Simulation and Control of System Dynamic of Water Pollutions based on Modeling of Differential Equations using Inverse GM

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Abstract

In order to create the model of differential equations of system using limited and poor datum in the simulation of system dynamic the inverse GM model is proposed in this paper. In the system with poor information the classic estimation of parameter is difficult to be used to create the model of differential equations for analysis and forecasting. For this the principle and process of inverse GM model are studied according the grey GM model. Then inverse GM model is used in the in simulation of system dynamic of water pollutions to create the differential equations and simulate and forecast the future behaviors. The application case verifies the method of inverse GM model.

Keywords: *Differential equations; poor history datum; simulation of system dynamic; water pollutions*

1. Introduction

How to create the differential equation is the most important and key content for modeling the system in system dynamic [1]. Classic modeling method of differential equation firstly uses the qualitative analysis method to ascertain the structure of differential equation then the parameter estimation method is used to estimate the value of parameter in the equation [2-4]. The parameter estimation in the application of differential equation is a key content and classic modeling method of differential equation requires much datum and information for the parameter estimation for many researches [5-7]. But there are some cases where only poor history can be gained for the complex system dynamic and classic method is difficult to estimate the parameter [8]. How to use poor history datum to create the differential equation for modeling of complex system dynamic is the key problem which should be solved. Grey GM modeling can use seldom datum to precisely forecast the future [10-12]. The objection of GM model is to forecast the future by inverse accumulated generating operation (IAGO) [13]. GM creates the differential equation of datum produced by AGO for generated datum [14]. But the differential equation of initial datum can not be created by GM model. In this paper the improved GM model called inverse GM model is proposed to create the differential equation for first order for the initial datum in complex system for simulation. Inverse GM model is different with the GM model. The objection of GM is to forecast the future by inverse accumulated generating operation (IAGO). The objection of inverse GM model is to create the differential equation of initial datum. The principle of inverse GM is that the initial datum satisfies the first order differential equation. The first order different equation is created by the initial datum. The inverse accumulated generating operation (IAGO) is used to produce the produced datum.

In following sections direct modeling of differential equations based on inverse GM model using poor history datum in simulation of complex system dynamic is proposed. In Section 2 the relative researches is reviewed. The inverse GM(1,1) is proposed in Section 3 and The inverse GM(1,N) is proposed in Section 4. The modeling and simulation of complex system dynamic method and case are studied based on inverse GM model to verify the validation of method in Section 5. The conclusions are given in Section 6.

2. Reviews of Relative Researches

In 1950's the professor Forrester proposed the system dynamic [1, 9]. After that system dynamic has been developed into excellent system analysis method for industry, society and economics. In system dynamic the system is modeled as a group of differential equations to simulate the behavior of system. Before this the process diagram and affection diagram is created to analysis the relation of system [2]. The differential equation in system commonly is represented as the following formation [1, 9]:

$$\frac{dX}{dt} = f(X_i, V_i, R_i, P_i) \qquad (1)$$

The difference formation of formula (1) is represented as the following formation:

$$X(t+\Delta t) = X_{(t)} + f(X_i, V_i, R_i, P_i) * \Delta t \qquad (2)$$

In above formula X is state variable. V is auxiliary variable. R is flow variable, P is parameter. t is the time of simulation. Δt is the step length of simulation.

In 1980's grey system theory was initiated by J. L. Deng in P. R. China [10]. As for successful resolutions of complex indeterminate problems grey systems can use only partially datum with small samples and poor information to analysis problems [11, 12]. GM represents grey differential equation model in grey theory including about GM(1,1) and GM(1,n) and so on. GM model can sue poor information to forecast the future even that the number is only 3 to 5 [11, 12]. GM(1,1) and GM(1,n) are the forecast models which use the first order accumulated generating operation(AGO) [13]

GM(1,1) is the model first order and one variable grey differential equation. The principle of modeling of GM(1,1) is in the following [13, 14]:

The initial data sequence $x^{(0)}$ is recorded as the following formation. $x^{(0)}$ is executed the first order accumulated generating operation(AGO) to gain the data sequence $x^{(1)}$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad \text{k=1,2,….., n}$$

$x^{(1)}$ is executed the first order even generation operation to generate x :

$$x(k) = -1/2(x^{(1)}(k-1) + x^{(1)}(k)) \quad \text{k=1, 2,3….., n}$$

GM（1, 1）can be represented as following formula:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u$$

