Design of Incremental Redundancy Hybrid-ARQ with Rate Compatible LDPC Codes

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Abstract

Mobile communication systems have been adopting link adaptive transmission schemes such as adaptive modulation and coding (AMC) and hybrid-ARQ (HARQ). Several wireless mobile communication systems have considered highly efficient incremental redundancy (IR) hybrid ARQ scheme. However IR-HARQ with LDPC codes has not been specified yet. In this paper, we propose an IR-HARQ scheme with rate compatible LDPC codes which adopts the notion of k-SR node and the propagation rate of soft output. Based on the design rule of rate compatible LDPC codes, we define the transmission priority of code bits and propose the sub-packet construction rule. We present the throughput performance of IR-HARQ with various modulation and coding and multi-antenna modes. Consequently, the proposed scheme provides the improvement of system throughput by elaborate link adaptation with CQI information.

Keywords: *IR-HARQ, LDPC, rate compatible LDPC codes*

1. Introduction

The benefits of adapting the transmission parameters in a wireless system to the changing channel conditions are well known. The process of modifying the transmission parameters to compensate for the variations in channel conditions is known as *link adaptation*. Hybrid ARQ (HARQ) is well known implicit link adaptation scheme which improves system throughput combining the error correcting codes and retransmission scheme. It has been adopted in several wireless mobile communication systems such as HSPA, 1xEV-DO, and LTE, *etc*. The HARQ is classified as Chase combining HARQ (type-III HARQ) and Incremental Redundancy (IR) HARQ (type-II HARQ) with rate compatible codes. The IR-HARQ transmits parity bits incrementally when the retransmission is requested. The IR-HARQ normally provides higher throughput than the Chase combining by adaptive transmission in accordance with channel variations.

For the IR-HARQ, rate compatible codes design is proactively necessary. There have been IR-HARQ schemes using rate compatible punctured convolutional codes (RCPC codes)[1] and rate compatible punctured turbo codes (RCPT codes) [2]. The design of rate compatible LDPC codes was proposed in [3-6]. Also, design of rate compatible LDPC codes for IR-HARQ was proposed in [8] and [9]. However, there has not been research on IR-HARQ scheme itself with LDPC codes. In this paper, we consider popular block LDPC codes with quasi-cyclic extension [7] and propose an IR-HARQ scheme which can provide adaptive transmission with various type of code rate, modulation level and MIMO modes.

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This paper is organized as follows. In Section 2, we will consider the incremental redundancy HARQ with rate compatible punctured block LDPC codes based on *k*-SR (step recoverable) nodes and propagation rate of soft output. We propose the IR-HARQ with sub-packet generation method in Section 3. Also we present the throughput performance with computer simulation in Section 4. Finally, conclusions are made in Section 5.

2. Incremental Redundancy Hybrid-ARQ Scheme

In this Section, we present design method of incremental redundancy HARQ (type-II HARQ) schemes based on rate compatible LDPC codes.

Generally, the incremental redundancy is another implementation of the HARQ technique wherein instead of sending simple repeats of the entire coded packet, additional redundant information is incrementally transmitted if the decoding fails on the first attempt. Type-III HARQ also belongs to the class of incremental redundancy ARQ schemes. However, with type-III HARQ, each retransmission is self-decodable, which is not the case with type-II HARQ. Chase combining involves the retransmission by the transmitter of the same coded data packet. The decoder at the receiver combines these multiple copies of the transmitted packet weighted by the received SNR. Diversity gain is thus obtained. In the type-III HARQ with multiple redundancy version different puncture bits are used in each retransmission.

In this paper, we consider type-II IR HARQ with rate compatible LDPC codes which efficiently supports link adaptation such as AMC based on CQI. The proposed HARQ system consists of encoder packet module, channel encoder module, transmission buffering module with grouping (or shuffling) and sub-packet generation module as shown in Figure 1. Encoder packet module constitutes an encoder packet from MAC PDU. The encoder packet is channel encoded with LDPC mother codes and generates code packet. Then the systematic and parity bits of code packet are classified with transmission priority and shuffled with interleaving pattern. This shuffling has an effect that the parity bits are punctured uniformly within the same priority. Finally, sub-packets are generated with allocated code rate and modulation index based on CQI information.

