

Mismatch Analysis between a Piezoelectric Actuator and Half-Length Sensor on a Beam

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Abstract

This paper investigates mismatch problem in the coupled response of the flexural and longitudinal motions between the piezoelectric actuator and half-length sensor on a beam. This problem is important to design a piezo sensor – actuator pair control system in terms of frequency response function. Piezoelectric transducers have been widely used in control of structures in both aspects: an actuator and a sensor. In this study a half-length piezosensor compared to the piezoactuator is considered and it is assumed bonded on the other side of the actuator. The frequency response functions (FRF) are investigated in theory and computer simulation when the mismatch ratios are 0 %, 15 %, 30 % and 50 % respectively between the sensor and the actuator. The simulation results show each FRF is quite similar except some frequency ranges. Also the flexural motion gives a big effect to the coupled motion of the sensor-actuator response. This paper could provide a better understanding on the interaction between piezosensor and piezoactuator which are bonded with some mismatch.

Keywords: *Integral-derivative feedback control, Lightly damped system, Non-minimum phase*

1. Introduction

Piezoelectric transducers have been widely used for the control of structures especially when the structures are less-damped [1-3]. These piezoelectric transducers can be used as an actuator as well as a sensor [4-8]. Some studies suggest the use of the piezoelectric sensor and piezoelectric actuator at the same time on structures for better control performance and stability [5,9]. For this reason, collocation and non-collocation of the piezo sensor and actuator are discussed in many references to control, for example, the travelling waves on beams [5,10]. Since the property of the piezoelectric transducer is very different from the conventional actuator and sensor, the response between the piezo sensor and actuator must be carefully investigated for using in structure control.

In this paper, a half-length piezo sensor compared to the piezo actuator is considered. When they are attached with some mismatch on either side of a beam, the frequency response function of the sensor and actuator will be critically important to design a controller for structure control such as vibration suppression or wave propagation cancellation [11]. Four different mismatched sensor and actuator arrangements will be analyzed in terms of the coupled response in detail for the coupled responses.

In section 2, a theoretical analysis on the flexural and longitudinal responses when the sensor and actuator are mismatched with some specific ratios will be described. After the

theory part, the results with an intensive computer simulation for the theoretical models will be discussed.

2. Matchness Problem between Piezoelectric Sensor and Actuator

2.1 Arrangement of Half-Length Matched Piezo Sensor and Actuator

Consider a piezoelectric actuator with the length of L_p and a piezoelectric sensor with the length $L_s = L_p / 2$ where the sensor and the actuator are "0 %" *half-length matched* each other on either side of a beam as shown in Figure 1(a). The frequency response function (FRF) between the piezo sensor and actuator is crucially important because the arrangement is widely adopted in many smart structures which use the piezoelectric transducers. The FRF offers fundamental controllability of the sensor-actuator pair for structure control especially against vibration or dynamic motions. In this study the other three arrangement of the sensor-actuator pair as plotted in Figure 1(b) for "15 %" *half-length matched*, Figure 1(c) for "30 %" *half-length matched* and Figure 1(d) for "50 %" *half-length matched* will be described.