The resolution of above differential equation is in the following:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{u}{a})e^{-ak} + \frac{u}{a} \quad (k=0,1,\ldots..)$$

In above formula the parameter a, u can be resolved by the least square method:

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} Y_N$$

Forecast model of $x^{(0)}$ can be gain by inverse accumulated generating operation (IAGO):

$$\hat{x}^{(0)}(k+1) = (1-e^{a})(x^{(0)}(1) - \frac{u}{a})e^{-ak} \quad (k=1,2,\ldots)$$

GM(1,n) is the model first order and multi-variables grey differential equation. The principle of modeling of GM(1,n) is in the following [15, 16]:

Assuming there are N datum sequences $X_i^{(0)}$. The accumulated generating operation is executed on $X_i^{(0)}$ to gain $X_i^{(1)}$.

$$\begin{aligned} X_i^{(1)} &= (X_i^{(0)}(1), \sum_{m=1}^{2} X_i^{(0)}(m), \cdots, \sum_{m=1}^{n} X_i^{(0)}(m)) \\ &= (X_i^{(1)}(1), X_i^{(1)}(1) + X_i^{(0)}(2), \cdots, X_i^{(1)}(n-1) + X_i^{(0)}(n)) \quad i = 1,2,\cdots,N \end{aligned}$$

The differential equation is recorded as GM（1,N） in the following:

$$\frac{dX_1^{(1)}}{dt} + aX_1^{(1)} = b_1 X_2^{(1)} + b_2 X_3^{(1)} + \cdots + b_{N-1} X_N^{(1)} \qquad (1)$$

The parameters is $\alpha = (a, b_1, b_2, \cdots b_{N-1})^T$ and $Y_N = (X_1^{(0)}(2), X_1^{(0)}(3), \cdots, X_1^{(0)}(n))^T$. The difference discrete and least square method are executed to gain the linear equation:

$$Y_N = B\hat{\alpha} \qquad (2)$$

$$\hat{\alpha} = (B^T B)^{-1} B^T Y_N$$

3. Direct Modeling of Differential Equations based on Inverse GM(1,1)

In order to create the differential equation of initial datum the inverse GM(1,1) is proposed in this section.

Inverse GM（1,1） model is different with the GM(1,1) model. The objection of GM(1,1) is to forecast the future by inverse accumulated generating operation (IAGO). GM(1,1) create the differential equation of datum produced by AGO. The differential equation of initial datum can not be created by GM(1,1).

The objection of GM(1,1) is to create the differential equation of initial datum. The principle of inverse GM(1,1) is that the initial datum satisfy the first order differential equation. The first order different equation is created by the initial datum. The inverse accumulated generating operation (IAGO) firstly is used to produce the produced datum.

The total process of inverse GM(1,1) is in the following:

The initial data sequence is recorded as the following formation:

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n))$$

It is assumed that the initial datum satisfies the first order differential equation. If the initial datum satisfies the first order differential equation it can be verified and preceded in the following. The following objection of proceeding is to produce the differential of initial datum $X^{(1)}$.

For this objection, the initial datum is executed the inverse accumulated generating operation (IAGO) to produce the following sequence $x^{(0)}$:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n))$$

In above sequence, $x^{(0)}$ is calculated by following formula:

$$x^{(0)}(k) = x^{(1)}(k+1) - x^{(1)}(k) \quad k=1,2,....., \quad n,$$

$x^{(0)}$ can be used in the following to calculate the parameter of differential equation.

$X^{(0)}$ and $X^{(1)}$ can be verified by the following method: $X^{(0)}$ is verified the standard smooth test and $X^{(1)}$ is verified the accurate index test.

Assuming

$$\rho(k) = \frac{x^{(0)}(k)}{x^{(1)}(k-1)} \quad k=2,3...n \qquad (3)$$

If

$$\rho(k) < 1 \qquad (3)$$

and

$$\rho(k) \in [0, \varepsilon] \ \varepsilon < 0.5 \qquad (3)$$

$\rho(k)$ has the decreasing trend. Then $X^{(0)}$ is called the standard smooth sequence. And $X^{(1)}$ has the regulation of accurate index.