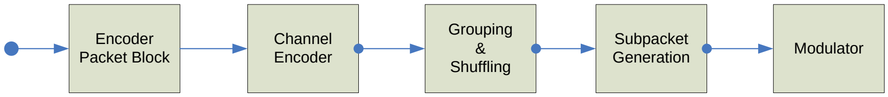

Figure 1. Block diagram of type-II HARQ system with rate compatible LDPC codes

2.1 Design of Rate Compatible Punctured Block LDPC Codes

In the previous work [9], we designed the rate compatible punctured BLDPC based on the notion of *k*-SR node. The *k*-SR node is the punctured parity bit node that can be recovered after *k*-th iteration at the receiver, where recovering means that the node is participating in the decoding process with non-zero soft output.

In the BLDPC, the parity check matrix **H** is defined as base matrix $\mathbf{H}^{\mathbf{b}}$ and (z x z) submatrix. For example, the parity check matrix **H** of BLDPC at a code rate 1/3, is shown in Figure 2. First, **H** is represented with (64 x 96) base matrix $\mathbf{H}^{\mathbf{b}}$, where $h^{b}_{\mathrm{i,j}}=1$ denotes the connection of *j*-th variable node block and *i*-th check node block in the bipartite graph. Each component of base matrix $\mathbf{H}^{\mathbf{b}}$ is substituted for (z x z) submatrix, where the submatrix is either zero matrix in case of $h^{b}_{\mathrm{i,j}}=0$ or the cyclic shift version of (z x z) identity matrix in case of $h^{b}_{\mathrm{i,j}}=1$.

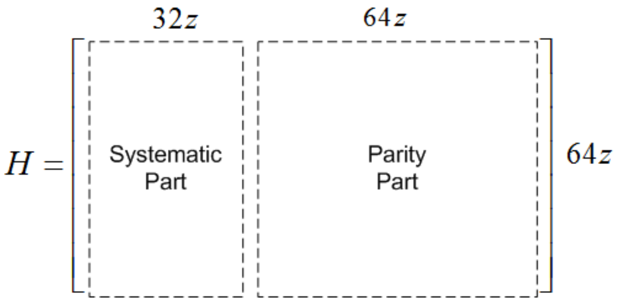

Figure 2. Parity check matrix of block LDPC, code rate r=1/3 (64z x 96z)

In Figure 3, the example of *k*-SR node block is presented at each puncturing stage. In non-punctured stage, no parity bit is punctured, i.e. every parity bit is transmitted and the code rate is lowest. In the 1st puncturing stage, every 1-SR node is punctured and the code rate gets higher than the non-punctured stage. Because every 1-SR node is punctured (*i.e.*, not transmitted), the decoder performs erasure decoding and can recover the punctured node after one iteration. In the 2nd punctured stage, every 2-SR node is additionally punctured (*i.e.*, not transmitted) and gets even higher than the 1st puncturing stage. In the next punctured stage, *k*-SR (*k*>2) nodes are additionally punctured likewise.

