Stable Walking of Qauadruped Robot by Impedance Control for Body Motion

Sooyeong Yi

*Department of Electrical and Information Engineering,
Seoul National University of Science and Technology, Republic of Korea
suylee@seoultech.ac.kr*

Abstract

General gait design for a walking robot has an assumption that the weight of a leg is negligible compared to that of body. Under the assumption, only the gravity center of the body is taken into consideration in the gait design since the total gravity center of whole robot coincides always with the gravity center of the body only. Roughly speaking, however, motion of a weighty leg has serious influence on stable walking of a quadruped robot in practical sense. In this paper, an impedance control for body sway motion is proposed to compensate the influence of a weighty leg on the total gravity center of the robot, so that the walking stability is secured in accordance with the pre-designed gait with consideration of the gravity center of the body only.

Keywords: *Impedance control, body sway motion, gait design, walking stability, quadruped robot*

1. Introduction

Gait design for a quadruped robot defines the sequence and phase of leg-transfer, so that the total gravity center of the robot always lies inside of the support polygon consisting of each support leg's tip position [1, 2]. One of the basic assumptions in the gait design is that the weight of a leg should be negligible compared to that of body, thus the total gravity center of the quadruped robot is not affected by motion of a leg. In extreme case of zero leg-weight, the total gravity center coincides exactly with the gravity center of the body. Since the body gravity center is constant in the moving coordinates of the robot and can be determined by geometry in advance, it is relatively easy to design a gait for stable walking guaranteeing the body gravity center to be inside of the support polygon always. Hereafter, it is denoted as an ideal robot for the case of the zero leg-weight, and a real robot for the other case. Roughly speaking, the pantograph type of walking robot can be classified into the ideal robot since all actuators are integrated into the body and the driving mechanism of a leg is simple and light compared to the body [3, 4]. On the contrary, the jointed-leg type of walking robot can be called as a real robot since each joint actuator is located directly at each joint, thereby a leg becomes relatively heavy. As a consequence, the total gravity center of the robot has fluctuation by the motion of a leg and the conventional gait design considering only the body gravity center cannot be successfully applicable.

In order to secure the walking stability for the real robot, it is necessary to compensate the fluctuation in the total gravity center. A compulsive body sway was proposed to drive the total gravity center of a walking robot into the support polygon [2, 5, 6] and an independent trunk mechanism was used to compensate the fluctuation in the total gravity center [7]. The first method has problems in determination of the sway direction and magnitude at each walking situation, and in the latter, together with an extra cost for an additional controller, the total weight of robot increases due to the trunk.

For a walking robot, the effect of the weighty leg on the total gravity center is reflected in the foot reaction force of each support leg. Thus, if the real robot is driven for the measured foot force to follow the desired foot force of the ideal robot, it is possible to realize the pre-designed stable walking for an ideal robot. Since the force-moment equation for a walking robot represents the relationship between the total gravity center and the foot force of each support leg, the body motion can be drawn for the foot force of the real robot to follow that of the ideal robot. The main aim of this paper is to propose an impedance control for the body motion of the jointed-leg type of walking robot. The effectiveness of the proposed algorithm is verified by extensive computer simulations. In Section 2, the influence of the weighty leg on the total gravity center of a walking robot and the corresponding foot reaction forces are described. And the body impedance control is proposed in Section 3. Simulation results of the proposed algorithm and the concluding remarks are presented in Section 4 and Section 5 respectively.

2. Effects of the weighty leg on the total gravity center of a walking robot

In order to inspect the effects of the weighty leg on the total gravity center, a quadruped robot is considered in this paper without loss of generality and each link element of the robot is modeled as a point mass as shown in Figure 1.