Else $X^{(0)}$ and $X^{(1)}$ is proceeded by the following method:

$$x'^{(0)}(k) = \frac{1}{n-k+1}(x(k) + x(k+1) + ... + x(n)) \quad k=1,2...n \qquad (4)$$

Ordering $x^{(0)}(k) = x'^{(0)}(k)$. $X^{(0)}$ is replaced by $X'^{(0)}$.

$x^{(1)}$ is executed the first order even generation operation to generate x:

$$x = (x(2), x(3), ...x(n))$$

In above sequence, $x(k)$ is calculated by the following method:

$x(k) = -1/2(x^{(1)}(k-1) + x^{(1)}(k))$ k=1, 2,3….., n

The least square method is used to calculated the following matrix:

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} Y_N \qquad (2)$$

In above matrix, B and Y_N is calculated in the following:

$$B = \begin{bmatrix} -x^1(2) & \\ -x^1(3) & \\ : & : \\ -x^1(n) & \end{bmatrix}$$

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ : \\ x^{(0)}(n) \end{bmatrix}$$

Y_N is calculated by the produced datum sequence $X^{(0)}$. B and Y_N is used to calculate the value of $\hat{a}$,a and u.

Then a, u is brought into the following formula to create the differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \qquad (10)$$

The formula (10) is the objection of inverse GM(1,1). It is the differential equation of initial datum sequence. The created differential equation of initial datum may be used in the system dynamic to induce the system behaviors.

The scope and verification are in the following:

(1) The scope of inverse GM(1,1)

When $-\hat{a} \leq 0.3$ the inverse GM(1,1) can produce behavior of system for the medium and long period. When $0.3 < -\hat{a} \leq 0.5$, the inverse GM(1,1) can produce behavior of system for the short period. When $0.5 < -\hat{a} \leq 0.8$ it should be cautiously used to analyze behavior of system for the short period. When $0.8 < -\hat{a} \leq 1$, residual correction should be sued. When $-\hat{a} > 1$, inverse GM(1,1) can not be used to create differential equation to analyze the behavior of system.

(2) The verification of inverse GM(1,1)

The sequence of residual is in the following:

$$\varepsilon^{(0)} = (\varepsilon(1), \varepsilon(2) \ldots \varepsilon(n)) = (x^{(0)}(1) - \hat{x}^{(0)}(1), x^{(0)}(2) - \hat{x}^{(0)}(2) \ldots x^{(0)}(n) - \hat{x}^{(0)}(n))$$

The even $\bar{\varepsilon}$ and variance S_{ε}^{2} of residual is calculated in the following:

$$\bar{\varepsilon}=\frac{1}{n}\sum_{k=1}^{n}\varepsilon(k)$$

$$S_{\varepsilon}^{2}=\frac{1}{n}\sum_{k=1}^{n}(\varepsilon(k)-\bar{\varepsilon})^{2}$$

The even $\bar{x}$ and variance S_{x}^{2} of $X^{(0)}$ is calculated in the following:

$$\bar{x}=\frac{1}{n}\sum_{k=1}^{n}x^{(0)}(k)$$

$$S_{x}^{2}=\frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k)-\bar{x})^{2}$$

Posterior ratio C is calculated by the following formula:

$$C=\frac{S_{e}}{S_{x}}$$

The probability of Small error p is calculated by the following formula:

$$p=P(\left|\varepsilon(k)-\bar{\varepsilon}\right|<0.6745S_{x})$$

In the above formulas it is good if the c is smaller and p is bigger. The reference table of levels of verification of precision

Table 1. Reference of levels of table of verification of precision

Index	Levels	C	P
one	Good	0.35	0.95
two	Qualified	0.50	0.80
three	Qualified with difficulty	0.65	0.70
four	not qualified	0.80	0.60

4. Direct Modeling of Differential Equations based on Inverse GM(1,n)

There are N initial datum sequences in the following:

$$X_{i}^{(1)}=(X_{i}^{(1)}(1),X_{i}^{(1)}(2),\cdots,X_{i}^{(1)}(n))\quad i=1,2,\cdots,N$$

The initial datum sequences are executed the inverse accumulated generating operation (IAGO) to produce the following sequence $x_{i}^{(0)}$:

$$x_{i}^{(0)}=(x_{i}^{(0)}(1),x_{i}^{(0)}(2),....,x_{i}^{(0)}(n))$$

In above sequence, $x^{(0)}$ is calculated by following formula:

$x_i^{(0)}(k) = x_i^{(1)}(k+1) - x_i^{(1)}(k)$ k=1,2,….., n,

$X_i^{(1)}$ is viewed as constant variable at time $t\,(k = 1,2,\cdots,n)$, $X_i^{(1)}$ is viewed as the function of $t: X_i^{(1)} = X_i^{(1)}(t)$. If $X_2^{(1)}, X_3^{(1)}, \cdots, X_N^{(1)}$ have the affection on variation ratio of $X_1^{(1)}$, then the following differential equation can be created:

$$\frac{dX_1^{(1)}}{dt} + aX_1^{(1)} = b_1 X_2^{(1)} + b_2 X_3^{(1)} + \cdots + b_{N-1} X_N^{(1)} \qquad (1)$$

The above differential equation is recorded as GM（1,N）.

The parameters are represented as $\alpha = (a, b_1, b_2, \cdots b_{N-1})^T$ in the differential equation. Ordering $Y_N = (X_1^{(0)}(2), X_1^{(0)}(3), \cdots, X_1^{(0)}(n))^T$, difference discrete is executed on this differential equation, the following linear equation groups can be gained:

$$Y_N = B\hat{\alpha} \qquad (2)$$

According to least square method the following formula can be induced:

$$\hat{\alpha} = (B^T B)^{-1} B^T Y_N \qquad (3)$$

The two- moving-average method is used to gain the following matrix:

$$B = \begin{pmatrix} -\frac{1}{2}(X_1^{(1)}(1) + X_1^{(1)}(2)) & X_2^{(1)}(2) & \cdots & X_N^{(1)}(2) \\ -\frac{1}{2}(X_1^{(1)}(2) + X_1^{(1)}(3)) & X_2^{(1)}(3) & \cdots & X_N^{(1)}(3) \\ \vdots & \vdots & & \vdots \\ -\frac{1}{2}(X_1^{(1)}(n-1) + X_1^{(1)}(n)) & X_2^{(1)}(n) & \cdots & X_N^{(1)}(n) \end{pmatrix}$$

Then $\hat{\alpha}$ can be resolved, the differential equation is also resolved。

If $n-1 < N$, The number of equations in linear equation groups（2）is few than unknown number. Then $B^T B$ is singular matrix, $\hat{\alpha}$ can not be resolved by the formula (3). So M is introduced to execute the weighted minimization of $\alpha^T \alpha$.

Ordering

$$M = diag(\alpha_1, \alpha_2, \cdots, \alpha_N)$$

Then $\hat{\alpha}$ can be calculated by the following formula:

$$\hat{\alpha} = M^{-1} B^T (BM^{-1} B^T)^{-1} Y_N$$

The relative verification method is similar to that method in inverse GM(1,1).

5. Case of Modeling and Simulation of System Dynamic based on Inverse GM Model using Poor History Datum

5.1 The case and modeling

The case is the appliction of system dynamic in the analysis of trend of govenment of water environment pollution for one toxic heavy metal pollution in one small city in the northeast region of China. For past some years toxic heavy metal pollution has made thousands of people sick and even lost their lives in this city. In order to protect the life safety the government constant to fund sufficiency input for this toxic heavy metal pollutions. The effection relation between the water pollution of environment and the captial invested is shown in the following affection digram. The main pollutant emissions comes from the enterprise in this city.

Now in the following tables show the relative datum which can be gained by the official statistics for this toxic heavy metal pollution. There are two methods to control pollutions. One is the one before the pollutions and another is the one after pollutions. The datum before 2007 is invalidating because the pollution is not heavy at that time. The government capital invested before pollution is shown in Table 1 for recently 6 years. The government capital invested after pollution is shown in Table 2 for recently 6 years. The pollutant emissions of enterprises for recently 6 years are shown in Table 3. The measurement value of environment for recently 6 years is given in Table 4. The other pollutant emissions which do not come from enterprise for recently 6 years are shown in Table 5. The objection of modeling and simulation is to analysis the future trend of pollution, environment and government capital invested and relation among them according the limited datum by the official statistics. The datum is not enough to execute the classic parameter estimation for differential equation.

In this case the government of higher level orders this city to finish the zero emissions for this toxic heavy metal pollution in future 10 years from 2013-2023. The objection of case is to analysis the future trend according to the history datum in Table 1-6. Whether the order of the government of higher level can be finished? If yes, which year the objection can be finished? What is the future developing trend for this pollution? According to this trend which method will be effective before pollution or after pollution for this city in the future?