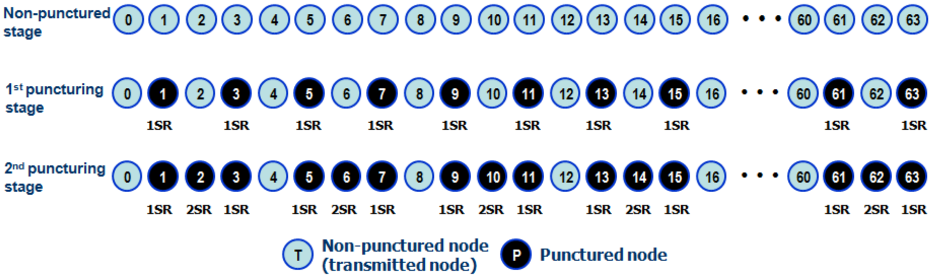

Figure 3. Parity bit block classification with *k*-SR node and puncturing order

3. Proposed Type-II HARQ System with Rate Compatible LDPC Codes

In this Section, we propose specific working procedures of type-II HARQ with rate compatible LDPC codes. The working procedures are shown in Figure 4. First, we constitute an encoder packet from MAC PDU or concatenated MAC PDUs which are input into the channel encoder module. In case of our simulation example, the size of encoder packet N_{EP} is as follows:

$$N_{EP} = 32z\,, \quad z \in \{3,6,9,12,18,24,36,48,54,72,96,108,144\}$$

When the size of MAC PDU does not match the size of encoder packet N_{EP}, padding or concatenation techniques of MAC PDUs are used.

In the LDPC encoding process, the encoder packet is encoded with LDPC mother codes and divided into systematic and parity blocks. In the systematic and parity blocks, there are classified kernel blocks which have different priorities each other. The kernel block classification is related to the transmission order of the packet. In other words, the systematic or parity bits that can have much influence on the decoding performance should be transmitted first. In other words, the kernel block whose transmission priority is higher will be interleaved in the preceding order in order to be transmitted first. In fact, the transmission order of the parity bit is opposite of the puncturing order, *i.e.*, *k*-SR node and (*k*-1)-SR node are transmitted sequentially and 1-SR node is transmitted last. However, there are many parity nodes with same transmission priority in a *k*-SR node group. Therefore, additional construction rule of sub-packet is required. We consider the propagation rate of soft output. For the fast propagation of soft output, the transmitted parity node should be uniformly distributed which can be proved in the bipartite graph. Considering the fast propagation of soft output, we propose the uniform construction rule of variable node group.

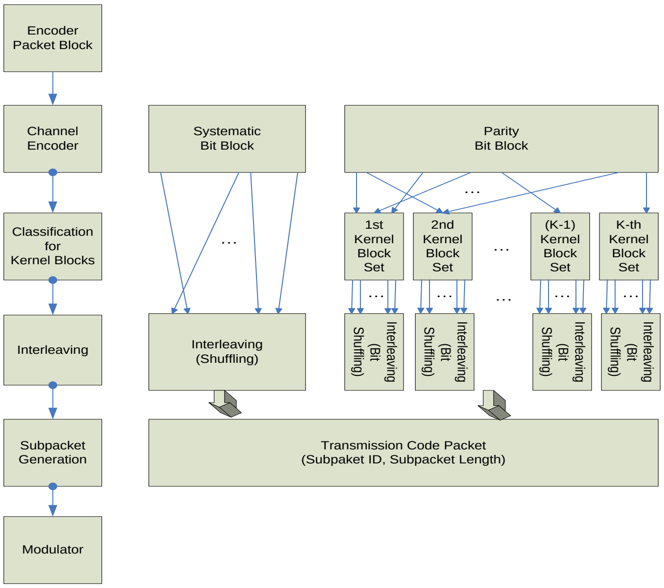

Figure 4. Working procedure of type-II HARQ system

3.1 Classification for Kernel Blocks

In BLDPC, systematic bits and parity bits are generated blockwise of which block size is z respectively. Encoded packet **C** is described as follows:

$$\mathbf{C} = \{\underline{\mathbf{s}}, \underline{\mathbf{p}}\} = \{\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_{31}, \mathbf{p}_0, \mathbf{p}_1, \cdots, \mathbf{p}_{63}\}$$

$$\underline{\mathbf{s}} = \{\mathbf{s}_0, \mathbf{s}_1, \cdots, \mathbf{s}_{31}\} = \{s_{0,0}, s_{0,1}, \cdots, s_{0,z-1}, s_{1,0}, s_{1,1}, \cdots s_{1,z-1}, \cdots, s_{31,0}, \cdots s_{31,z-1}\}$$

