with the help of LMI toolbox in Matlab we get a group of matrices for the solution to guarantee the stochastic stability for the system as follows (For simplicity, we only list the matrices for P_i, Q_j).

$$\begin{aligned} P_1 &= \begin{bmatrix} 0.8604 & -0.9435 \\ -0.9435 & 2.1713 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.1423 & -0.5869 \\ -0.5869 & 1.7253 \end{bmatrix} \\ Q_1 &= \begin{bmatrix} 0.8998 & -0.6011 \\ -0.6011 & 1.6542 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1.9452 & -0.4873 \\ -0.4873 & 2.0454 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 1.8418 & -0.1236 \\ -0.1236 & 2.0157 \end{bmatrix} \end{aligned}$$

Finally, in order to show the validity of the method, we show the trajectory of the system state under an initial state in the following Figure 1. It can be seen that the nominal neutral Markovian jump system of **Example 1** has reached the steady state in 10 seconds under the initial state $x_0 = (-2, 0.05)^T$.

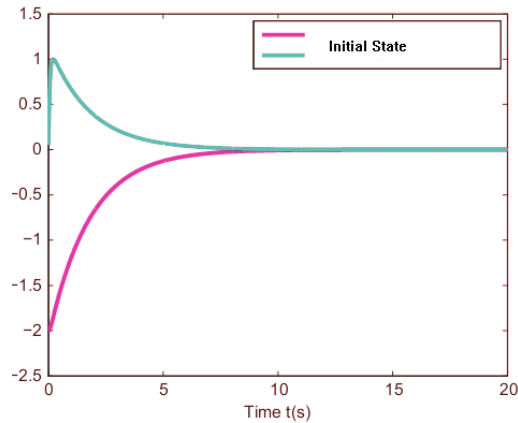


Figure 1. The trajectory of the system state under the initial data $x_0 = (-2, 0.05)^T$

Example 2: Consider the uncertain neutral Markovian jump system with $N=2$ and the following parameters are used:

$$\dot{x}(t) - (C_i + \Delta C_i(t))\dot{x}(t - \tau) = (A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))x(t - d(t))$$

where

$$\begin{aligned} [\pi_{ij}] &= \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}, \quad i, j \in S = \{1, 2\} \\ A_1 &= \begin{bmatrix} -1.2 & 0 \\ 0.1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & 0.1 \\ -0.1 & -1 \end{bmatrix} \\ B_1 &= \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.4 & 0.4 \\ -1.2 & -0.6 \end{bmatrix} \\ C_1 &= \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix} \\ H_1 = H_2 &= \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad F_1 = F_2 = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix} \\ E_{A1} = E_{A2} &= \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix}, \quad E_{B1} = E_{B2} = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix} \\ E_{C1} = E_{C2} &= \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \end{aligned}$$

We assume that $d_1 = 0.05$, $d_2 = 0.17$, $\mu = 2.6$ and apply the criteria in Sun et.al. [6], Qiu et al., [7] and **Theorem 3.2** in this paper and obtain the allowed upper bound of τ . We show the results in the following Table 1:

Table 1. The allowed upper bound of τ

Method	$d_2=0.17, \mu=2.6$
Qiu et.al.[7]	0.60
Sun et.al.[6]	0.62
Theorem 3.2 in this paper	0.65

From the above table, we know that the maximum upper bound of delay $\tau = 0.65$ in this paper by setting $d_2=0.17$, $\mu=2.6$, while the maximum upper bound of delay $\tau = 0.60$ for [7], $\tau = 0.62$ for [6]. It can be seen that our method is less conservative than some existing ones.

5. Conclusions

In this paper, the problem of stochastic stability for uncertain neutral Markovian jump systems with mixed delays is considered. The main contribution of this paper contains the following twofold: one is the extension of delay-dependent stochastic stability conditions for Markovian jump delay systems to uncertain neutral Markovian jump systems and the other is less conservative by the new method which has utilized new types of Lyapunov functional.

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