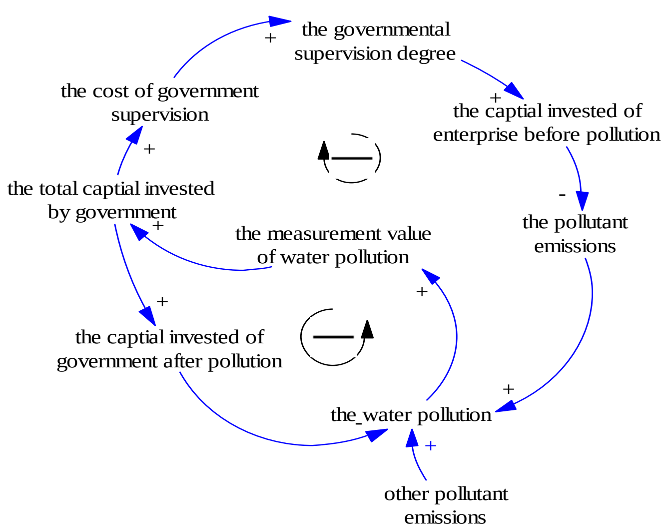

Figure 1. The effection digram of water pollution and government

Table 1. The enterprise captial invested before pollutions for recent 6 years

Year	2007	2008	2009	2010	2011	2012
Government captial invested before pollution(thousand yuan)	49489.3	100652.8	156464.0	214151.9	279751.9	357728.4

Table 2. The total government captial invested for recent 6 years

Year	2007	2008	2009	2010	2011	2012
Captial invested after pollutions(thousand yuan)	363461.35	824623.91	1381516.14	1979523.09	2675223.09	3473210.62

Table 3. The pollutant emissions of enterprises for recent 6 years

Year	2007	2008	2009	2010	2011	2012
Pollutant emissions(mg/l)	5970.3	5560.8	5170.5	4990	4700	4500.5

Table 4. The measured value of water pollution for recent 6 years

Year	2007	2008	2009	2010	2011	2012
Measurement value of environment(mg/l)	4030.47	3801.32	3502.56	3303.2	3205.371	3012.23

Table 5. The other pollutant emissions for recent 6 years

Year	2007	2008	2009	2010	2011	2012
Other pollutant emissions(mg/l)	2.43	6.02	10.02	15.54	29.74	48.45

Table 6. The government supervision degree

Year	2007	2008	2009	2010	2011	2012
Other pollutant emissions	0.35	0.77	1.32	1.95	2.67	3.38

In this case the proposed method inverse GM model in this paper is used to create the system dynamic model to analysis the behavior of system. Considering the datum in the above effecting diagram in Figure 1 may be simplified. The flow diagram of simplified loop clockwise is created in the following Figure 2. From the Figure 2 and above table it can be seen:

The differential equation of other pollutant emissions can be created by the inverse GM(1,1). And the differential equations of captial invested of enterprise before pollution, the pollutant emissions and total captial invested of government can be created by the inverse GM(1,N).