$$\underline{\mathbf{p}} = \{\mathbf{p}_0, \mathbf{p}_1, \cdots, \mathbf{p}_{63}\} = \{p_{0,0}, p_{0,1}, \cdots, p_{0,z-1}, p_{1,0}, p_{1,1}, \cdots p_{1,z-1}, \cdots, p_{63,0}, \cdots p_{63,z-1}\}$$

where $\underline{\mathbf{s}}=\{\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{31}\}$ and $\underline{\mathbf{p}}=\{\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{63}\}$ are systematic bit block and parity bit block, respectively. Also, *i* th parity bit block $\mathbf{p}_i=\{p_{i,0}, p_{i,1}, \ldots, p_{i,z-1}\}$, i=0~63, is composed of z code bits.

Parity bit blocks of code packet are classified as *k*-th kernel blocks. The (*k*-1)-st kernel block is regarded as having higher priority than the *k*-th kernel block. Also, the parity bits in the same kernel blocks are regarded as same priority. Parity bit blocks which are included in *k*-th kernel block set $\mathbf{\Psi}_k$, (k=1,2,…5) are given as follows.

1st kernel blocks: $\mathbf{\Psi}_1=\{\mathbf{P}_{\mathbf{B1(n)}}\}, \quad n = 0,1,2,3$

2nd kernel blocks: $\mathbf{\Psi}_2=\{\mathbf{P}_{\mathbf{B2(n)}}\}, \quad n = 0,1,2,3$

3rd kernel blocks: $\mathbf{\Psi}_3=\{\mathbf{P}_{\mathbf{B3(n)}}\}, \quad n = 0,1,2,3,\cdots,7$

4th kernel blocks: $\mathbf{\Psi}_4=\{\mathbf{P}_{\mathbf{B4(n)}}\}$, $n = 0,1,2,3,\cdots,15$

5th kernel blocks: $\mathbf{\Psi}_5=\{\mathbf{P}_{\mathbf{B5(n)}}\}$, $n = 0,1,2,3,\cdots,31$

The indices of parity bit blocks for *k*-th kernel block set $\mathbf{\Psi}_k$ are defined as $\mathbf{B}_k(\boldsymbol{n})$,

$$B_1(n) = 16n\,,\quad n = 0,1,2,3$$

$$B_2(n) = 16n + 8\,,\quad n = 0,1,2,3$$

$$B_3(n) = 8n + 4\,,\quad n = 0,1,2,3,\cdots,7$$

$$B_4(n) = 4n + 2\,,\quad n = 0,1,2,3,\cdots,15$$

$$B_5(n) = 2n + 1,\quad n = 0,1,2,3,\cdots,31\,.$$

3.2 Parity Bit and Systematic Bit Shuffling

Parity bits $\underline{\mathbf{p}}=\{\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{63}\}$ of code packet are shuffled before sub-packet generation. Shuffling (or interleaving) process performs block-wise and block-wise shuffling is related with kernel block classification and represented as $\mathbf{B}_k(\boldsymbol{n})$. We define $q(i)$ as i-th shuffled bit of code packet $i \in \{0,1,\cdots,(64z-1)\}$,

$$q(i) = p_{B_1(n),J_1(i)}\,,\quad n = BRO_2(i \bmod 4)\,,\quad i \in \{0,1,\cdots,(4z-1)\}$$

$$q(i) = p_{B_2(n),J_2(i)}\,, n = BRO_2(i \bmod 4)\,,\quad i \in \{4z, 4z+1,\cdots,(8z-1)\}$$

$$q(i) = p_{B_3(n),J_3(i)}\,,\quad n = BRO_3(i \bmod 8)\,,\quad i \in \{8z, 8z+1,\cdots,(16z-1)\}$$

$$q(i) = p_{B_4(n),J_4(i)}\,,\quad n = BRO_4(i \bmod 16)\,,\quad i \in \{16z, 16z+1,\cdots,(32z-1)\}$$

$$q(i) = p_{B_5(n),J_5(i)}\,,\quad n = BRO_5(i \bmod 32),\quad i \in \{32z, 32z+1,\cdots,(64z-1)\}$$

where $\mathrm{BRO}_m(i)$ means m bit reverse ordered value of i. Also, $J_k(i)$ is the representation of bitwise shuffling.

