

Direct Robust Adaptive Control of High-Speed Train Based on Nonlinear and Time-Varying Models

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Abstract

This paper investigates the tracking control problem of high-speed trains, and a direct robust adaptive control strategy is designed to deal with the uncertainties and time-varying properties occurring in the train dynamics. The longitudinal dynamics of the train is a typical uncertain nonlinear presentation, whose parameters, especially the coefficients of the resistance cannot be acquired precisely and varying due to the complexity running conditions, and the impacts of these uncertainties are becoming even more serious as the speed increases. Although adaptive algorithm can improve a controller performance with parameterized uncertainties where the plant parameters are unknown but constant for all time, the stability properties can no longer be guaranteed when the plant parameters are varying with time even the speed of the variations are sufficiently small. In this paper, a traditional adaptive controller is designed based on the time-invariant assumption, and then, a kind of robust adaptive law is designed to improve the robustness when the plant parameters are varying slowly with time. Both the rigorous analysis and the simulation results are all verified the effectiveness of the proposed control scheme.

Keywords: Robust adaptive control, tracking control, High-speed train, ATO

1. Introduction

After the implementation of the CTCS (Chinese Train Control System) standards in 2002, Chinese high-speed railway systems have gone through a massive phase of upgrading and expansion. It is estimated that by the end of 2020, China will have 18, 000 km high-speed railway lines that will cover most areas of the whole country, with an operating speed of over 300 km/h [1]. Inevitably, Chinese next-generation railway systems will face with many challenges such as security techniques, operational strategies, riding comfort of passengers, and energy consumption. Whether or not a railway system can be operated safely and efficiently most depends on the performance of its ATC (Automatic Train Control) systems.

As an essential part of ATC, the ATO (Automatic Train Operation) system is designed to drive the train automatically and effectively, which is one of the most important trends in development of high-speed trains. Because the "distance-to-go speed control mode" has the shortest braking distance in general, if the deviations between train's actual running trajectory and its desired target speed curve derived from the distance-to-go speed curves are sufficiently small, the performance of the railway system's safety, efficiency and energy consumption will be greatly improved. So improving the tracking performance becomes the most important issue for the ATO system.

Because the dynamics of high-speed train is a typical nonlinear system, which is working under time-varying and uncertain conditions, accurate tracking performance is not easily

achieved for the ATO system, so it draws extensive attentions among domestic and overseas scientific and technical researchers [2-11]. Due to the simplicity of their implementations, PID based algorithms have been widely applied to the automatic train operation. Although the control precision is improved, but the frequent switching between acceleration and deceleration affects the riding comfort seriously, which also increases energy consumptions [1, 2]. With the development of artificial intelligent, many intelligent control strategies have been studied for the ATO system [3-6]. The fuzzy controller proposed in Ref. [3] can drive a train as skillfully as experienced human drivers, but the operating experience needed by the fuzzy logic is hard to summarize comprehensively and accurately [17]. Although the adaptive and neural network techniques are introduced to enhance the learning abilities of the fuzzy controllers [4-6], the tracking accuracy is hardly achieving the high-speed train's requirements. Subsequently, some advanced control theories have been studied for the ATO system. Ref. [7] developed a TILC controller for accurate station stop control of the train, and it can achieve extremely precision performance after enough learning cycles, but it may not meet the real-time requirement of the high-speed train. Employing the predictive control theory, Ref. [8, 9] developed the predictive-like controllers applied to the automatic train control, and the robust and safety performance is improved, but the massive data processing leads to an increasing cost for application. Recently, aiming to overcome the increasing impacts caused by the uncertainties exists during train operation, adaptive control strategies have been studied for the ATO [10, 11], but Ref. [10] is based on the time-invariant assumptions, and the convergence speed of the neural network used in Ref. [11] remains to improve and may fall into the local optimal point.

In this paper, the inherent nonlinearities and various uncertainties are discussed adequately, and a direct robust adaptive control scheme is developed to improve the tracking performance of high-speed train's tracking performance. The remainders are sectioned as follows: in Section 2, the dynamics of high-speed train are discussed and presented in a compact form. In Section 3, the traditional adaptive controller is designed in a direct manner based on the general time-invariant assumption. In Section 4, the actual time-varying condition is considered, and a robust adaptive law is used to improve the robust stability of the proposed control strategy. The conclusion is made in Section 5.

2. Dynamic Model of High-speed Train

During train operation, the dynamics are quite complicated, involving such as starting, speeding, braking, and stopping, not to mention the complex status under different loading and weather conditions. When coming to high-speed trains, the situation is more complicated and even intractable: the train as a whole is a time-varying high-dimensional nonlinear dynamical system with rich and complex dynamical behaviors. Therefore, accurately modeling the dynamics of high-speed train is by no means an easy task. In this paper, our main concerning is the dynamics in longitudinal direction, and the force diagram is shown in Figure 1.

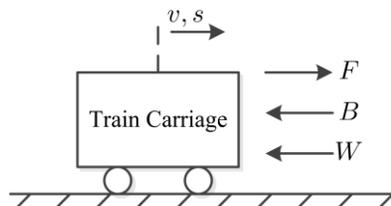


Figure 1. The longitudinal dynamics of high-speed train

where $v(t)$, $s(t)$ denote the train speed and location, $F(t)$ is the traction force, $B(t)$ is the braking force, $W(t)$ represents the running resistance suffered by the train. The traction force $F(t)$ and the braking force $B(t)$ are managed by ATO to regulate the train speed $v(t)$ tracking with the target speed accurately, and the running resistance $W(t)$ always exists during the whole operation and not controlled by the ATO.