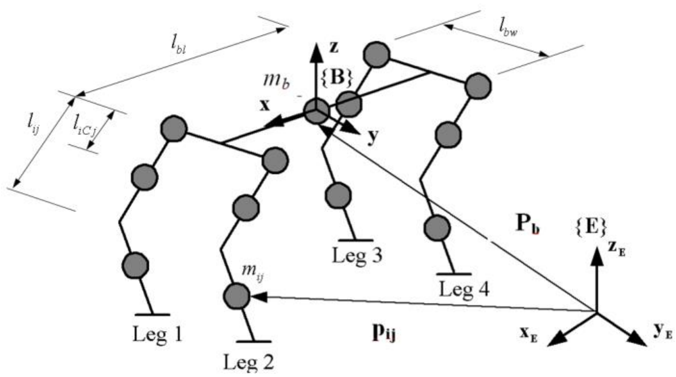


Figure 1. The point mass model for a quadruped robot

Then, the position of the total gravity center $\mathbf{p}_{cg} = (x_{cg}, y_{cg})$ of the robot satisfies the following (1)

$$m_b(\mathbf{p}_b - \mathbf{p}_{cg}) + \sum_{i \in LEG} \sum_{j \in LINK} m_{ij}(\mathbf{p}_{ij} - \mathbf{p}_{cg}) = 0 \quad (1)$$

where m_b and m_{ij} denote the body mass and the j^{th} link mass of the i^{th} leg respectively. The vectors, $\mathbf{p}_b$ and $\mathbf{p}_{ij}$, represent the positions of m_b and m_{ij} with respect to the earth-fixed reference coordinate frame, $\mathbf{E}$, and LEG and $LINK$ are the sets of all legs and all links of a leg respectively. Note that the body gravity center, $\mathbf{p}_b$, is constant in the moving coordinates of body, $\mathbf{B}$, during walking and it coincides with the geometric center of body in general. Hereafter, it is denoted the body gravity center as the body center without loss of generality.

Eq. (1) implies that the total gravity center, $\mathbf{p}_{cg}$, departs from the gravity center of body, $\mathbf{p}_b$, due to m_{ij}, while $\mathbf{p}_{cg}$ coincides exactly with $\mathbf{p}_b$ in the ideal case of $m_{ij} = 0$, $\forall\, i,\, j$. Therefore, the walking stability of the conventional gait design taking into account only the body center is not applicable for the real robot. In order to investigate the relationship between the total gravity center and the foot reaction force of each support leg, it is considered the following force-moment equation with respect to $\mathbf{p}_{cg} = (x_{cg}, y_{cg})$ in the reference coordinates.

$$\begin{aligned}
&\sum_{i \in SLEG} f_i (x_i^{tip} - x_{cg}) = \sum_{i \in LEG} \sum_{j \in LINK} m_{ij}(x_{ij} - x_{cg}) + m_b (x_b - x_{cg}) \\
&\sum_{i \in SLEG} f_i (y_i^{tip} - y_{cg}) = \sum_{i \in LEG} \sum_{j \in LINK} m_{ij}(y_{ij} - y_{cg}) + m_b (y_b - y_{cg}) \\
&\sum_{i \in SLEG} f_i = \sum_{i \in LEG} \sum_{j \in LINK} m_{ij} + m_b = W
\end{aligned} \tag{2}$$

where W denotes the total mass of the robot, and $\mathbf{p}_i^{tip} = (x_i^{tip},\ y_i^{tip})$ is the tip position of the i^{th} leg, and the set, $SLEG$, has the index i for all support legs as its element. Combining (2) with (1) gives the following (3)[6].

$$\begin{aligned}
&\sum_{i \in SLEG} f_i (x_i^{tip} - x_{cg}) = 0 \\
&\sum_{i \in SLEG} f_i (y_i^{tip} - y_{cg}) = 0 \\
&\sum_{i \in SLEG} f_i = W
\end{aligned} \tag{3}$$

Eq. (3) can be written in vector form as

$$\begin{bmatrix} x_{cg} \\ y_{cg} \end{bmatrix} = \begin{bmatrix} x_1^{tip} & x_2^{tip} & x_3^{tip} & x_4^{tip} \\ y_1^{tip} & y_2^{tip} & y_3^{tip} & y_4^{tip} \end{bmatrix} \begin{bmatrix} f_1/W \\ f_2/W \\ f_3/W \\ f_4/W \end{bmatrix} \equiv \mathbf{p}_{cg} = \mathbf{A}\mathbf{f}_n \tag{4}$$

where

$\mathbf{A} = \begin{bmatrix} x_1^{tip} & x_2^{tip} & x_3^{tip} & x_4^{tip} \\ y_1^{tip} & y_2^{tip} & y_3^{tip} & y_4^{tip} \end{bmatrix}$ and $\mathbf{f}_n = [\ f_1/W \quad f_2/W \quad f_3/W \quad f_4/W\ ]^t$.