Systematic bit block of code packet is interleaved. We define $v(i)$, (i=0,1,…,32z-1) as *i*-th shuffled systematic bit.

$$v(i) = s_{n,J_5(i)}\,,\quad n = BRO_5(i \bmod 32)\,,\quad i \in \{0,1,\cdots,(32z-1)\}$$

3.3 Sub-packet Construction

In Figure 5, we present the example of sub-packet construction rule (parity nodes only) of the proposed IR-HARQ when the parity check matrix H is (96z x 64z) shown in Figure 2.

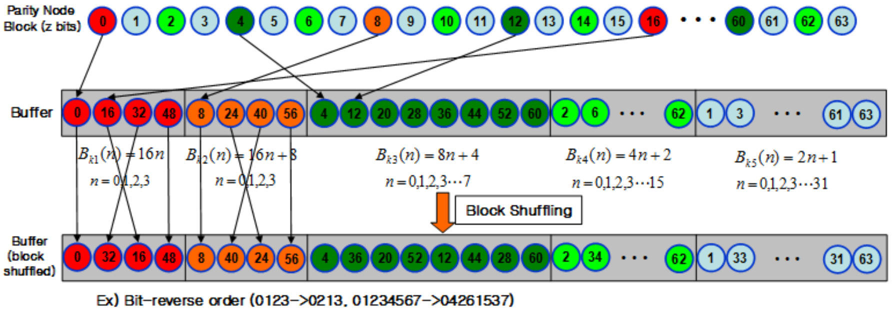

Figure 5. Sub-packet construction rule (parity nodes only) of proposed IR-HARQ

The shuffled systematic bits $x_s(i)$ and shuffled parity bits $x_p(i)$ are constructed. The sub-packet generation module concatenates $x_s(i)$ and $x_p(i)$ and generates a shuffled code packet $x(i)$

$$x(i)=\begin{cases} x_s(i), & 0\le i<32z \\ x_p(i-32z), & 32z\le i<96z \end{cases} .$$

Then it constructs sub-packet which is transmitted via physical subchannel with allocated subchannel number and modulation index. As shown in Figure 6, IR-HARQ uses circular transmission buffer in which the starting point and sub-packet length of k-th sub-packet are defined as sub-packet ID SPID$_k$ and sub-packet length SPL$_k$, respectively in accordance with various modulation and coding, MIMO modes.