The running resistance $W(t)$ experienced by the train comes from all fronts: steel rails, bow net and the air. It is quite difficult to derive an accurate formula or equation to precisely describe such compound resistance characteristic of high-speed train. Lots of experiments have shown that the total resistance $W(t)$ is proportional to the gross mass of the train, and the widely used empirical formula divided $W(t)$ into two parts: the mechanical resistance $W_r(t)$ and the aerodynamic drag $W_a(t)$ [12, 13], which can be shown as

$$w(t) = \underbrace{c_0(t) + c_v(t)v(t)}_{w_r(t)} + \underbrace{c_a(t)v^2(t)}_{w_a(t)} \quad (1)$$

where $w(t)$ denotes the running resistance on per mass unit, $c_0(t)$, $c_v(t)$, $c_a(t)$ are resistance coefficients obtained by wind tunnel test. The mechanical resistance $W_r(t)$ is dominant in the low speed range, while the aerodynamic drag is dominant in the high speed range. As the train speed increases, the aerodynamic drag $W_a(t)$ becomes the major part of the total running resistance. Because the resistance dynamics can be changed by the different loading conditions and the tough railway lines such as ramps, curves and tunnels, the precise value of the coefficients $c_0(t)$, $c_v(t)$, $c_a(t)$ are not only difficult to obtain but also varying with complex running conditions.

Theoretically, the train operation consists of three types of operation modes: traction mode, braking mode, and coasting mode. In the traction mode, the train are experienced by the traction force $F(t)$ and the resistance force $W(t)$, while in the braking mode, the composition force suffered by the train is mainly consists the braking force $B(t)$ and the resistance force $W(t)$, and in the coasting mode, the train is only experienced by the running resistance $W(t)$. Therefore, as for the traction force $F(t)$ and the braking force $B(t)$, only one of them is taken effect at the same time at most. If we treat $u(t)$ as the control action of the ATO, and defined as

$$u(t) = F(t) - B(t) \quad (2)$$

According to Newton's second law, the basic dynamics of high-speed train can be described as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \frac{1}{m}u(t) - \theta^T(t)\phi(x(t)) \end{cases} \quad (3)$$

with

$$\theta(t) = [c_0(t), c_v(t), c_a(t)]^T, \quad \phi(x(t)) = [1, x_2(t), x_2^2(t)]^T$$

where $x_1(t) = s(t)$, $x_2(t) = v(t)$ represent the train location and speed respectively, $\theta(t)$ is the running resistance coefficients vector which is unknown and time-varying parameters, $\phi(x(t)) : \mathbb{R}_+^2 \rightarrow \mathbb{R}^3$ is the nonlinear mapping vector associated by the running resistance, m is the equivalent mass of the train which is also a unknown

parameter due to the changing load conditions but it is constant in an independent operations.

Remark 2.1: For facilitating the design expression, the time domain parameter t will be omitted in the situations where confusions will not be caused.

3. Traditional Adaptive Controller

The objective of ATO is to generate the appropriate control action $u(t)$, so that the running trajectory of high-speed train can tracking with the target speed curve accurately. For simplicity of the exposition, we shall first treat the general ideal time-invariant assumption which is commonly used in the domain of adaptive control, and the detail will be given in the following.

Assumption A: The model parameters m and $\theta(t)$ in plant (3) are unknown but constant for all t .

In this section, a direct regular adaptive controller is designed where Assumption A holds. By using the backstepping technique, the control structure is developed based on the CE (Certainty-Equivalent) principal, where the SPR-Lyapunov [14] design approach is employed to derive the adaptive law in the straightforward manner. The proposed algorithm guarantees the closed-loop system is asymptotic stable, and both the rigorous theoretical analysis and the simulation results are all verified the effectiveness of the proposed algorithm.

3.1. Design procedure

The backstepping method can transform the tracking problem to a stabilizing problem and is able to deal with the nonlinear properties in plant (3) effectively [18]. Let x^* be the desired location information in the target curve, which is second order differentiable and assume that $\dot{x}^*, \ddot{x}^* \in \mathcal{L}_\infty$. Because the dynamic model of the train is a typical second-order system, two procedures are needed to derive the adaptive controller according to its relative order.

Step 1: The derivative of the tracking error $z_1 = x_1 - x^*$ is

$$\dot{z}_1 = x_2 - \dot{x}^* \quad (4)$$

Because x_2 is the train speed which can be measured directly, we treat it as a “virtual control”. Obviously, the following feedback control law

$$x_2 = -c_1 z_1 + \dot{x}^* \quad (5)$$

where $c_1 > 0$ is a controller parameter, can stabilize the system (4). Because the derivative of the Lyapunov function $V_1(z) = (1/2)z_1^2$ along the trajectory of (4) and (5) is

$$\dot{V}_1 = -c_1 z_1^2 \leq 0 \quad (6)$$

But x_2 is not the actual input of the plant (3). According to the idea of backstepping, let z_2 be the error between the actual and desired control law of x_2

$$z_2 = c_1 z_1 - \dot{x}^* + x_2 \quad (7)$$

Substituting (7) into (4), we obtain

$$\dot{z}_1 = -c_1 z_1 + z_2 \quad (8)$$

The discussion of Step 1 is completed, and we obtained the results (8) and (6) which will be used in the next step.