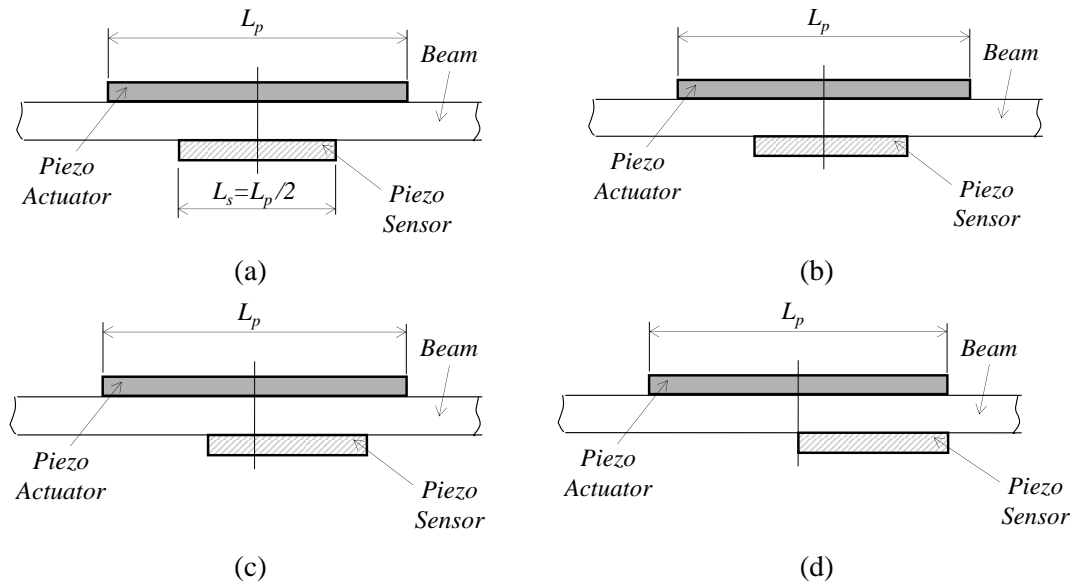


Figure 1. Arrangement of half-length matched piezo sensor and actuator. The vertical line in the middle of the actuator is the half-length position. (a) "0 %" half-length mismatch. (b) "15 %" half-length mismatch. (c) "30 %" half-length mismatch. (d) "50 %" half-length mismatch

2.2 Flexural Wave Generation by Piezo Actuator

As shown in Figure 2, the piezoelectric actuator generates eight different flexural waves on a beam. They are four near-field evanescent waves A_{f1} , A_{f2} , B_{f1} and B_{f2} and four propagating waves A_{f3} , A_{f4} , B_{f3} and B_{f4} due to the two bending moments M_A and M_B at both ends of the piezo actuator as illustrated in Figure 3 [10, 11].

Among the eight waves, the piezo sensor will detect only four waves A_{f2} , B_{f1} , A_{f4} and B_{f3} because the sensor will be positioned between $x = 0$ and $x = L_p$. Since the magnitudes of the two bending moments M_A and M_B are the same but the signs are

different each other ($M = -M_A = M_B$), the amplitudes of the four waves in the sensor region, as plotted in Figure 2, have the following relationship as

$$A_{f2} = B_{f1} = -A_{f4} = -B_{f3} \quad (1)$$

where the amplitude can be defined as, for example $A_{f2} = M / 4Y_b I_b k_f^2$ [10]. In which Y_b , I_b and k_f are Young's modulus, 2nd moment of bending and the flexural wavenumber respectively.

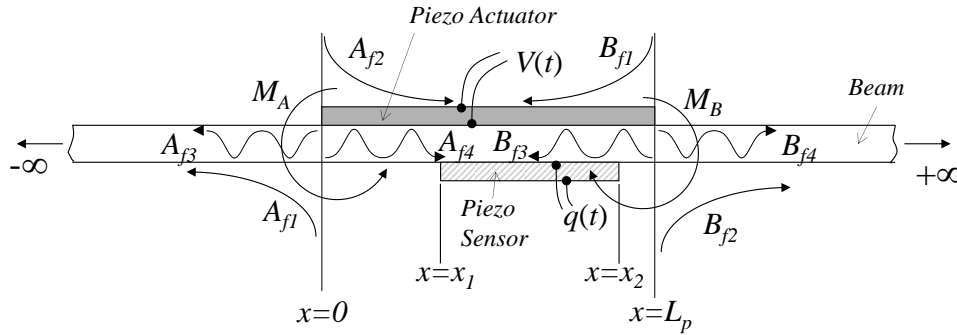


Figure 2. Eight different flexural waves generated by a piezoelectric actuator. Among them four flexural waves can be detected by the piezoelectric sensor which is half-length mismatched

2.3 Longitudinal Wave Generation by Piezo Actuator

The piezoelectric actuator generates four longitudinal waves A_{o1} , A_{o2} , B_{o1} and B_{o2} as illustrated in Figure 3. Since the two longitudinal forces are $F = -F_A = F_B$, the amplitudes of the two longitudinal waves in the sensor region, as plotted in Figure 2, have the following relationship as [10]

$$A_{o2} = -B_{o1} \quad (2)$$

where the amplitude of the wave A_{o2} can be defined by $A_{o2} = jF / 2Y_b A_b k_o$ [10].