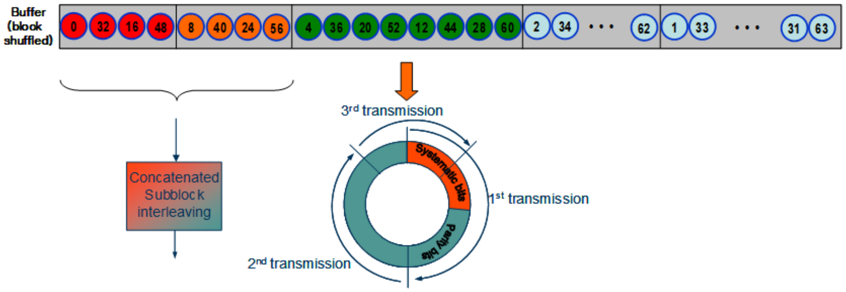

Figure 6. Circular Transmission Buffer

4. Simulation Results

In this Section, we present simulation results with proposed IR-HARQ scheme with rate compatible BLDPC codes. We assume a (64z x 96z) parity check matrix **H** given in Figure 2, at a code rate 1/3. Also, we assume 2x4 MIMO and 4x4 MIMO systems. In 2x4 MIMO, we consider both Alamouti coding (R1) and spatial multiplexing with ML detection (R2) scheme. Also, in 4x4 MIMO we just consider spatial multiplexing scheme. There are 9 possible transmission packet formats with various spectral efficiencies. With such transmission packet formats the link adaptation can be possible according to the channel status. The specific modulation type and code rate are given in Table 1. The transmission PF0 to PF2 provide low spectral efficiency and those PFs is appropriate for bad channel condition. On the other hands, PF6 and PF7 provide very high spectral efficiency with 64QAM modulation and higher code rate. Such PFs are usually adopted as transmission type for good channel condition.

Table 1. Spectral efficiency and AMC parameters on PF

PF	Spectral Efficiency	Mod	Code Rate
0	0.25	2(QPSK)	0.125
1	0.5	2(QPSK)	0.25
2	1	2(QPSK)	0.5
3	1.5	2(QPSK)	0.75
4	2	4(16QAM)	0.5
5	2.5	4(16QAM)	0.625
6	3	4(16QAM)	0.75
7	4	6(64QAM)	0.666667
8	5	6(64QAM)	0.833333

We analyze data throughput with various transmission packet formats. In Figure 7, the throughput performance of the proposed IR-HARQ is presented. In every transmission packet format, the proposed IR-HARQ provides data throughput gain at given combined E_b/N_o. Especially at a good channel condition, *i.e.*, PF6 or PF7, the performance improvement gets tremendously larger.

Based on the data throughput performance curve, we obtain so called “hull curve” which is the set of highest data throughput at the given combined E_b/N_o. Based on this hull curve, the wireless communication system assigns the most appropriate AMC set to the user and can improve the system throughput. With Figure 7, the hull curve consists of just the proposed IR-HARQ system.

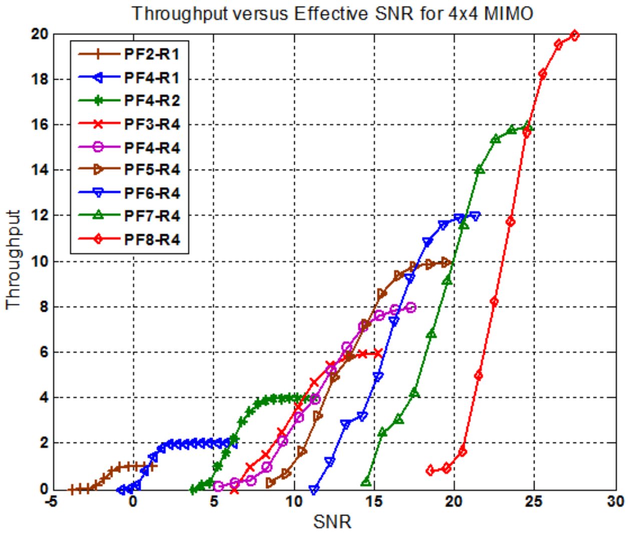

Figure 7. Data throughput with various transmission packet formats with MIMO

Consequently, our proposed IR-HARQ scheme with rate compatible block LDPC is a good solution to the wireless communication system.

5. Conclusions

In this paper, we proposed novel IR-HARQ scheme with rate compatible LDPC codes which adopts the notion of *k*-SR node and the propagation rate of soft output. In the proposed IR-HARQ scheme, we presented the subpacket construction rule considering the impact on the performance of decoding. The subpacket construction rule is related to the transmission order of the packet. In other words, the systematic or parity bits that can have much influence on the decoding performance should be transmitted first.

We presented the performance comparison of IR-HARQ. In every transmission packet format, the proposed IR-HARQ provides data throughput gain at given combined E_b/N_o. Especially at a good channel condition, the performance improvement gets tremendously larger. Consequently, the proposed scheme provides the improvement of system throughput by elaborate link adaptation with CQI information.

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