Step 2: Using (7), the derivative of z_2 can be expressed as

$$\dot{z}_2 = -c_1^2 z_1 + c_1 z_2 - \ddot{x}^* + \frac{1}{m} u - \theta^T \phi(x) \quad (9)$$

Adding and subtracting $c_2 z_2$, we rewrite (9) as

$$\dot{z}_2 = -c_2 z_2 + \rho u - \theta^T \phi(x) - \varphi(z, x^*) \quad (10)$$

with

$$\rho = \frac{1}{m}, \quad \varphi(z, x^*) = c_1^2 z_1 - (c_1 + c_2) z_2 + \ddot{x}^*$$

where $c_2 > 0$ is arbitrary. The actual input u of the plant (3) is already appearing in the right side of (10). If the model parameters are known exactly, a candidate control law is

$$u = m [\theta^T \phi(x) + \varphi(z, x^*)] \quad (11)$$

where $\rho = 1/m$ is used. Because the plant parameters of m and θ are unknown, the control law (11) cannot be implemented. Based on the CE principal, the control law (11) can be implemented together with appropriate adaptive laws for the unknown parameters. Direct adaptive algorithm is studied in this paper, and the CE form of the control law (11) is given by

$$u = \hat{m}(t) [\hat{\theta}^T(t) \phi(x) + \varphi(z, x^*)] \quad (12)$$

where $\hat{m}(t)$, $\hat{\theta}(t)$ are the estimates of m and θ at time t respectively. And then, the rest task is searching an adaptive law to generate these estimates on-line. We re-express (10) to its PDO (Polynomial Differential Operator) form as

$$\begin{aligned} z_2 &= \frac{1}{s + c_2} \cdot \rho \{ u - m [\theta^T \phi(x) + \varphi(z, x^*)] \} \\ &= \frac{1}{s + c_2} \left\{ \rho \cdot \tilde{m} [\hat{\theta}^T \phi(x) + \varphi(z, x^*)] + \tilde{\theta}^T \phi(x) \right\} \end{aligned} \quad (13)$$

where $s = d/dt$ is the differential operator, $\tilde{m} = \hat{m} - m$, $\tilde{\theta} = \hat{\theta} - \theta$ are the estimation errors of the unknown parameters, and the direct control law (12) is used. Based on the CE principal, the estimation of z_2 can be generated by

$$\hat{z}_2 = \frac{1}{s + c_2} \cdot \hat{\rho} \left\{ u - \hat{m} [\hat{\theta}^T \phi(x) + \varphi(z, x^*)] \right\} = 0 \quad (14)$$

where $\hat{\rho}$ is the estimate of ρ , and the same control law (12) is used. From equation (14), we can obtain that the output estimation error, defined as $\varepsilon \triangleq z_2 - \hat{z}_2$ is the same as the error state z_2 . So there is no need to generate \hat{z}_2 , and furthermore, the estimate $\hat{\rho}$ of ρ is not required, which implies that the output estimation error $\varepsilon = z_2$ satisfies the following derivative equation

$$\dot{z}_2 = -c_2 z_2 + \rho \cdot \tilde{m} [\hat{\theta}^T \phi(x) + \varphi(z, x^*)] + \tilde{\theta}^T \phi(x) \quad (15)$$

Theorem 1: If the control law (12) together with the following adaptive law

$$\begin{aligned}\dot{\hat{\theta}} &= -\Gamma\phi(x)z_2, \\ \dot{\hat{m}} &= -\gamma \left[\hat{\theta}^T\phi(x) + \varphi(z, x^*) \right] z_2.\end{aligned}\tag{16}$$

where $\gamma > 0$, $\Gamma = \Gamma^T > 0$ are arbitrary which have suitable dimensions, are applied to system (3), and subject to Assumption A, guarantees the following properties:

- i). All signals in the closed-loop system are bounded;
- ii). The tracking error $z_1(t)$ converges to zero exponentially fast.

Proof: Consider the following Lyapunov-like function candidate

$$V_2(z, \tilde{m}, \tilde{\theta}) = V_1(z) + \frac{1}{2}z_2^2 + \frac{\rho}{2\gamma}\tilde{m}^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}\tag{17}$$

where $\rho = 1/m > 0$ is used. The time derivative of V_2 along the trajectory of (8) and (15) is given by

$$\begin{aligned}\dot{V}_2 &= -c_1z_1^2 - c_2z_2^2 + \frac{\rho}{\gamma}\tilde{m} \left\{ \dot{\hat{m}} + \gamma \left[\hat{\theta}^T\phi(x) + \varphi(z, x^*) \right] z_2 \right\} \\ &\quad + \tilde{\theta}^T\Gamma^{-1} \left[\dot{\hat{\theta}} + \Gamma\phi(x)z_2 \right]\end{aligned}\tag{18}$$

If the adaptive laws (16) are used, we can obtain

$$\dot{V}_2 = -c_1z_1^2 - c_2z_2^2 \leq 0\tag{19}$$

which implies that $V_2, \tilde{m}, \hat{m}, \tilde{\theta}, \hat{\theta} \in \mathcal{L}_\infty$ and $z_1, z_2 \in \mathcal{L}_\infty \cap \mathcal{L}_2$. Together with the definition of z_1 and z_2 , we can obtain $x_1, x_2 \in \mathcal{L}_\infty$, which implies that $\dot{z}_1, \dot{z}_2, u \in \mathcal{L}_\infty$. Therefore, all the signals in the closed-loop system (8), (15) and (16) are bounded. Furthermore, due to $z_1 \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\dot{z}_1 \in \mathcal{L}_\infty$, we can establish that $\lim_{t \rightarrow \infty} z_1(t) = 0$ through Babalat lemma. This completes the proof of Theorem 1.