2.4 Flexural Wave Sensing by Piezo Sensor

As can be seen from Figure 2, the piezoelectric sensor is located between x_1 and x_2 to be partly matched with the piezoelectric actuator on the beam. The partly matched sensor detects six flexural waves: three near-field evanescent waves (A_{f2} , B_{f1} and B_{f2}) and three propagating waves (A_{f4} , B_{f3} and B_{f4}). The flexural displacements

$w(x_1)$ at $0 \leq x_1 \leq L_p$ and $w(x_2)$ at $L_p/2 \leq x_2 \leq L_p$ in the beam using can be expressed with

$$\begin{aligned} w(x_1) &= A_{f2}e^{-k_f x_1} + A_{f4}e^{-jk_f x_1} + B_{f1}e^{k_f(x_1-L_p)} + B_{f3}e^{jk_f(x_1-L_p)} \\ w(x_2) &= A_{f2}e^{-k_f x_2} + A_{f4}e^{-jk_f x_2} + B_{f1}e^{k_f(x_2-L_p)} + B_{f3}e^{jk_f(x_2-L_p)} \end{aligned} \quad (3)$$

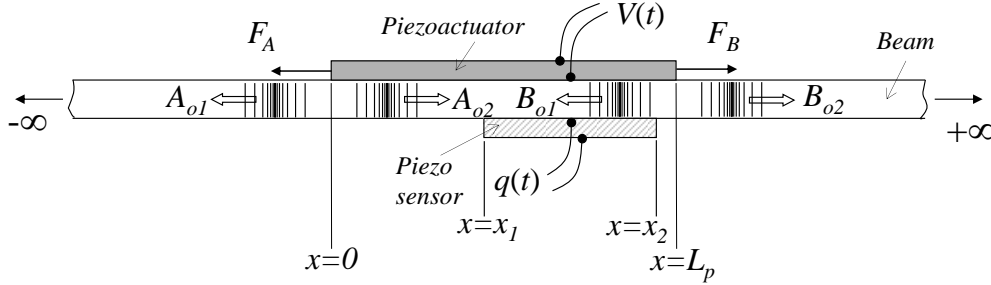


Figure 3. Arrangement of half-length matched piezo sensor and actuator. The vertical line in the middle of the actuator is the half-length position: (a) "0 %" half-length mismatch; (b) "5 %" half-length mismatch; (c) "20 %" half-length mismatch; (d) "50 %" half-length mismatch

Because the amplitudes of waves has the relationship of $A_{f2} = B_{f1} = -A_{f4} = -B_{f3} = M / 4Y_b I_b k_f^2$ [5,10], the charge output q_f of the piezoelectric sensor by the transverse wave motion is expressed with [2]

$$q_f = -\int_{x_1}^{x_2} e_{31} h_{sen} B \frac{d^2 w}{dx^2} dx = -e_{31} h_{sen} B [w'(x_2) - w'(x_1)], \quad (4)$$

the slopes $w'(x_1)$ and $w'(x_2)$ of the beam can be expressed as

$$\begin{aligned} w'(x_1) &= \frac{M}{4Y_b I_b k_f} \left[(-e^{-k_f x_1} + e^{k_f(x_1-L_p)}) + j(e^{-jk_f x_1} - e^{jk_f(x_1-L_p)}) \right] \\ w'(x_2) &= \frac{M}{4Y_b I_b k_f} \left[(-e^{-k_f x_2} + e^{k_f(x_2-L_p)}) + j(e^{-jk_f x_2} - e^{jk_f(x_2-L_p)}) \right] \end{aligned} \quad (5)$$