Note that f_i is equal to zero when the i^{th} leg is swinging, and the corresponding $(x_i^{tip},\ y_i^{tip})$ in (4) is meaningless as a consequence

3. The body impedance control for walking stabilization

3.1 The walking stability of the static wave gait for a quadruped robot

The definition of the static stability of a walking robot is the minimum distance between the total gravity center of the robot and each edge of the support polygon during walking [1, 2]. In case that the total gravity center is inside of the support polygon, the walking robot has positive stability, and has negative stability in the other case. The negative stability implies that the walking robot may fall down. Thus, the stable gait design requires the proper choice of the sequence for the leg-transfer and the trajectory control for each leg which should guarantee the total gravity center to be inside of the support polygon. Since the total gravity center coincides exactly with the body center in case of the ideal robot, it is relatively easy to design an off-line stable gait in geometric way. For example, the minimum distance, S , between the body center and the support polygon of the well-known wave gait for an ideal quadruped robot is given in analytic form as

$$S = (\beta - \frac{3}{4})\lambda \tag{5}$$

where β is the duty factor, *i.e.*, the time ratio between the support interval of a leg and a whole walking period, and λ implies the stride of a leg. Thus, choosing only the duty factor as $\beta > \frac{3}{4}$ guarantees the statically stable walking for the ideal robot. But in case of the real robot having the considerable weighty leg, the total gravity center has fluctuation according to the motion of a leg and does not coincides with the body center. As a consequence, even though the body center stays inside of the support polygon, the total gravity center can be out of the polygon and the analytic stability in (5) is not valid anymore. Thus, it is required a compensation scheme to keep the total gravity center inside of the support polygon, so that to secure the walking stability.

3.2 The impedance control for body motion

It is assumed that the leg sequence, together with the trajectory of each leg according to the desired body trajectory is pre-designed for the ideal robot corresponding to a given real robot. Here, the corresponding ideal robot implies the walking robot with zero leg-weight having the same total weight with the real robot. Note that, in the pre-designed walking trajectory, the body center is kept inside of the support polygon always during walking. Since the walking stability is defined using the gravity center, in order to make the walking trajectory stable, it is necessary to generate the body motion in which the total gravity center of the real robot follows the pre-designed trajectory for the body center of the ideal robot. As described in Section 2, eq. (4) implies that the fluctuation in the total gravity center is reflected in the foot reaction force of each support leg. Therefore, if the real robot is driven to reduce the difference between the measured foot force and the reference foot force of the corresponding ideal robot, it is possible for the real robot to achieve the walking stability of the pre-designed walking trajectory for the ideal robot. The reference foot force of the ideal robot can be computed by using (3) for given walking trajectory, which will be explained in Section 3.3.

At first, it is assumed that the walking robot maintains its attitude parallel to the ground. It is noted here that the right-hand-side of (4) multiplied by W is the moment in $x-y$ plane, *i.e.*, the direction and the magnitude of the force exerted on the body causing the position change of the total gravity center, $\mathbf{p}_{cg}$. Then, it is defined the following equation for the body impedance model shown in Figure 2 [8, 9].

$$k_a(\ddot{\mathbf{p}} - \ddot{\mathbf{p}}_p) + k_v(\dot{\mathbf{p}} - \dot{\mathbf{p}}_p) + k_p(\mathbf{p} - \mathbf{p}_p) = \mathbf{A}\left\{ \mathbf{f}_n(\mathbf{p}) - \mathbf{f}_n(\mathbf{p}_p) \right\}, \quad k_a, k_v, k_p > 0 \tag{6}$$