3.2. Numerical simulation

A corresponding numerical simulation is conducted to verify the effectiveness of the proposed adaptive control scheme (12) and (16) where Assumption A holds. The target speed curve used in this simulation is shown in Figure 2.

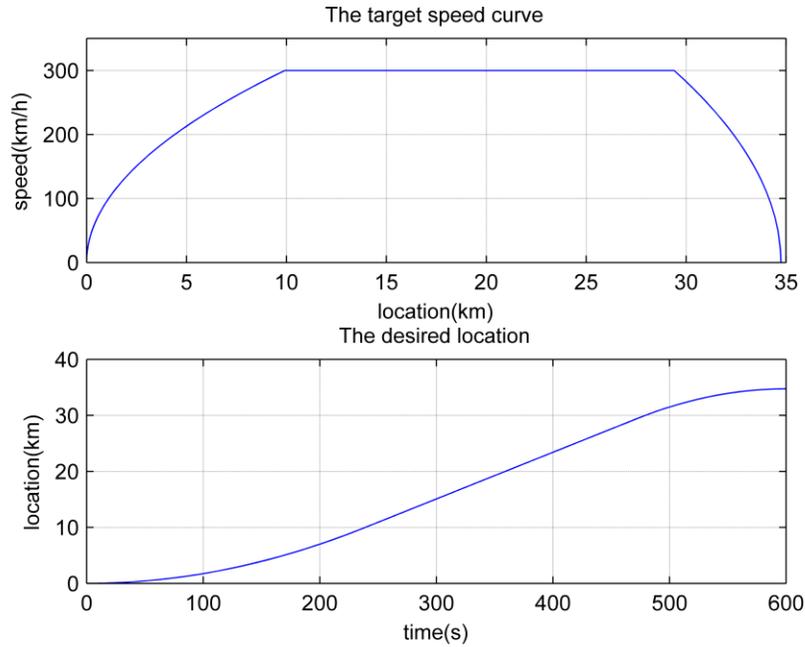


Figure 2. The target speed curve

The train model used here is similar to the CRH-300 series which have been operated on the Beijing-Tianjin intercity railway for years. The vehicle parameters of CRH-300 together with the controller ones are shown in Table 1.

Table 1. The parameters for simulation

| Category | Description | Unit | Value |
|------------|----------------|--------|-----------------|
| CRH3 | m (mass) | t | 510 |
| | c_0 | N/t | 7.75 |
| | c_v | null/t | 0.06327 |
| | c_a | null/t | 0.00128 |
| Controller | c_1 | null | 0.1 |
| | c_2 | null | 0.1 |
| | γ | null | 1×10^5 |
| | $\Gamma_{1,1}$ | null | 1×10^4 |
| | $\Gamma_{2,2}$ | null | 1 |
| | $\Gamma_{3,3}$ | null | 0.01 |

where Γ is chosen as a diagonal matrix.

The simulation results are given by Figure 3 ~ Figure 5. As shown in Figure 3, the estimates of the unknown parameters are converging to the steady state rapidly. The asymptotic convergence of the tracking errors are verified by Figure 4. The control action and the resulting acceleration are presented in Figure 5, from which we can obtain that the control strategy generated by Theorem 1 is smooth and flat.