Therefore the transfer function between the charge output from the piezoelectric sensor against the applied electric field E_3 to the piezoelectric actuator in terms of the flexural waves is defined by

$$\frac{q_f}{E_3} = -\frac{e_{31}d_{31}h_{sen}B^2\Delta_f}{4k_f} [(-e^{-k_f x_2} + e^{k_f(x_2-L_p)} + e^{-k_f x_1} - e^{k_f(x_1-L_p)}) + j(e^{-jk_f x_2} - e^{jk_f(x_2-L_p)} - e^{-jk_f x_1} + e^{jk_f(x_1-L_p)})] \quad (6)$$

2.5 Longitudinal Wave Sensing by Piezo Sensor

The piezoelectric sensor is located between x_1 and x_2 as illustrated in Figure 3, but the piezoelectric actuator is attached between 0 and L_p . Hence the half-length matched piezoelectric sensor detects two longitudinal waves: one right-going waves (A_{o2}) and one left-going wave (B_{o1}).

Thus, the longitudinal displacements of the beam $u(x_1)$ and $u(x_2)$ at $0 \leq x_1 \leq L_p$ and $L_p/2 \leq x_2 \leq L_p$ respectively, as shown in Figure 3, are governed by the two waves and can be expressed with

$$u(x_1) = A_{o2}e^{-jk_o x_1} + B_{o1}e^{jk_o(x_1-L_p)} \quad u(x_2) = A_{o2}e^{-jk_o x_2} + B_{o1}e^{jk_o(x_2-L_p)} \quad (7)$$

As the piezoelectric sensor is a rectangular patch, the electric charge output of the piezoelectric sensor in this case can be described by [2]

$$q_o = \int_{x_1}^{x_2} e_{31}B \frac{du}{dx} dx = e_{31}B[u(x_2) - u(x_1)] \quad (8)$$

Thus the transfer function between the charge output from the piezoelectric sensor against the applied electric field E_3 to the piezoelectric actuator in terms of the longitudinal waves is given by

$$\frac{q_o}{E_3} = \frac{je_{31}d_{31}B^2\Delta_o}{2k_o} [e^{-jk_o x_2} - e^{jk_o(x_2-L_p)} - e^{-jk_o x_1} + e^{jk_o(x_1-L_p)}] \quad (9)$$

2.6 Coupled Wave Sensing by Piezo Sensor

From the above sections, the separate flexural and longitudinal responses are discussed and formulated. In practice, the applied voltage to the piezo actuator will induce the flexural and the longitudinal waves at the same in the beam. So the piezo sensor will not detect these waves separately but simultaneously. Thus the two responses will be coupled, the transfer function between the coupled response q_c against the input electric field can be defined as

$$\begin{aligned} \frac{q_c}{E_3} = \frac{q_f}{E_3} + \frac{q_o}{E_3} = & -\frac{e_{31}d_{31}h_{sen}B^2\Delta_f}{4k_f} [(-e^{-k_f x_2} + e^{k_f(x_2-L_p)} + e^{-k_f x_1} - e^{k_f(x_1-L_p)}) \\ & + j(e^{-jk_f x_2} - e^{jk_f(x_2-L_p)} - e^{-jk_f x_1} + e^{jk_f(x_1-L_p)})] \\ & + \frac{je_{31}d_{31}B^2\Delta_o}{2k_o} [e^{-jk_o x_2} - e^{jk_o(x_2-L_p)} - e^{-jk_o x_1} + e^{jk_o(x_1-L_p)}]. \end{aligned} \quad (10)$$

3. Computer Simulation Results and Discussions

3.1 Flexural Responses

Figure 4 shows the FRF of the flexural response between the sensor and actuator as defined in equation (6). The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements as defined in Figure 1. In spite of the mismatch ratio increases, the magnitude doesn't change much especially below about 3,000 Hz. However, the difference of magnitude response between each mismatch ratio becomes eminent above that frequency.