where $\mathbf{p}$ denotes the modified trajectory for the body center of the real robot and $\mathbf{p}_p$ represents the pre-designed trajectory for the body center of the ideal robot. The measured foot force and the reference foot force are represented by $\mathbf{f}_n(\mathbf{P})$ and $\mathbf{f}_n(\mathbf{P}_p)$ respectively. The subscript, n, implies the normalization with the total weight, W. Eq. (6) can be understood by the spring-damper system between the body center of the real and the ideal robot. When the difference between the gravity center and the body center occurs, the moment, $\mathbf{A}\left\{\mathbf{f}_n(\mathbf{p})-\mathbf{f}_n(\mathbf{p}_p)\right\}$ is generated, so that $\mathbf{p}$ is pushed away until $\mathbf{f}_n(\mathbf{P})$ approaches to $\mathbf{f}_n(\mathbf{P}_p)$. Thus, the solution, $\mathbf{p}$, of (6) is the trajectory on which the foot force of the real robot becomes same as that of the ideal robot, which implies the achievement of the walking stability pre-designed for the ideal robot.

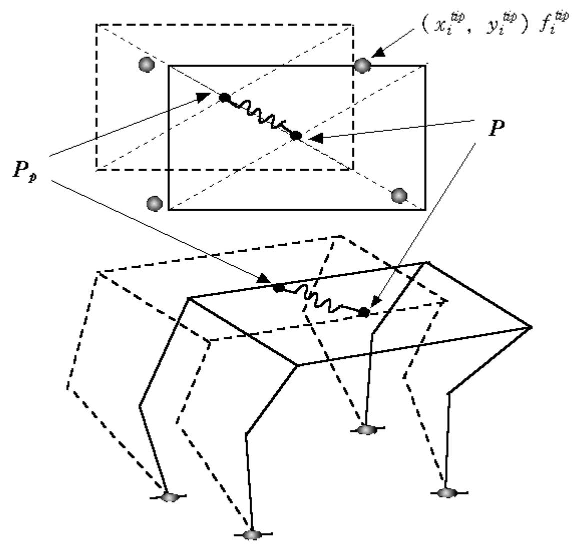


Figure 2. The body impedance model

3.3 Computation of the reference foot force

In case of the ideal robot, since the total gravity center, $\mathbf{p}_{cg}$, coincides with the body center, $\mathbf{p}_b$, always, the relationship (3) between the total gravity center and the foot forces becomes as follows

$$\begin{aligned}&\sum_i f_i(\mathbf{p}_p)(x_i^{tip}-x_b)=0\\&\sum_i f_i(\mathbf{p}_p)(y_i^{tip}-y_b)=0\\&\sum_i f_i(\mathbf{p}_p)=W\end{aligned} \tag{7}$$

The tip position of support leg, $\mathbf{p}_{cg}^{tip}$ and the body trajectory, $\mathbf{p}_p(=\mathbf{p}_b)$, are known in advance of course. For example, in case that 1^{st}, 2^{nd}, 4^{th} and legs are supporting, eq. (7) becomes

$$\begin{aligned} & f_1 \cdot (x_1^{tip} - x_b) + f_2 \cdot (x_2^{tip} - x_b) + f_4 \cdot (x_{14}^{tip} - x_b) = 0 \\ & f_1 \cdot (y_1^{tip} - y_b) + f_2 \cdot (y_2^{tip} - y_b) + f_4 \cdot (y_{14}^{tip} - y_b) = 0 \\ & f_1 + f_2 + f_4 = W \end{aligned} \tag{8}$$

which can be rewritten in vector form as follows:

$$\begin{bmatrix} f_1 \\ f_2 \\ f_4 \end{bmatrix} \cdot \begin{bmatrix} x_1^{tip} - x_b & x_2^{tip} - x_b & x_{14}^{tip} - x_b \\ y_1^{tip} - y_b & y_2^{tip} - y_b & y_{14}^{tip} - y_b \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} \tag{9}$$