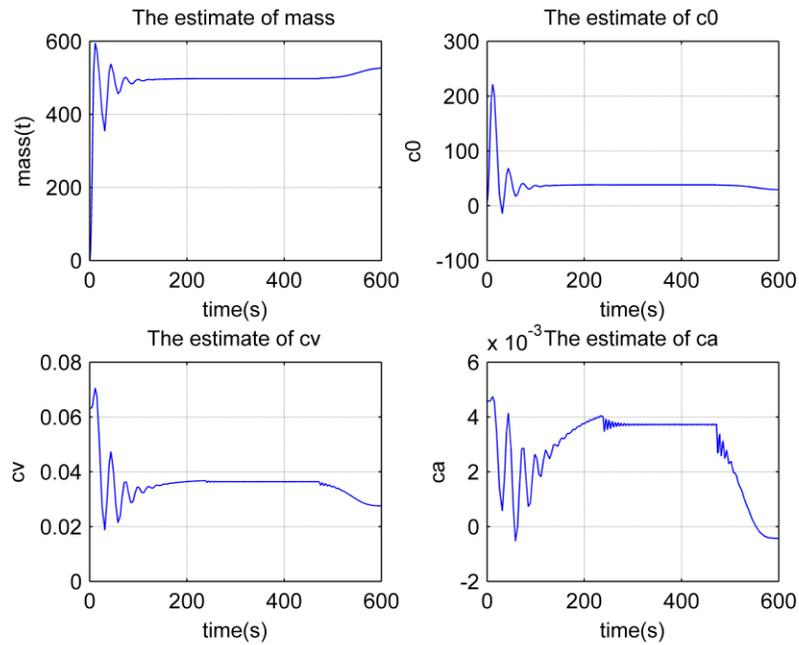


Figure 3. The estimates of the unknown parameters under Theorem 1

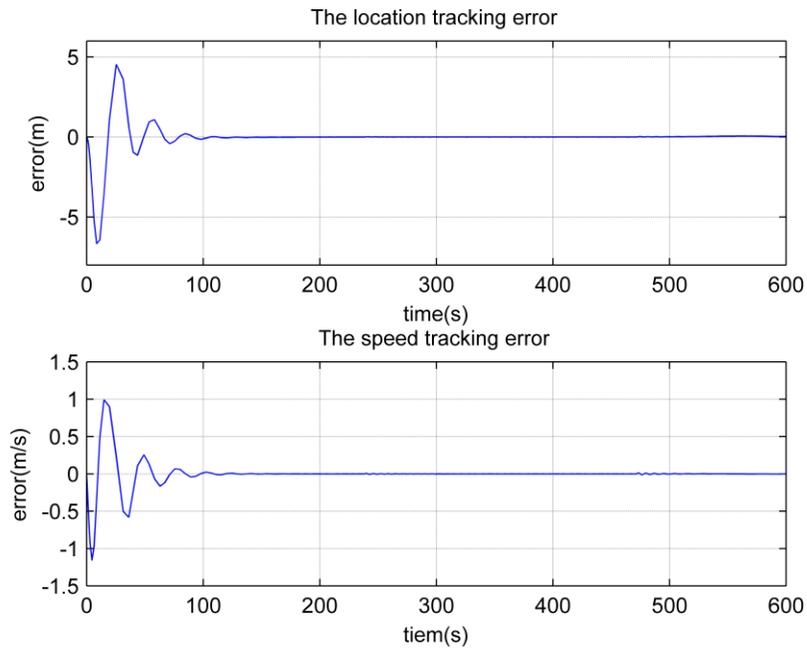


Figure 4. The tracking errors under Theorem 1

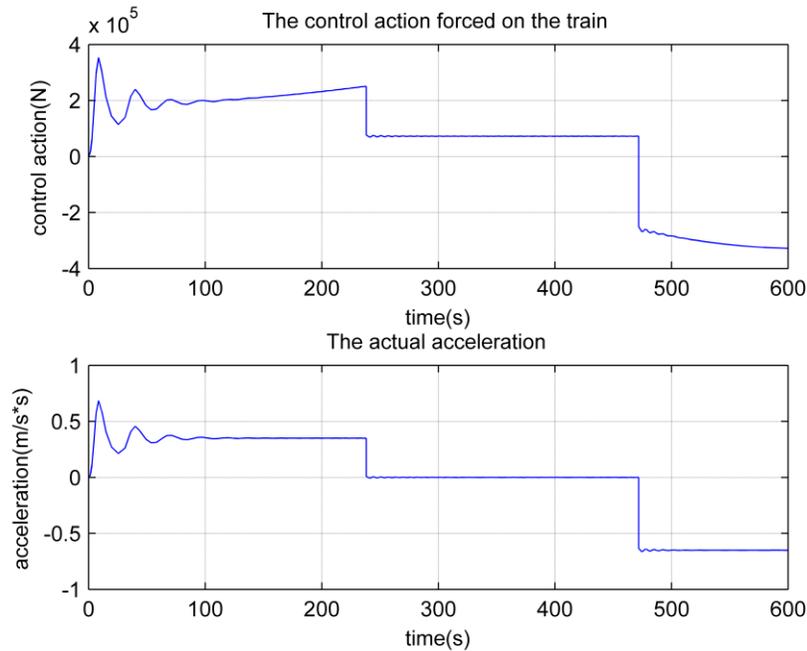


Figure 5. The control action and the actual acceleration under Theorem 1

4. Robust modification

Because adaptive control is nonlinear certainly makes it inherently much more difficult to ascertain stability robustness, so from the perspective of practice, the stability robustness of adaptive control is a major concern. Up to now, many literatures have revealed that the adaptive controller designed for the time-invariant models will no longer guarantee the stability properties when the plant parameters are varying with time, even the variations are sufficiently small [14-16].

In this section, the impacts of the time-varying characteristics possessed by the resistance coefficients vector $\theta(t)$ are discussed first, and we can see that the stability results achieved in the Theorem 1 are no longer valid. And then, the standard projection algorithm is used to improve the robustness of the traditional adaptive controller. Rigorous theoretical analysis shows that the projected adaptive scheme can ensure the stability of the closed-loop system (8) and (15) where the parameters' variations are sufficiently small, and the tracking error will converge to a residual set which is related to the variations of the parameters. And when the time-varying property of the parameters is disappeared, the tracking error will regain the asymptotic convergence. At last, a series of numerical simulations has been conducted to verify the proposed control strategy.

4.1. Impacts of the time-varying parameters

In the actual operation environment, the resistance coefficients vector $\theta(t)$ is varying with the complicated running conditions, and the variation forms cannot be assumed. But an upper bound $\theta(t)$ and the boundedness of its variations can be assumed due to the dynamic characteristics of high-speed train and the sufficient priori knowledge acquired by the on-board signal equipment. Without loss of generality, we give the following assumptions.

Assumption B: There exists a known convex set $S \subset \mathbb{R}_+^3$ which is given by

$$S = \{\theta \in \mathbb{R}_+^3 | g(\theta) = (\theta - \theta^*)^T (\theta - \theta^*) \leq M_0^2\} \quad (20)$$

where θ^* , M_0 are the nominal value and the variation span of $\theta(t)$ respectively, such that the plant parameter $\theta(t)$ are constraint to lie in S for all t .

Assumption C: The variation of $\theta(t)$ is bounded and sufficiently small, which implies that there exists $\mu > 0$ such that

$$\|\dot{\theta}(t)\| \leq \mu \quad (21)$$

for all t , where μ is sufficiently small.

Remark 3.1: Assumption B and C give the constraint for the estimates $\hat{\theta}(t)$ of the parameters $\theta(t)$, and imply that there exists $k > 0$ such that $\|\hat{\theta}_1 - \hat{\theta}_2\| \leq k$ for all $\hat{\theta}_1, \hat{\theta}_2 \in S$.