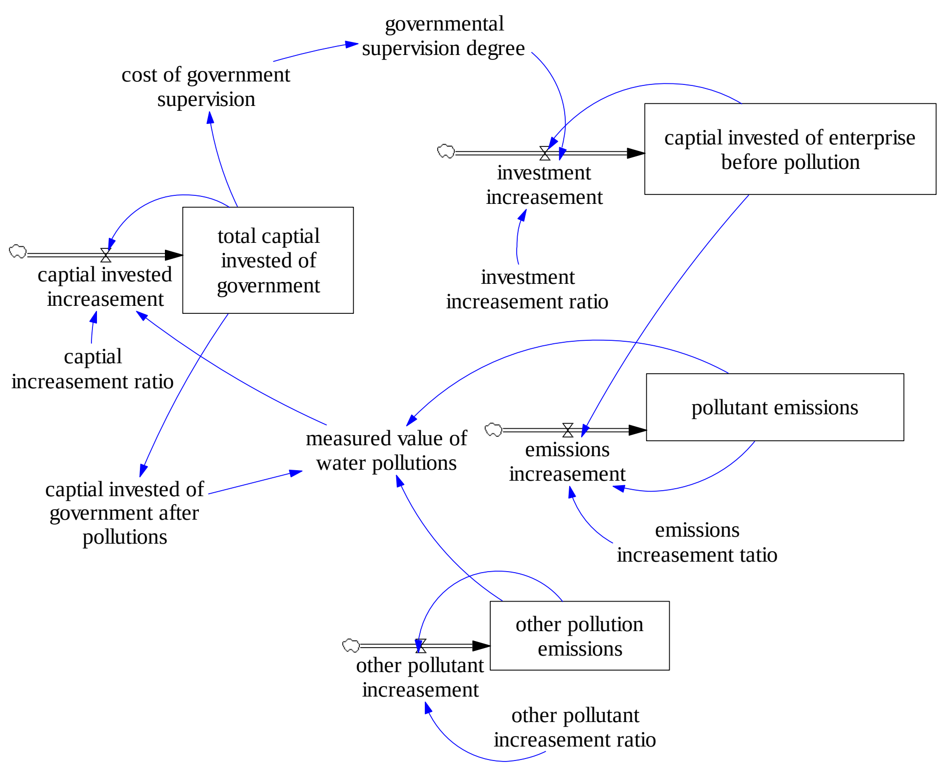

Figure 2. The flow digram of water pollution and government

Inverse GM(1,1) modeling proposed in this paper can be used for the other pollutant emissions for recent 6 years in Table 5 to created the following differential equation:

$$\hat{a} = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} Y_N = \begin{bmatrix} -0.3834 \\ 0.3517 \end{bmatrix} \text{ and } \frac{dx}{dt} \text{-} 0.3834x = 0.3517 .$$

According to this differential equation it can be used to set the value of variables in the flow Figureure:

Other pollutant increasement ratio= 0.3834

Other pullutant increasement = other pollution emissions * 0.3834 + 0.3517

Inverst GM(1,n)(Ordering n=2) model proposed in this paper can be used for the modeling of pollutant emissions recorded as x_1 in Table 3 and the captial invested of enterprise before pollution recorded as x_2 in Table 1.

$$\frac{dx_1}{dt} - 1.5304 x_1 = -0.1103 x_2$$

According to this differential equation it can be used to set the value of variables in the flow Figureure:

Emissions increasement = captial invested of enterprise before pollution * (-0.0101) + 1.4601 * pollutant emissions

Inverst GM(1,n)(Ordering n=2) model proposed in this paper can be used for the modeling of pollutant emissions recorded as x_1 in Table 3 and the government supervision degree recorded as x_3 in Table 6.

$$\frac{dx_1}{dt} \text{-} 4.0891 x_1 = -358240 x_2$$

According to this differential equation it can be used to set the value of variables in the flow Figureure:

investment increasement = captial invested of enterprise before pollution * 4.0891-358240* government supervision degree

Inverst GM(1,n)(Ordering n=2) model proposed in this paper can be used for the government total captial invested for recent 6 years in Table 2 recorded as x_4 and the measurement value of water pollution for recent 6 years recorded as x_5 in Table 4.

$$\frac{dx_4}{dt} \text{-} 0.1607 x_4 = 100.4323 x_5$$

According to this differential equation it can be used to set the value of variables in the flow Figureure:

captial invested increasement = captial invested of government after pollution * 0.1607+100.4323* measurement value of water pollution.

After that according to the proportion of government expenditure : the cost of government supervision=the total captial invested of government*0.05, captial invested of government after pollutions=the total captial invested of government*0.95.

According to the government evalution the future regulation from 2013 between the cost of government supervision and the governmental supervision degree is adjusted as following:

IF THEN ELSE(cost of government supervision <2*100000, 0.9 , (IF THEN ELSE(cost of government supervision <4*100000, 1.2, (IF THEN ELSE(cost of government supervision <6*100000, 1.65, (IF THEN ELSE(cost of government supervision <7*100000, 2.36, (IF THEN ELSE(cost of government supervision <8*100000, 2.89, (IF THEN ELSE(cost of government supervision <9*100000, 3.62, (IF THEN ELSE(cost of government supervision <10*100000, 3.66, (IF THEN ELSE(cost of government supervision

<11*100000, 3.68, 4.2))))))))))))))). The the governmental supervision degree is divided as 5 levels which represented the real distance number between 1 and 5.

According to the government evalution and reality checking forecast measured value of water pollutions is represented by following formula:

Measured value of water pollutions=other pollution emissions + pollutant emissions - (captial invested of government after pollutions/100000).