Also the phase responses present the half-length mismatchness doesn't give serious effect to the sensor-actuator interaction on the beam. Since the region of the sensor placement is restricted with the region of actuator as illustrated in Figure 1, the FRFs of each arrangement of the sensor-actuator look quite similar.

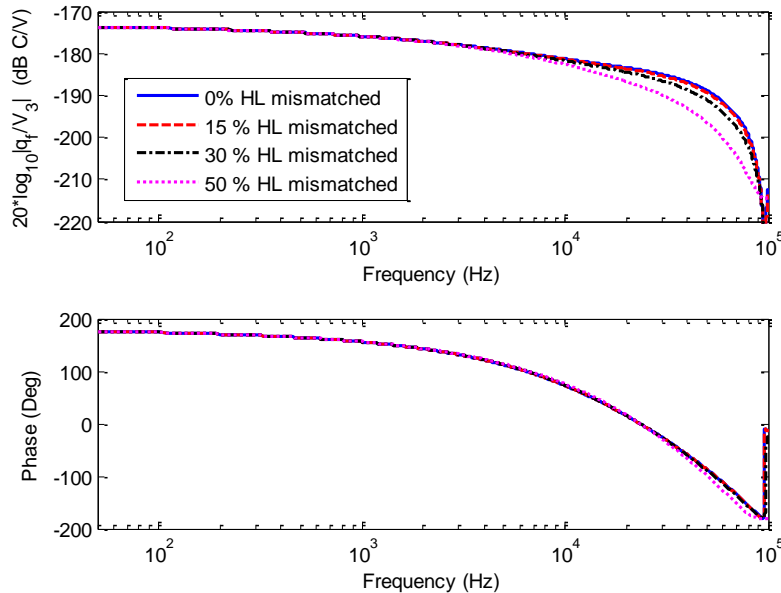


Figure 4. FRFs of the flexural motion between the half-length mismatched piezo sensor and actuator. The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements

3.2 Longitudinal Responses

Figure 5 shows the FRF of the longitudinal response between the sensor and actuator as defined in equation (9). The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements as defined in Figure 1. These FRFs are very similar in both terms of magnitude and phase except the magnitude response at the frequencies above 20,000 Hz as can be seen from Figure 5.

The magnitude response of the longitudinal motion maintains a constant value below 20,000 Hz. And the phase response keeps an unchanging 0 ° below 1,000 Hz and stays from -180 ° to 0 °. So it is noted that the longitudinal response of the half-length sensor which is bonded any location within the extent of the actuator placement can be perturbed in a little amount. Especially the phase responses are almost the same with the four different mismatchness of the sensor and actuator.

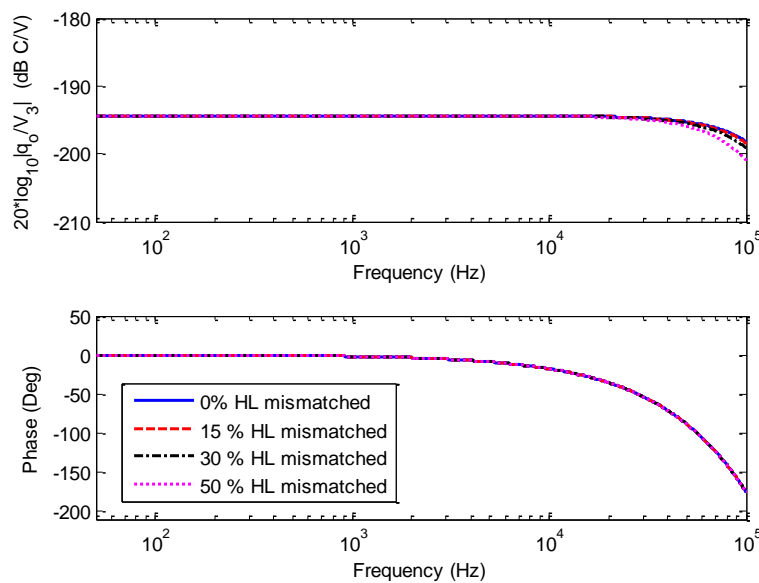


Figure 5. FRFs of the longitudinal motion between the half-length mismatched piezo sensor and actuator. The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements

3.3 Coupled Responses

Figure 6 shows the FRF of the coupled response (flexural + longitudinal motions) between the sensor and actuator as defined in equation (10). The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements as defined in Figure 1. These coupled FRFs are very similar in both terms of magnitude and phase except the magnitude response at the frequencies above 3,000 Hz as can be seen from Figure 6. This because the flexural response showed different magnitude at each arrangement above around that frequency as shown in Figure 4.

The coupled phase response exists between -180 ° to +180 ° although the longitudinal phase stays -180 ° to 0 °. Since the magnitude of the flexural motion is much larger than that of the longitudinal motion, the coupled response is affected mainly by the inclusion of the flexural motion rather than the longitudinal motion.

It is very important that the mismatchness of the half-length piezo sensor against the placement of the piezo actuator on the either side of the beam cannot give a big difference only if the half-length sensor attaches within the extent of the actuator placement.

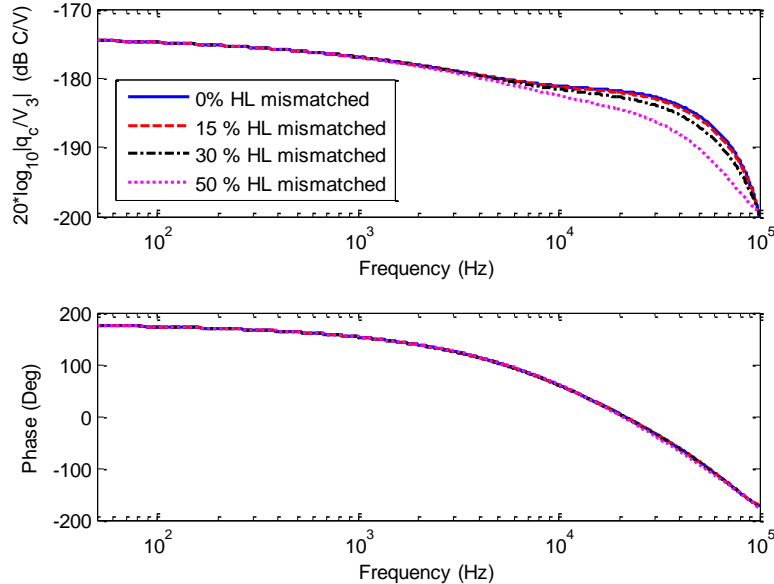


Figure 6. FRFs of the coupled motion between the half-length mismatched piezo sensor and actuator. The solid line, dashed line, dashed-dotted line and dotted line indicate 0 %, 15 %, 30 % and 50 % half-length mismatched sensor-actuator arrangements

4. Conclusions

In this paper, a half-length piezo sensor compared to the piezo actuator is considered. When they are attached with some mismatch on either side of a beam, the frequency response function of the sensor and actuator will be critically important to design a controller for structure control such as vibration suppression or wave propagation cancellation. Four different mismatched sensor and actuator arrangements will be analyzed in terms of the coupled response in detail for the coupled responses. The coupled phase response exists between -180° to $+180^\circ$ although the longitudinal phase stays -180° to 0° . Since the magnitude of the flexural motion is much larger than that of the longitudinal motion, the coupled response is affected mainly by the inclusion of the flexural motion rather than the longitudinal motion. It is very important that the mismatchness of the half-length piezo sensor against the placement of the piezo actuator on the either side of the beam cannot give a big difference only if the half-length sensor attaches within the extent of the actuator placement.

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