Holding Assumption B and C, the time derivatives of V_2 given by (18) changed as

$$\begin{aligned} \dot{V}_2 = & -c_1 z_1^2 - c_2 z_2^2 + \frac{\rho}{\gamma} \tilde{m} \left\{ \dot{\tilde{m}} + \gamma [\varphi(z, x^*) + \hat{\theta}^T \phi(x)] z_2 \right\} \\ & + \tilde{\theta}^T \Gamma^{-1} [\dot{\hat{\theta}} + \Gamma \phi(x) z_2] - \rho \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}, \end{aligned} \quad (22)$$

If Theorem 1 is still used, then

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 - \rho \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} \quad (23)$$

Because the sign of the third term in right side of (23) is unknown, the boundedness of $V_2, z_1, z_2, \tilde{m}, \tilde{\theta}$ cannot be assured from the properties of V_2 and its derivative given by (17) and (23) no matter how small $\dot{\theta}(t)$ are.

4.2. Robust adaptive law

Holding Assumption B, the standard projection algorithm is used to modify the adaptive law for $\hat{\theta}(t)$ as

$$\dot{\hat{\theta}} = \text{Pr}(-\Gamma \phi z_2) = \begin{cases} -\Gamma \phi z_2, & \text{if } \hat{\theta} \in S_0 \text{ or} \\ -\Gamma \phi z_2 + \Gamma \frac{\nabla g \nabla g^T}{\nabla g^T \Gamma \nabla g} \Gamma \phi z_2, & \text{if } \hat{\theta} \in S_\delta \text{ and } (\Gamma \phi z_2)^T \nabla g \geq 0 \\ -\Gamma \phi z_2, & \text{if } \hat{\theta} \in S_\delta \text{ and } (\Gamma \phi z_2)^T \nabla g < 0 \end{cases} \quad (24)$$

with

$$S_0 = \{\hat{\theta} \in \mathbb{R}_+^3 | g(\hat{\theta}) < M_0^2\}, \quad S_\delta = \{\hat{\theta} \in \mathbb{R}_+^3 | g(\hat{\theta}) = M_0^2\}.$$

where, $\nabla g = \hat{\theta} - \theta^*$ is the gradient of $g(\hat{\theta})$ at point of $\hat{\theta}$, S_0 is the interior of S , S_δ is the boundary of S .

Theorem 2: The same control schemes expressed in Theorem 1 replaced by the modified adaptive law for $\hat{\theta}(t)$ given by (24) guarantees that

- i). All signals in the closed-loop system are bounded;

ii). The tracking error z_1 converges to the following residual

$$D = \left\{ z_1 \in \mathbb{R} \mid z_1^2 \leq \frac{\lambda_M}{2\alpha}(\alpha k^2 + 2\mu k) \right\} \quad (25)$$

where λ_M is the maximum eigenvalue of the symmetric matrix Γ^{-1} , the value of α will be clarified in the following contents, exponentially fast.

iii). If the time-varying properties of $\theta(t)$ are disappeared, the tracking error $z_1(t)$ will regain the asymptotic convergence.

Proof: It follows from (24) that whenever $\hat{\theta} \in S_\delta$, we have $(\Gamma\phi z_2)^T \nabla g \geq 0$, which implies that the vector $\dot{\hat{\theta}} = -\Gamma\phi z_2$ points either towards S_0 or along the tangent plane S_δ at the point of $\hat{\theta}$, and in the case of $(\Gamma\phi z_2)^T \nabla g < 0$, $\hat{\theta}$ will be ensure to stay in S through the projection operator. Because $\hat{\theta}(0)$ is chosen to lie in S , it follows that $\hat{\theta}(t)$ will never leave S , i.e., $\hat{\theta}(t) \in S, \forall t > 0$, so $\hat{\theta}(t), \tilde{\theta}(t) \in \mathcal{L}_\infty$ can be established.

The modified adaptive law (24) has the same form as the one without projection except for the additional term

$$Q = \begin{cases} 0, & \text{if } \hat{\theta} \in S_0 \text{ or if } \hat{\theta} \in S_\delta \text{ and } (\Gamma\phi z_2)^T \nabla g \geq 0 \\ \Gamma \frac{\nabla g \nabla g^T}{\nabla g^T \Gamma \nabla g} (\Gamma\phi z_2), & \text{if } \hat{\theta} \in S_\delta \text{ and } (\Gamma\phi z_2)^T \nabla g < 0 \end{cases} \quad (26)$$

which will also introduce an corresponding term in the time derivatives of V_2 as

$$\tilde{\theta}^T \Gamma^{-1} Q = \begin{cases} 0, & \text{if } \hat{\theta} \in S_0 \text{ or if } \hat{\theta} \in S_\delta \text{ and } (\Gamma\phi z_2)^T \nabla g \geq 0 \\ \frac{\tilde{\theta}^T \nabla g}{\nabla g^T \Gamma \nabla g} \nabla g^T (\Gamma\phi z_2), & \text{if } \hat{\theta} \in S_\delta \text{ and } (\Gamma\phi z_2)^T \nabla g < 0 \end{cases} \quad (27)$$