Taking into account the under-determinate case of the 4-leg-supporting, the general pseudo inverse solution for (9) is given as follows [10, 11]

$$\mathbf{f}(\mathbf{p}) = \mathbf{B}^{\mathbf{t}}(\mathbf{B}\mathbf{B}^{\mathbf{t}})^{-1}\mathbf{W} \tag{10}$$

where

$$\mathbf{B} = \begin{bmatrix} x_1^{tip} - x_b & x_2^{tip} - x_b & x_{14}^{tip} - x_b \\ y_1^{tip} - y_b & y_2^{tip} - y_b & y_{14}^{tip} - y_b \\ 1 & 1 & 1 \end{bmatrix} \text{ and } \mathbf{W} = [\,0 \quad 0 \quad W\,]^t .$$

4. Computer simulations

In order to verify the effectiveness of the proposed body impedance control on the walking stabilization, extensive computer simulations are carried out. The link parameters used in this simulation study are tabulated in Table 1.

Table 1. Link parameters of a jointed-leg type walking robot (in MKS)

m_b	m_{i1}	m_{i2}	m_{i3}	l_{ic1}	l_{ic2}	l_{ic3}
36.0	3.393	6.04	5.30	0.0344	0.1861	0.2475

l_{bw}	l_{bl}	l_{i0}	l_{i1}	l_{i2}	l_{i3}	
0.4	0.56	0.06	0.09	0.37	0.495	

As shown in Table 1, the total weight of the robot is $W = m_b + 4 \times (m_{i1} + m_{i2} + m_{i3}) = 94.932$ kg. Thus the corresponding ideal robot is set as $m_b = 94.932$ kg and $m_{i1} = m_{i2} = m_{i3} = 0$ kg $\forall\, i = 1, \cdots, 4$. As the reference walking trajectory, the well-known straight line wave gait is used. The condition for the duty factor is $\beta > 0.75$ for stable walking as stated before. The leg sequence is chosen as $4-2-3-1$. As a prerequisite study, the static stabilities according to β for several body-to-leg weight ratios in Table 2 are evaluated in order to investigate the influence of the weighty leg on the static stability. Through the computer simulations, the minimum distances between the total gravity center computed by (4) and the support polygon are obtained while the quadruped robot is driven by the pre-designed wave gait for the ideal robot without the body impedance control. Here, the trajectory for a leg is generated through the Bezier curve with the stride $\lambda = 0.5m$ and the height $h = 0.1\ m$. The reference trajectory for the body center and the gait diagram for, *e.g.*, $\beta > 0.76$ are illustrated in Figure 3 and Figure 4 respectively.

Table 2. Link parameters for various body-to-leg weight ratios (MKS)

case	m_b	m_{i1}	m_{i2}	m_{i3}	W
m_0	94.932	0	0	0	94.932
m_1	89.039	0.339	0.604	0.530	94.932
m_2	65.466	1.697	3.020	2.650	94.932
m_3	6.536	5.090	9.060	7.950	94.932
m_4	36.000	6.040	3.393	5.300	94.932

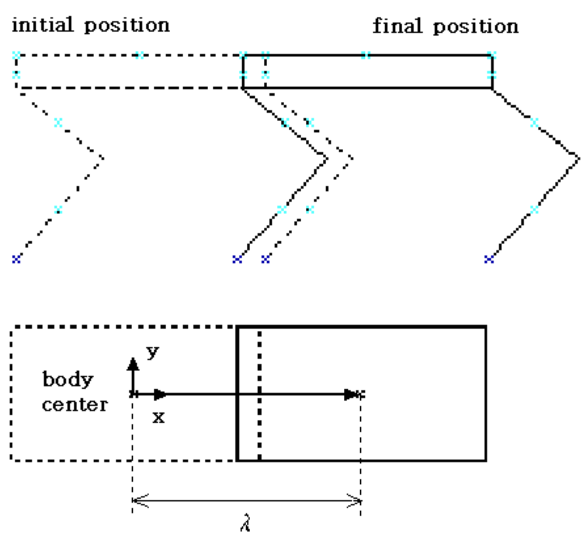


Figure 3. The reference trajectory for the body center when $\lambda = 0.5m$

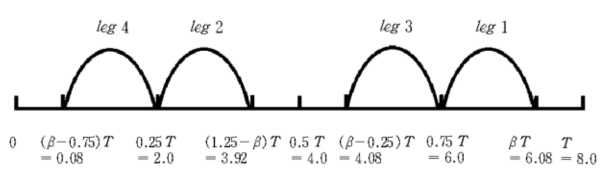


Figure 4. Gait diagram with $\beta = 0.76$ **and** $T = 8\,\text{sec}$.