5.2 Result and analysis

After modeling the system dynamic model is executed to gain the following results. The Figure 3 shows the measured value of water pollutions of toxic heavy metal pollution in the future. The objection of zero pollution will be achieved before 2020.

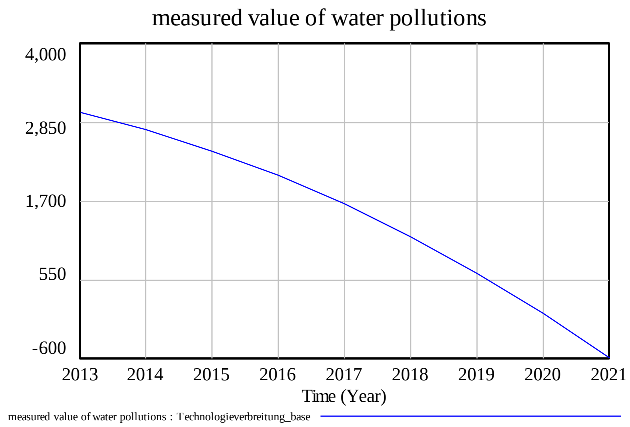

Figure 3. Measured value of water pollutions

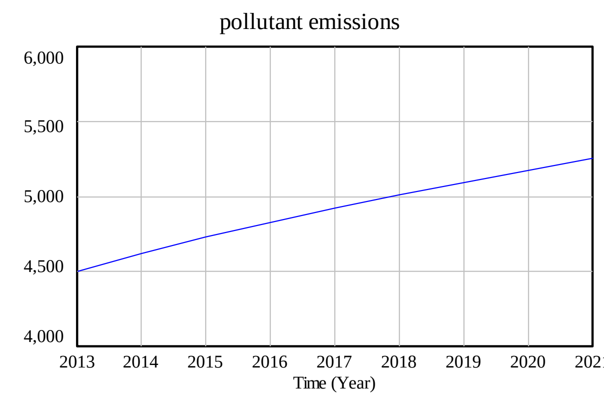

Figure 4. Pollution emissions

Figure 4 shows the trend of pollutant emissions. It is serious that the pollutant emissions are increasing in the future. The enterprise is not validating to present the trend of pollution.

The Figure 5 shows the trend of total capital invested by government. The total capital quantity is very huge in 2020. It can be seen that in order to achieve the objection the huge capitals and efforts should be input. The Figure 6 shows the governmental supervision degree. It can be seen that the governmental supervision degree is all increasing.