Let us analyze the second case, due to the convex property of S and the facts $\theta, \theta^* \in S$, we have $\tilde{\theta}^T \nabla g = (\hat{\theta} - \theta)^T (\hat{\theta} - \theta^*) \geq 0$ when $\hat{\theta} \in S_\delta$. Because the second case of (27) implies $\nabla g^T (\Gamma\phi z_2) = (\Gamma\phi z_2)^T \nabla g < 0$, it follows that $\tilde{\theta}^T \Gamma^{-1} Q \leq 0$. Therefore, the additional term $\tilde{\theta}^T \Gamma^{-1} Q$ introduced by the projection algorithm can only make \dot{V}_2 more negative. So, by using the modified adaptive law (24), we can obtain

$$\dot{V}_2 \leq -c_1 z_1^2 - c_2 z_2^2 - \tilde{\theta}^T \Gamma^{-1} \dot{\theta} \quad (28)$$

Adding and subtracting αV_2 , we re-express (28) as

$$\dot{V}_2 \leq -\alpha V_2 - \frac{1}{2}(2c_1 - \alpha)z_1^2 - \frac{1}{2}(2c_2 - \alpha)z_2^2 + \frac{\alpha}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} - \tilde{\theta}^T \Gamma^{-1} \dot{\theta} \quad (29)$$

If we choose $0 < \alpha \leq \min\{2c_1, 2c_2\}$, we can obtain

$$\dot{V}_2 \leq -\alpha V_2 + \frac{\alpha}{2}\lambda_M k^2 + \lambda_M \mu k \quad (30)$$

under assumption B and C. Which implies that for $V_2 \geq V_0 = \frac{\lambda_M}{2\alpha}(\alpha k^2 + 2\mu k)$, $\dot{V}_2 < 0$. So $V_2, z_1, z_2 \in \mathcal{L}_\infty$, which implies $x_1, x_2 \in \mathcal{L}_\infty$. Furthermore, we can establish that

$\dot{z}_1, \dot{z}_2, u, \dot{\theta} \in \mathcal{L}_\infty$. Therefore, all signals in the closed-loop system (8) and (15) are bounded. In addition, we can establish by integrating (30) that

$$V_2 \leq e^{-\alpha t} V_2(0) + \frac{\lambda_M}{2\alpha} (\alpha k^2 + 2\mu k) \quad (31)$$

which implies that the tracking error z_1 converges to the residual set \mathcal{D} exponentially.

Instead of (30), integrating both side of (28), we obtain

$$c_1 \int_0^T z_1^2 dt + c_2 \int_0^T z_2^2 dt \leq \lambda_M \mu k T + [V_2(0) - V_2(T)] \leq \lambda_M \mu k T + k_0 \quad (32)$$

$\forall T > 0$ where $k_0 = \sup_{T>0} [V_2(0) - V_2(T)]$. Letting $\beta = \min\{c_1, c_2\}$, from (32) we can obtain

$$\int_0^T z_1^2 dt + \int_0^T z_2^2 dt \leq \frac{\lambda_M}{\beta} \mu k T + \frac{k_0}{\beta} \quad (33)$$

which implies that z_1, z_2 are μk -small in the mean square sense, i.e., $z_1, z_2 \in \mathcal{S}(\mu k)$. If the time-varying property of $\theta(t)$ is disappeared, i.e., $\dot{\theta}(t) = 0$, which implies that $\mu k = 0$, we can obtain that the tracking error $z_1(t)$ is zero-small in the mean, so the same convergence property can be established as in Theorem 1. Therefore, the projected adaptive law (24) does not destroy any of the ideal properties of the unmodified adaptive law (16). This completes the proof of Theorem 2.

4.3. Numerical simulation

In this section, a numerical simulation is conducted for verifying the effectiveness of Theorem 2. The configurations employed in this simulation, such as the target speed curve, the parameters of the vehicle and controller are the same as in Section 3.2, but the time-varying property of $\theta(t)$ is considered, and assumed in the following form

$$\begin{aligned} c_0(t) &= c_0^* + 0.3 \cdot c_0^* \sin(0.1s), \\ c_v(t) &= c_v^* + 0.2 \cdot c_v^* \sin(0.1s), \\ c_a(t) &= c_a^* + 0.1 \cdot c_a^* \sin(0.1s). \end{aligned} \quad (34)$$

where c_0^*, c_v^*, c_a^* are the nominal values of $c_0(t), c_v(t)$ and $c_a(t)$ respectively. And the priori knowledge about the resistance coefficients used here is given by Table 2.

Table 2. Prior knowledge of resistance coefficients

| Category | Nominal value | Variation span |
|----------|---------------|----------------|
| c_0 | 7.85 | 3.138 |
| c_v | 0.6318 | 0.01417 |
| c_a | 0.00127 | 0.000152 |

The simulation results are shown in Figure 6 ~ Figure 8.