The simulation results for walking stability are represented in Figure 5 where m_0 corresponds to the ideal robot and the solid m_4 does to the real robot in Table 1. It is noted in Figure 5 that the walking stability is positive always in the ideal case of m_0 , the uppermost dotted line if $\beta > 0.75$ as (5). However in the other cases, the stability in (5) is not valid and has negative value even if $\beta > 0.75$.

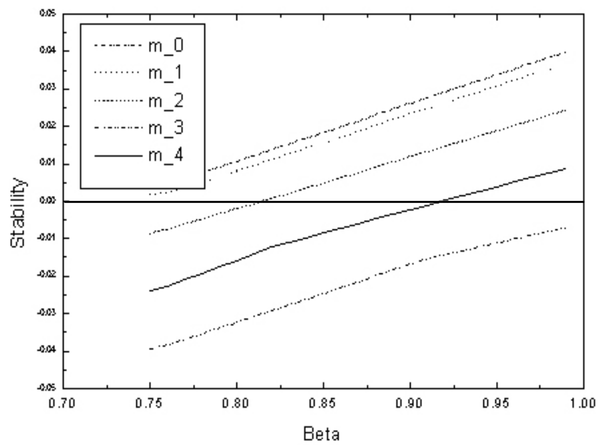


Figure 5. The walking stabilities according to various body-to-leg weight ratios

The reference trajectory for the body center and the corresponding trajectory of the total gravity center of the real robot, m_4 during a period are shown in Figure 6. Here, the leg-sequence is chosen as 4-2-3-1, the duty factor, β, is set as 0.76, and the body impedance control is not applied. Note that, since all legs are bent in front as shown in Figure 3, the total gravity center has positive deviation in x direction from the body center and has fluctuation according to the motion of a leg during walking.

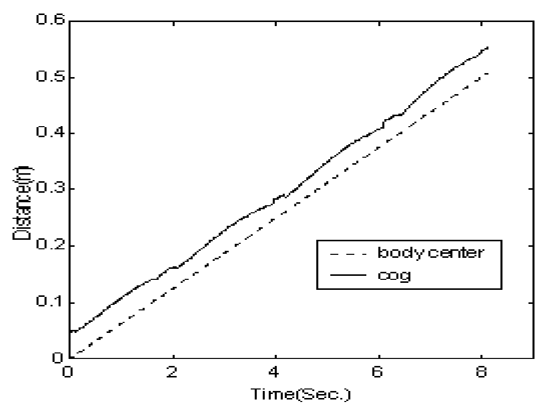
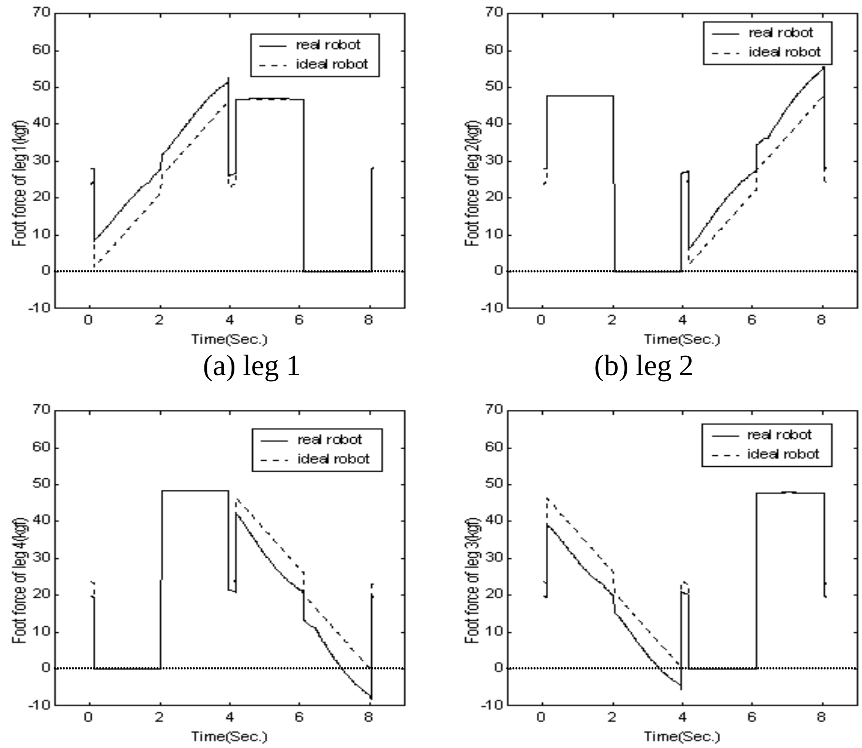