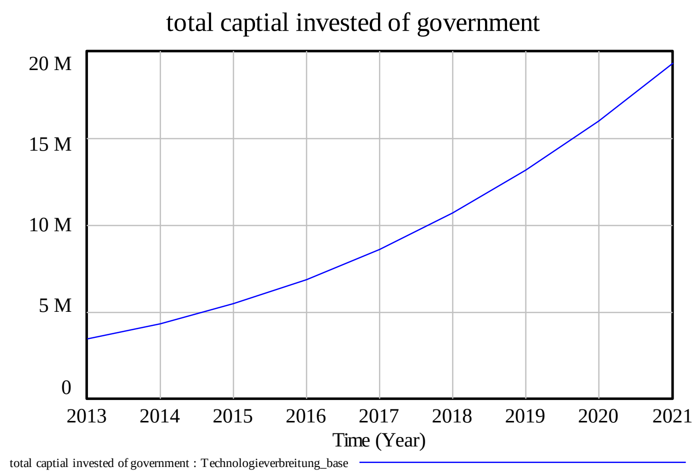

Figure 5. The trend of total captial incested of government

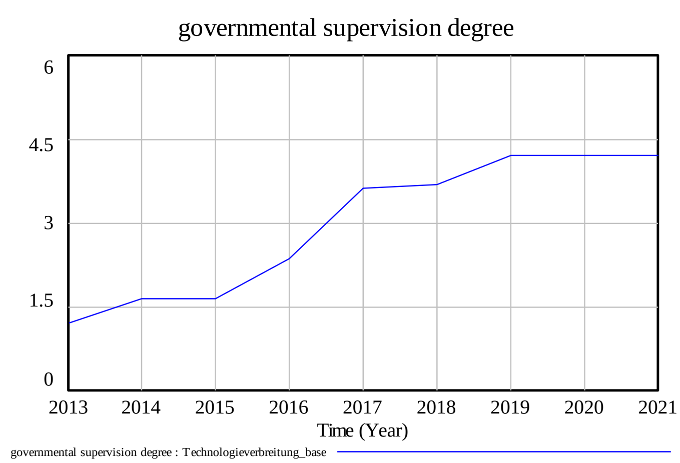

Figure 6. The governmental supervision degree

The Figure 7 show the captial of enterprises before pollution. The total capital quantity is very huge in 2020. It can be seen that in order to achieve the objection the huge capitals and efforts should be input. But there is a strange regulation in this result. At the same time it can be seen that the captial of enterprises before pollution is descending as showed the governmental supervision degree the Figure 6 is increasing. The enterprise escapes the responsibility of governing pollution to put it to the governments. The reason maybe is that the law is lacking in China for forcing the enterprise to take the responsibility of governing pollution.

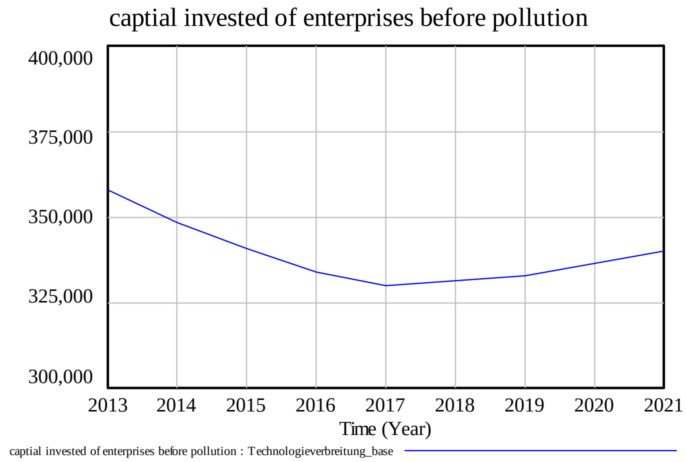

Figure 7. The trend of captial incested of enterprise

From the cantrast in Figure 4 and Figure 7 it can be seen the zero pollution is mainly attained by the method after control. The method before pollution is invalidate. It is strange for it. But this may be explained by the situation for law and enterpsises in China. the pollutions is serious but the government is the main power to elimite the pollution. Some enterpsises is out of control because some enterprises even take the cheap method to fool the conturies and people for their owners economic profits.

In Figure 8 the trend of other emissions are shown. It can be seen that the other emissions is increasing but the quantity is smaller. In the futrue other emissions should be noticed to control if the quantity is increasd.

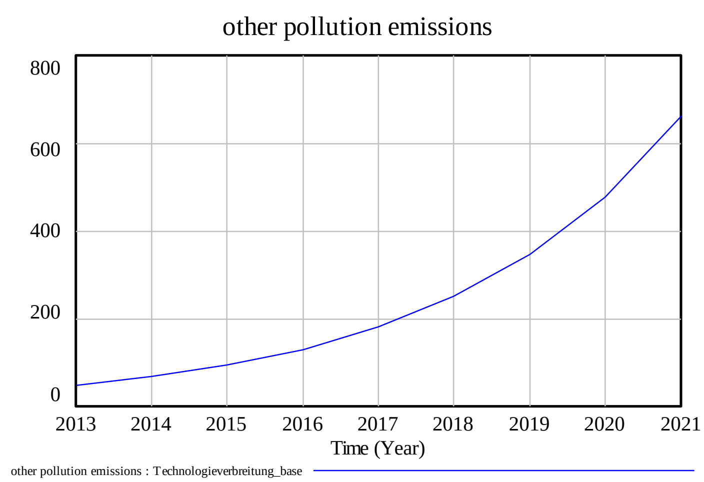

Figure 8. The trend of other emissions

6. Conclusions

In this paper invered GM modeling method of differential equations is proposed and be used in the application of simulation of system dynamic of water pollution of toxic heavy metal. Invered GM modeling method is validate in inducing the future behaviors for the system. It can create the ststem model using the limited history datum (The datum before

2007 is invalidating because the pollution is not heavy at that time.). And the model can be used to induce the future trend and forecast and verify the behavior of system. So the invered GM modeling method is a excelletn modeling method of differential equations for system.

Acknowledgements

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