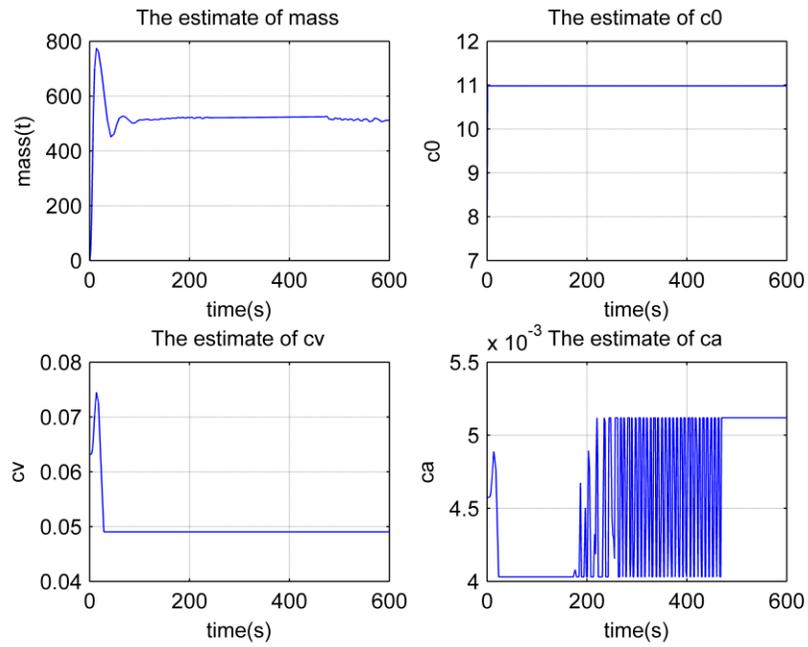


Figure 6. The estimates of the unknown parameters under Theorem 2

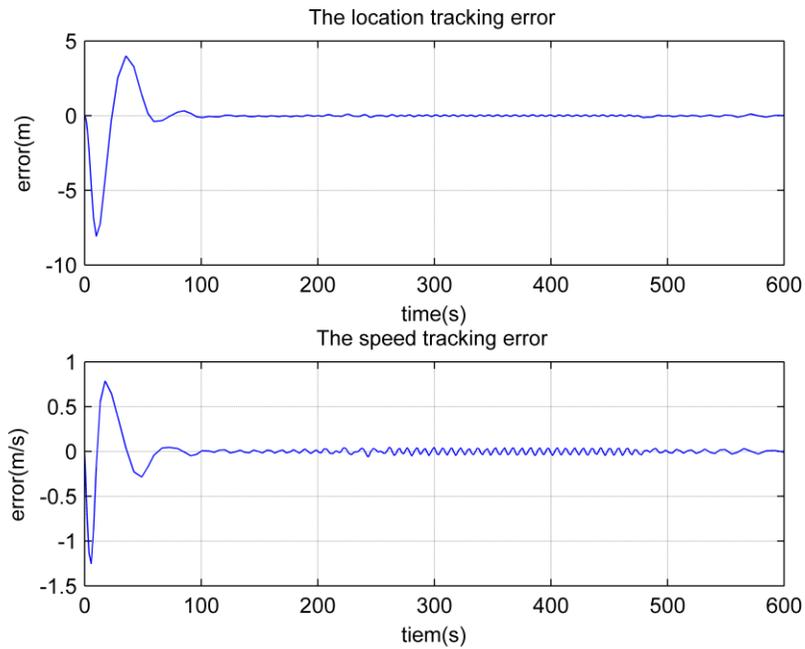


Figure 7. The tracking errors under Theorem 2

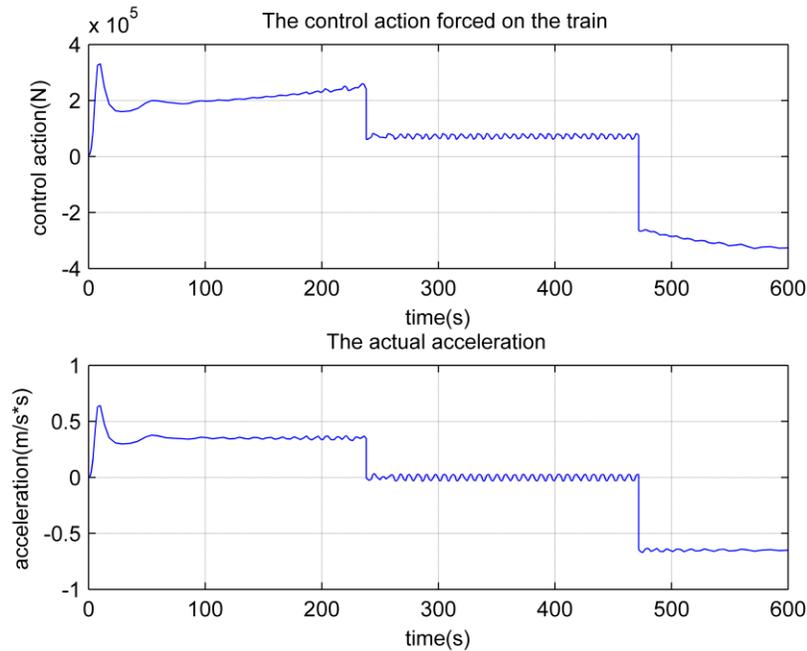


Figure 8. The control action and the actual acceleration under Theorem 2

As shown in Figure 6, due to the projection operator, the estimates of the unknown parameters are ensured to be uniformly bounded. Figure 7 shows that the asymptotic convergence of the tracking errors is no longer guaranteed under Assumption B and C, but they are converging to the small residual set exponentially. Although the tracking performance is excellent enough, but the chattering phenomenon exists in the control action and the resulting acceleration is needed to be improved, as shown in Figure 8.

5. Conclusion

In this paper, a direct robust adaptive controller is developed to improve the tracking performance of high-speed train. Aiming to deal with the parameterized uncertainties and nonlinearities in the dynamic model of high-speed train, a traditional adaptive controller is designed through the backstepping method under the general time-invariant assumption. And then, the standard projection algorithm is used to improve the robustness of the proposed control algorithm when the plant parameters are considered to be time-varying, but the variations are assumed to be sufficiently small. Both the rigorous theoretical analysis and the simulation results are all verified that the propose algorithm can guarantee the stability of the closed-loop system, and the tracking error will converge to a residual set exponentially. Furthermore, if the variation properties of the unknown parameters are disappeared, the convergence of the tracking error can be guaranteed to be asymptotically.

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