Figure 6. The reference trajectory for the body center and the corresponding trajectory of the total gravity center without the body impedance control

The foot reaction force of each leg is shown in Figure 7 in this case. The dotted lines in the figure represent the foot forces of the corresponding ideal robot. The real robot, m_4, has the negative stability as shown in Fig. 5 at $\beta = 0.76$, which is verified in Figures 7 (c) and (d) where the foot forces of the 3^{rd} and the 4^{th} legs are negative in 3~4 sec. and 7~8 sec. respectively. The negative value of the foot force is not actual of course, but in the computer simulation, it implies the unstable situation as like the ground pushes up the foot.



(a) leg 1 (b) leg 2

(c) leg 3 (d) leg 4

Figure 7. Foot force of each leg during a walking period without the body impedance control

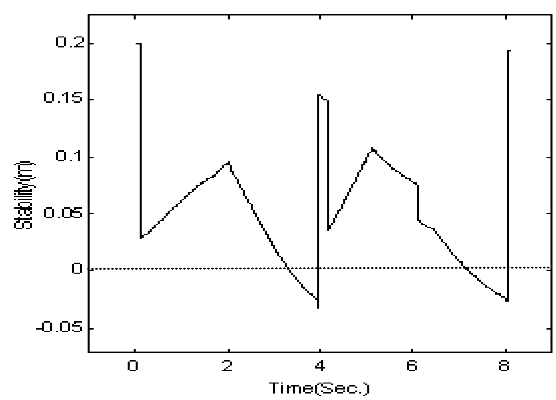


Figure 8. Instant walking stability during a whole walking period without the body impedance control

Figure 8 showing the instant stability also accounts for the unstable situation where the instant stability has negative value at 3~4 sec. and 7~8 sec. The instant stability denotes the minimum distance between the total gravity center and the support polygon at each time instant here.

When the body impedance control is used on the contrary, the resultant trajectory of the body center and the total gravity center are shown in Figure 9. Note that the trajectory of the total gravity center is almost same as that of the body center in Figure 6, which is the stable walking reference trajectory. Thus, the walking stability is achieved for the real robot by using the proposed body impedance control. The stable walking can be verified also in Figure 10 showing the foot reaction force of each leg. In the figure, the foot force of the real robot conforms that of the ideal robot. Figure 11 shows the instant walking stability which is positive always during the whole walking period.

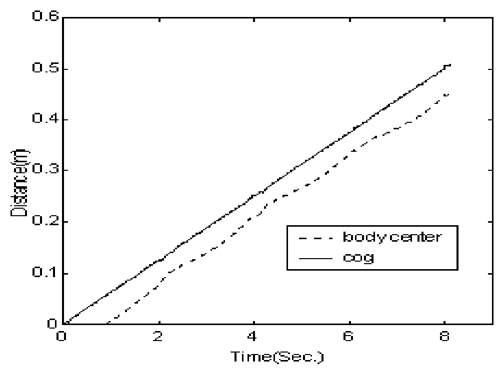


Figure 9. The reference trajectory for the body center and the corresponding trajectory of the total gravity center with the body impedance control

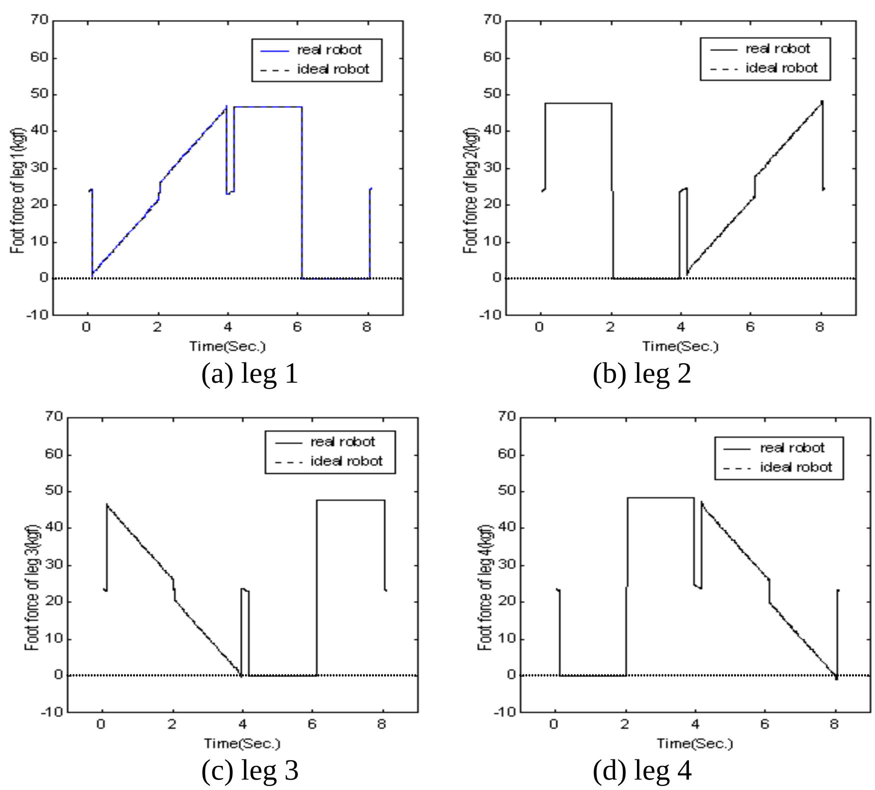


Figure 10. Foot force of each leg during a walking period with the body impedance control

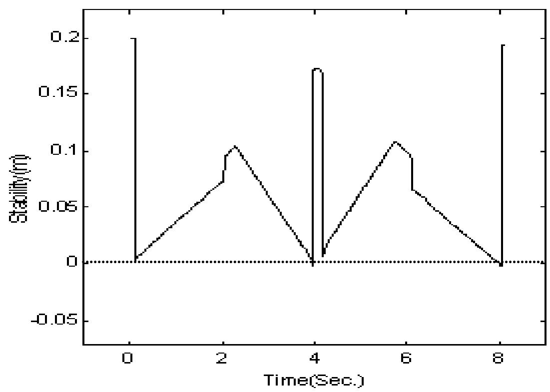


Figure 11. Instant walking stability during a walking period with the body impedance control

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Author

Sooyeong Yi

Address: 172 Gongreung-Dong, Nowon-Gu, Seoul, 139-743, Korea
Seoul National University of Science and Technology

Education & Work experience:

Sooyeong Yi received the M.S. and Ph.D. degrees in Electrical Engineering from Korea Advanced institute of Science and Technology in 1990 and 1994 respectively. During 1995-1999, he stayed in Human Robot Research Center in Korea Institute of Science and Technology as a senior researcher. He was a professor in the Div. of Electronics and information Engineering, Chonbuk national University, Korea from Sept. 1999 to Feb. 2007. He also was a post doctorial researcher in the Dept. of Computer Science, Univ. of Southern California, Los Angeles in 1997 and a visiting researcher in the Dept. of Electrical and Computer Engineering, Univ. of Illinois at Urbana-Champaign in 2005. He is now with the Dept. of Electrical and Information Engineering in Seoul National University of Science and Technology, Korea. His primary research interest is in the area of service robot, robot vision, and intelligent control theory.

Tel: +082-02-970-6407

E-mail: suylee@seoultech.ac.kr